Breaking avalanches in the limit of high disorder of the fiber bundle model

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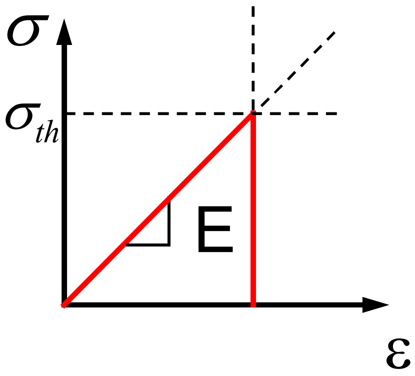
The role of disorder

0 disorder

Perfectly brittle fracture



The role of disorder

0 disorder

Perfectly brittle fracture

Macroscopic properties

Lower strength

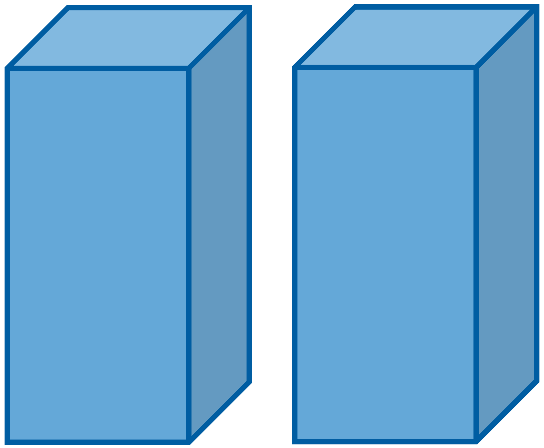
$$\sigma_c \ll E$$

Concrete: $E \sim GPa$ $\sigma_c \sim MPa$

Varying strength

$p(\sigma_c)$ Weibull distribution

$$P(\sigma_c) = 1 - e^{-\left(\frac{\sigma_c}{\lambda}\right)^m}$$



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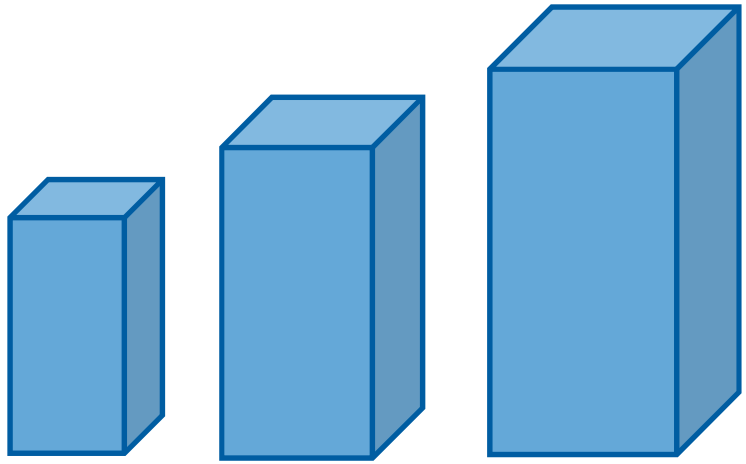
Size effect

$\sigma_c \sim 1/\log L$ Decreasing strength



The role of disorder

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Perfectly brittle fracture

Gradual breaking

Precursors

Macroscopic properties

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$$\sigma_c \ll E$$

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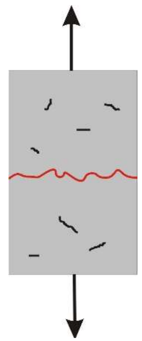
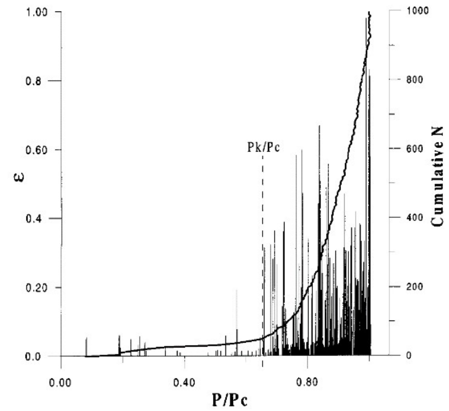
Microscopic dynamics

Acoustic emissions



cracks

elastic waves



A. Guarino et al., Eur. Phys. J. B **6**, 13 (1998)

The role of disorder

0 disorder → Strong disorder

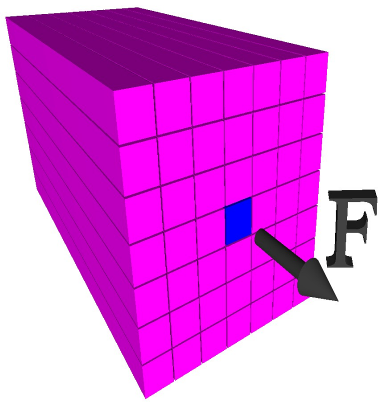
Perfectly brittle fracture

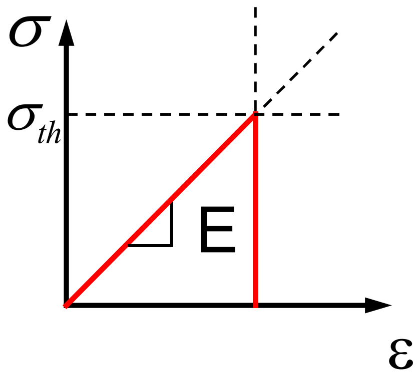
Gradual breaking

Precursors

?

The fiber bundle model





- ❖ Discrete set of parallel fibers
- ❖ The same Young modulus E
- ❖ Load parallel to fibers
- ❖ Perfectly brittle behaviour
- ❖ Breaks instantaneously, if $\sigma_i > \sigma_{th}^i$

Stochastic failure thresholds

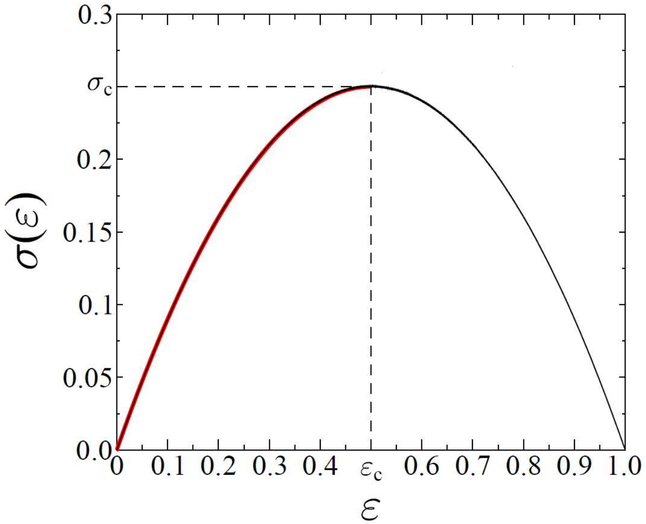
$$g(\sigma_{th})$$

- ❖ Quasi-static loading
- ❖ Equal load redistribution

F. Kun, H. A. Carmona, J. S. A. Jr, and H. J. Herrmann, Phys. Rev. Lett. **100**, 094301 (2008)

Intermediate disorder

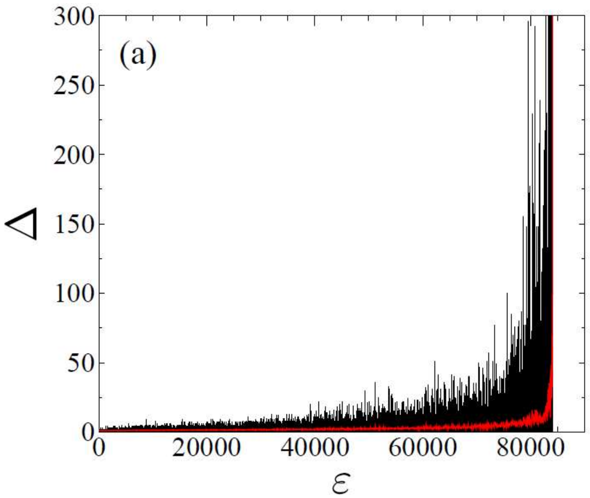
Macroscopic response



$$\langle\sigma_c\rangle(N) = \sigma_c(\infty) + AN^{-2/3}$$

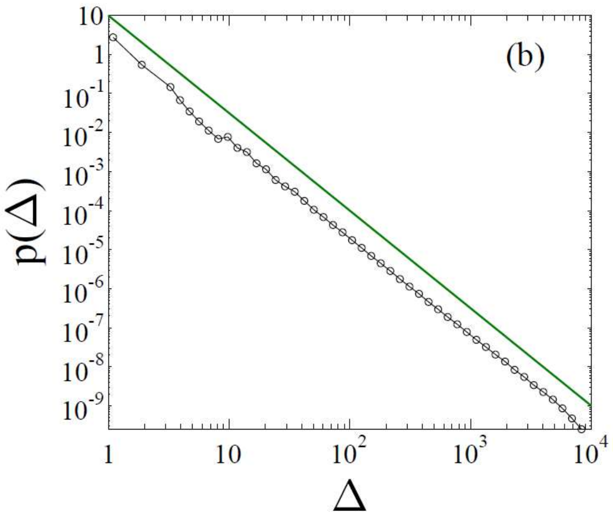
$$\langle\varepsilon_c\rangle(N) = \varepsilon_c(\infty) + BN^{-2/3}$$

Time series of bursts



Δ: Burst size

Burst size distribution



$$p(\Delta) \sim \Delta^{-\tau}$$

$$\tau = 5/2$$

V. Kadar TDK disszertation

Realization of strong disorder

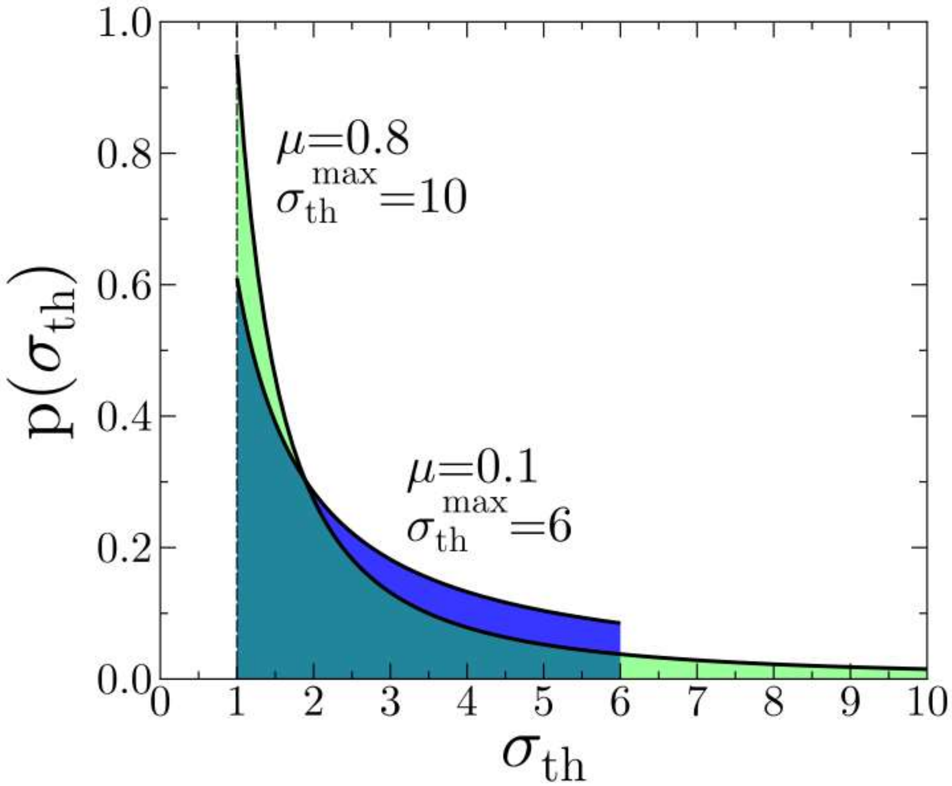
Fat tailed distribution of the failure thresholds

$$p(\sigma_{th}) = \begin{cases} 0, & \sigma_{th} < \sigma_{th}^{min} \\ A\sigma_{th}^{-(1+\mu)}, & \sigma_{th}^{min} \le \sigma_{th} \le \sigma_{th}^{max} \\ 0, & \sigma_{th}^{max} < \sigma_{th} \end{cases}$$

$$\sigma_{th}^{min} = 1$$

$$\sigma_{th}^{min} \le \sigma_{th}^{max} \le +\infty$$

$$0 \le \mu < 1$$



V. Kadar, Zs. Danku, F. Kun, Physical Review E 96, 033001 (2017).

Macroscopic response and size effect

Macroscopic behaviour

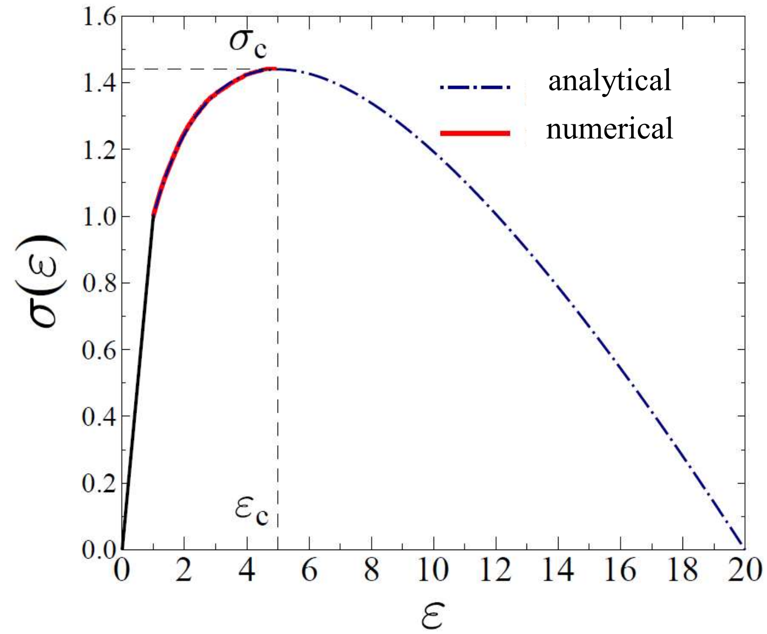
Constitutive equation

$$\sigma(\varepsilon) = \begin{cases} \varepsilon, & 0 \leq \varepsilon \leq \varepsilon_{min} \\ \dfrac{\varepsilon\left(\varepsilon^{-\mu} - \varepsilon_{max}^{-\mu}\right)}{\varepsilon_{min}^{-\mu} - \varepsilon_{max}^{-\mu}}, & \varepsilon_{min} \leq \varepsilon \leq \varepsilon_{max} \\ 0, & \varepsilon_{max} < \varepsilon \end{cases}$$

$$\varepsilon_{min} = \sigma_{th}^{min}/E$$

$$\varepsilon_{max} = \sigma_{th}^{max}/E$$

$$\mu = 0.5, \sigma_{th}^{max} = 20$$



Macroscopic strength

Critical stress

σ_c

Critical strain

ε_c

V. Kadar, Zs. Danku, F. Kun, Physical Review E 96, 033001 (2017).

V. Kadar TDK disszertation

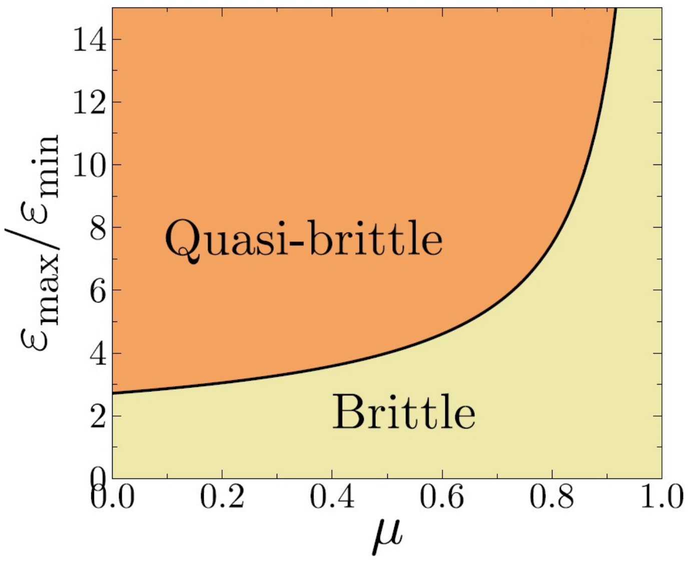
Macroscopic behaviour

Phase boundary

$$\varepsilon_{max}^{c} = \frac{\varepsilon_{min}}{(1-\mu)^{1/\mu}}$$

$\varepsilon_{max} < \varepsilon_{max}^{c}$ Brittle

$\varepsilon_{max} > \varepsilon_{max}^{c}$ Quasi-brittle



$\lambda = \varepsilon_{max}/\varepsilon_{max}^{c}$

$\lambda \geq 1$

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

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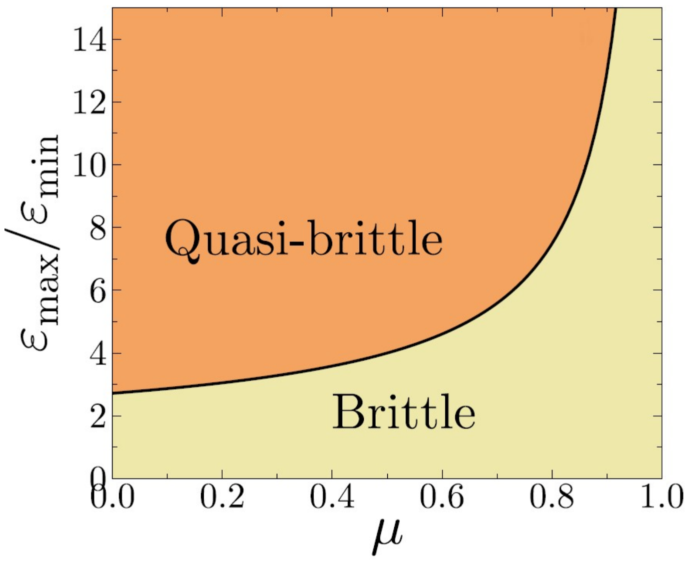
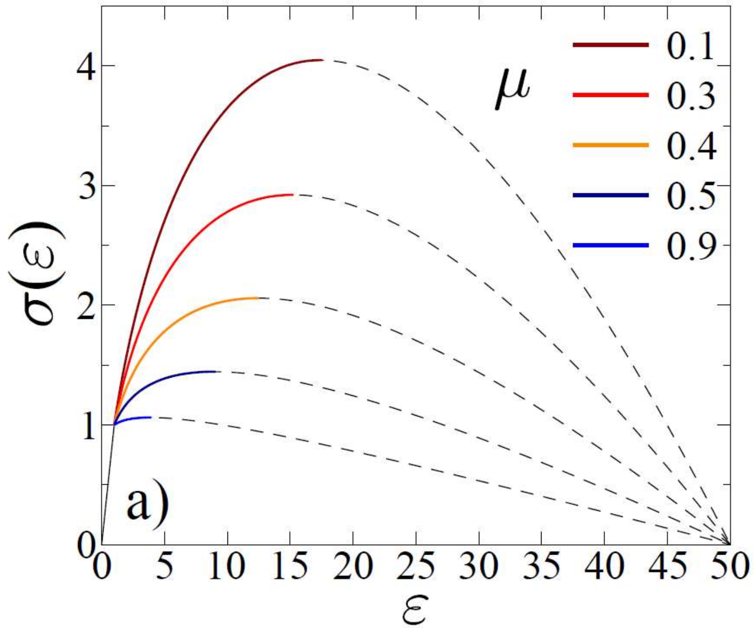
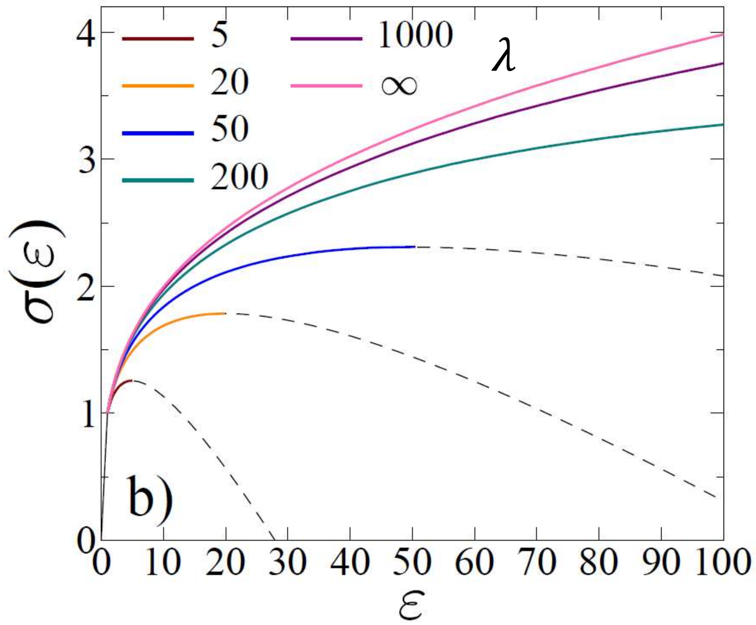


Macroscopic response

$\varepsilon_{max} = 50$



$\mu = 0.7$



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V. Kadar, Zs. Danku, F. Kun, Physical Review E 96, 033001 (2017).

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

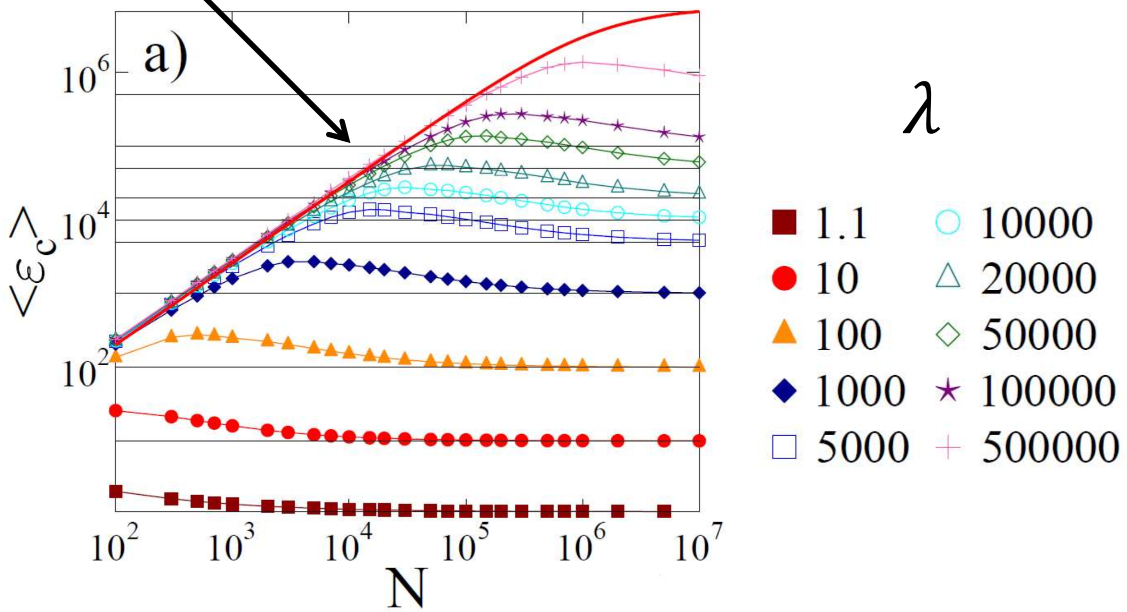
Size scaling of failure strength

Characteristic system size $N_c \sim \varepsilon_{max}^{\mu}$

$N < N_c$ $\langle \varepsilon_c \rangle(N) = \langle \varepsilon_{th}^{max} \rangle_N = P^{-1}\left(1 - \frac{1}{N+1}\right)$

$$\langle \varepsilon_c \rangle = \left[\left((\varepsilon_{th}^{max})^{-\mu} - (\varepsilon_{th}^{min})^{-\mu}\right)\left(1 - \frac{1}{N+1}\right) + (\varepsilon_{th}^{min})^{-\mu}\right]^{-\frac{1}{\mu}}$$

$\langle \varepsilon_c \rangle(N) \sim N^{1/\mu}$



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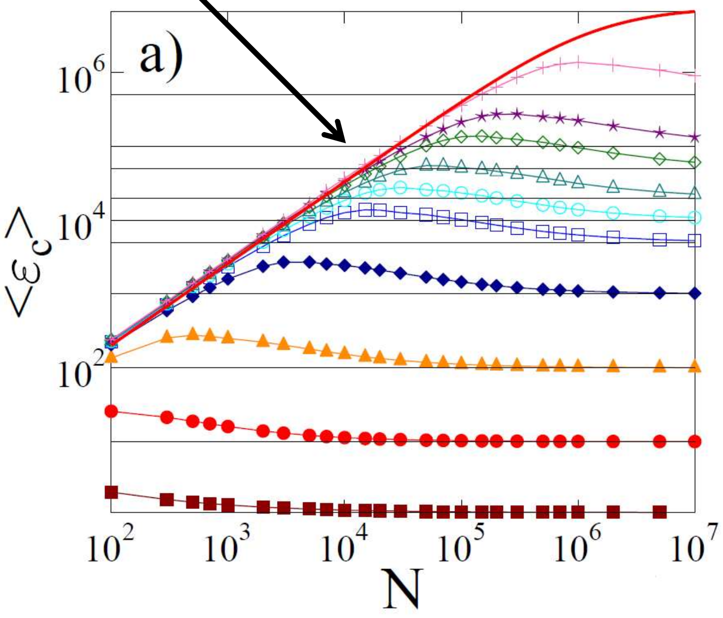
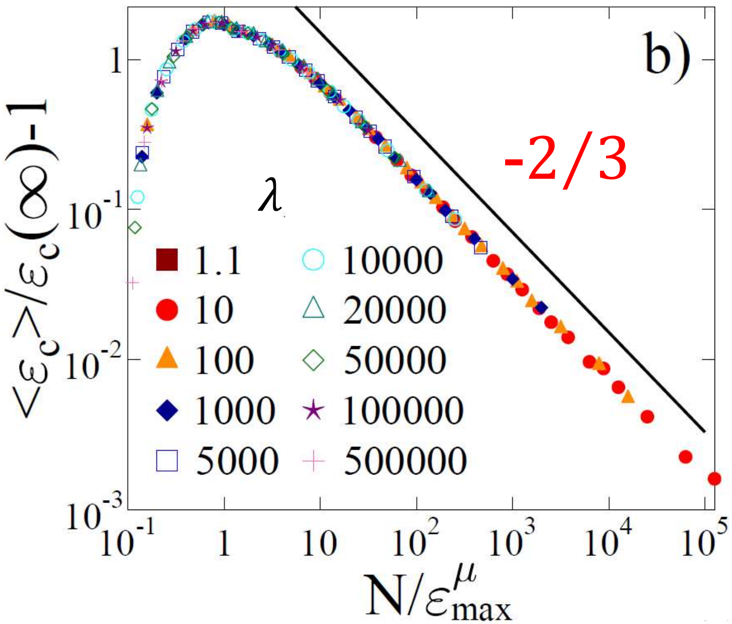
$$\langle \varepsilon_c \rangle = \left[\left((\varepsilon_{th}^{max})^{-\mu} - (\varepsilon_{th}^{min})^{-\mu} \right) \left(1 - \frac{1}{N+1} \right) + (\varepsilon_{th}^{min})^{-\mu} \right]^{-\frac{1}{\mu}}$$

$$\langle \varepsilon_N \rangle(N) = \varepsilon_c(\infty) + AN^{-2/3}$$

$$\langle \varepsilon_c \rangle(N) \sim N^{1/\mu}$$



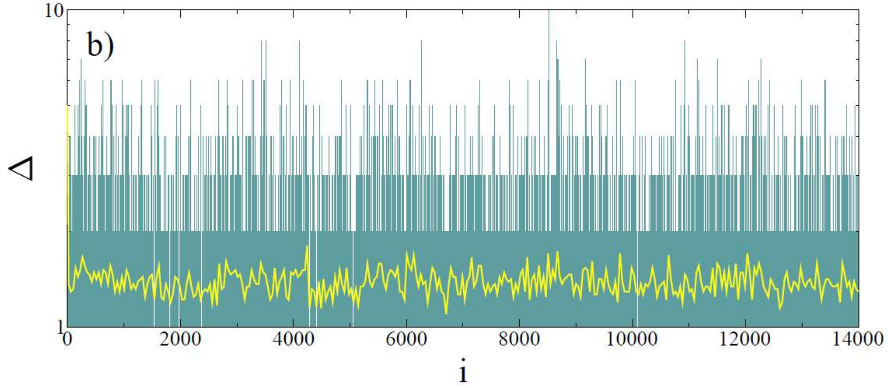


V. Kadar, Zs. Danku, F. Kun, Physical Review E 96, 033001 (2017).

Avalanche statistics in the limit of high disorder

Microscopic dynamics of fracture, $\lambda = +\infty$

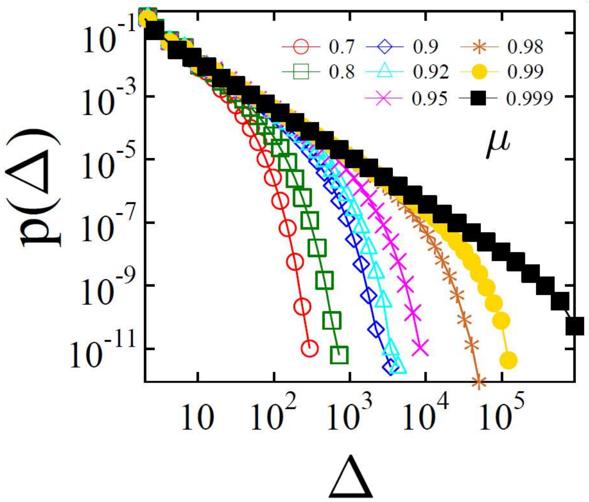
Stationary time series $N = 10^5, \mu = 0.8$



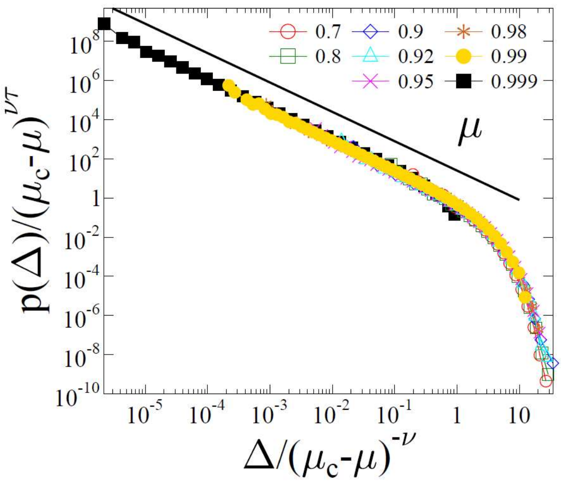
No signs of macroscopic failure

Burst size distribution

$N = 10^6, \varepsilon_{max} = +\infty$



Scaling



$$\frac{p(\Delta)}{N} \cong \Delta^{-3/2} e^{-\Delta/\Delta^*}$$

$$\Delta^* = \frac{1}{\mu - 1 - \ln \mu}$$

$$\Delta^* \sim (\mu_c - \mu)^{-\upsilon}$$

$$\upsilon = 2$$

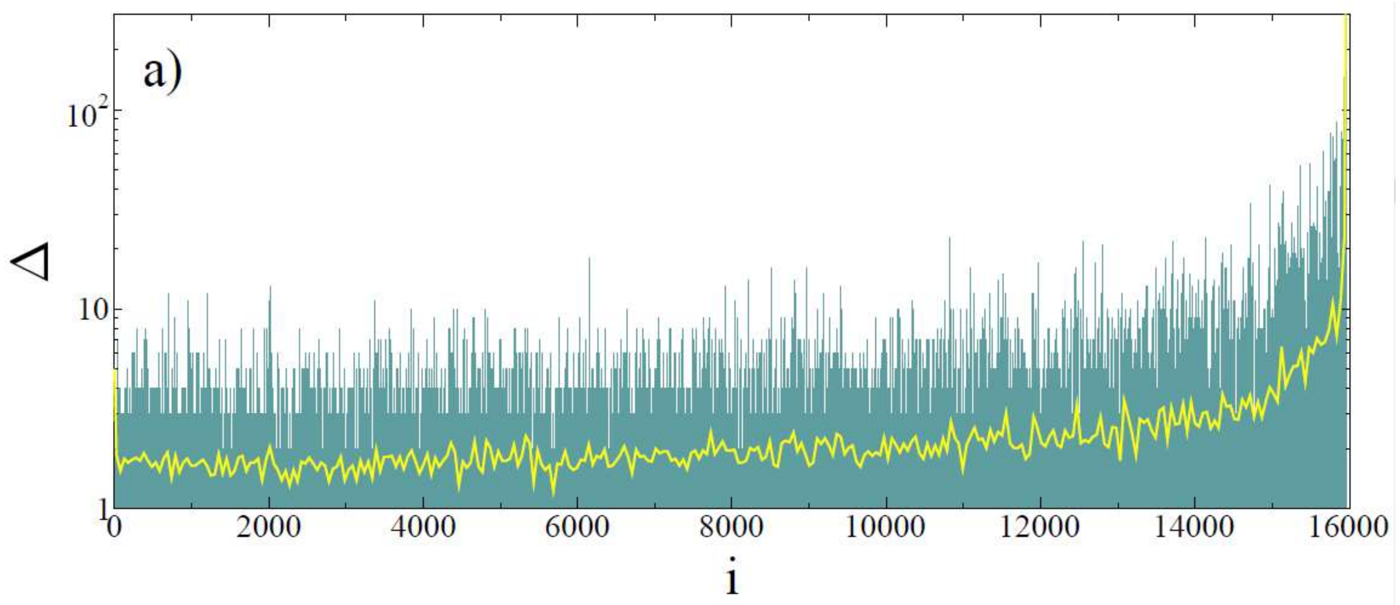
$\boldsymbol{\tau = 3/2 \ll \tau_{ELS} = 5/2}$

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Microscopic dynamics of fracture, finite cutoffs

Time series

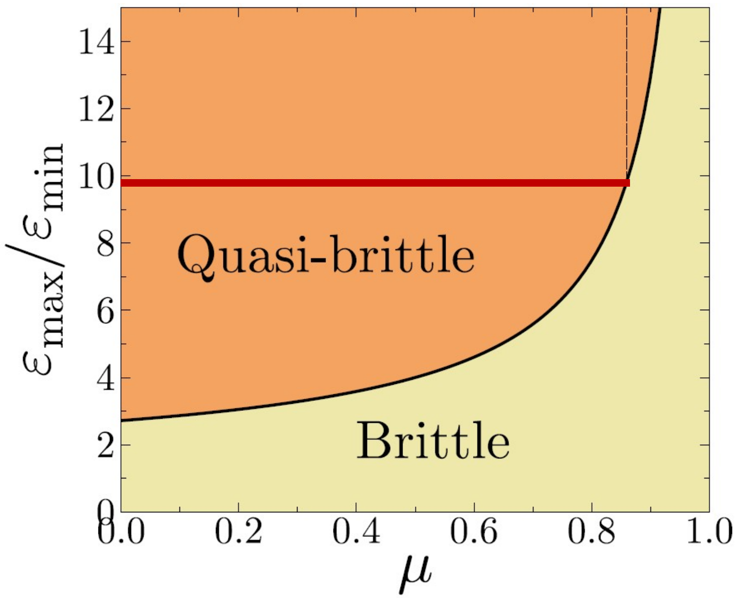
$N = 10^5, \mu = 0.8, \lambda = 100$



Acceleration towards failure

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Burst size distribution – finite cutoffs



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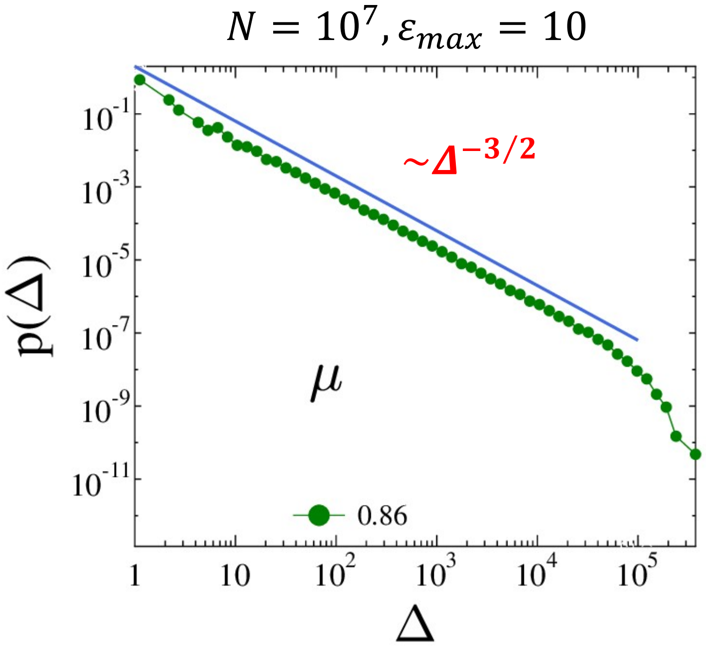
Burst size distribution – finite cutoffs



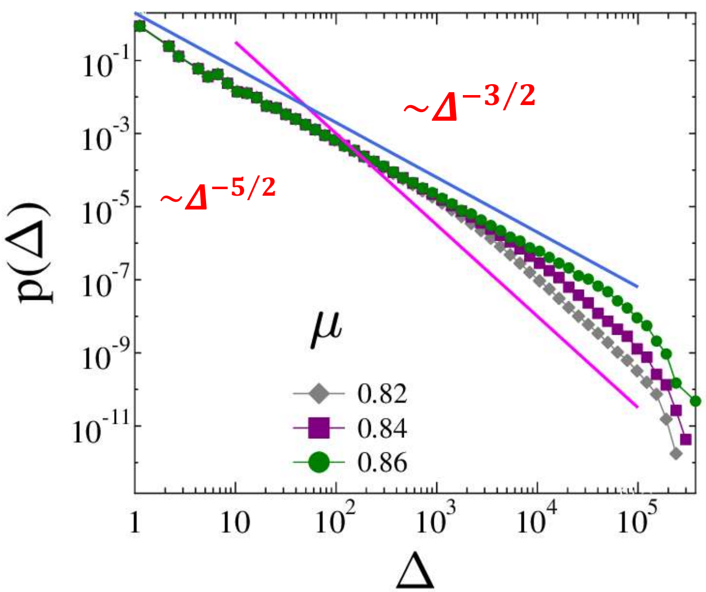
$\mu \to 0$

τ: 3/2

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Burst size distribution – finite cutoffs

$N = 10^7, \varepsilon_{max} = 10$



$\mu \to 0$

$\tau: 3/2 \to 3/2 + 5/2$

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

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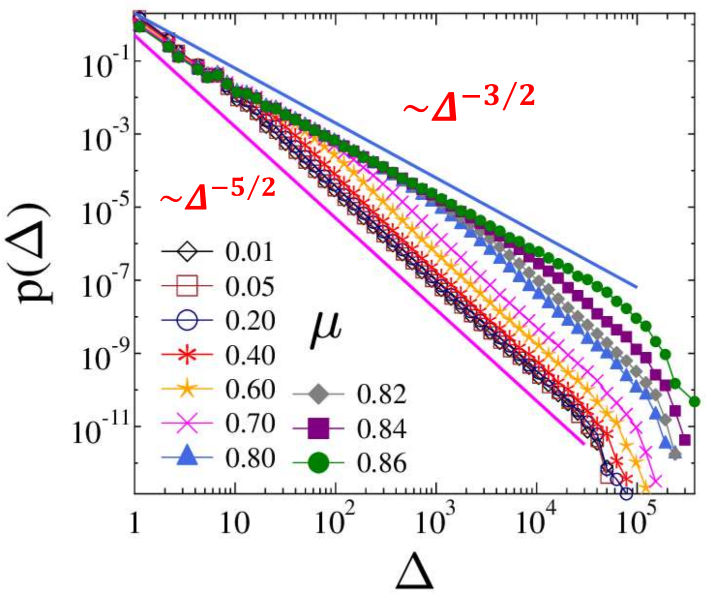
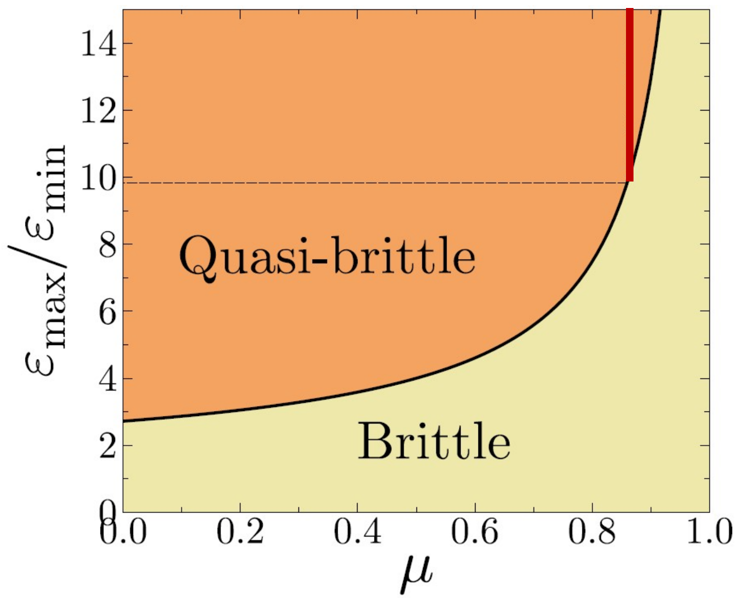




$\mu \to 0$

τ: $3/2 \to 3/2 + 5/2 \to 5/2$

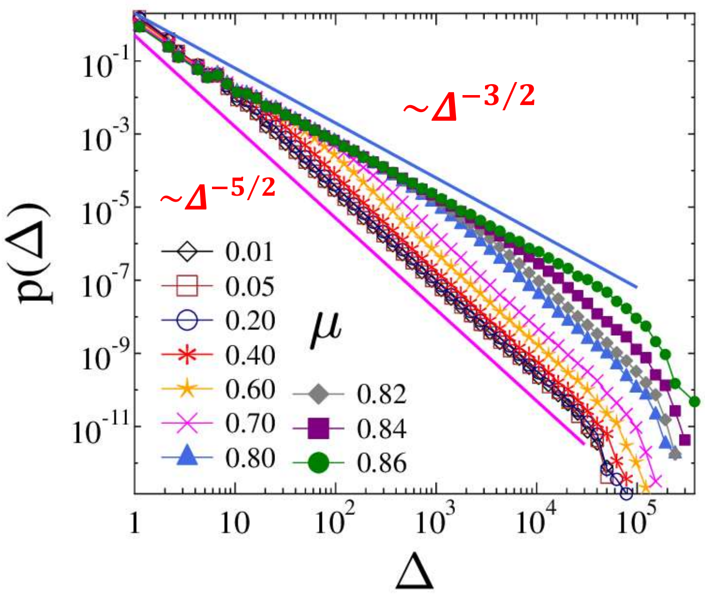
Δ_0: crossover burst size

$\mu \to \mu_c(\varepsilon_{max})$ $\quad \Delta_0 \sim (\mu_c - \mu)^{-\gamma}, \quad \gamma = 2$

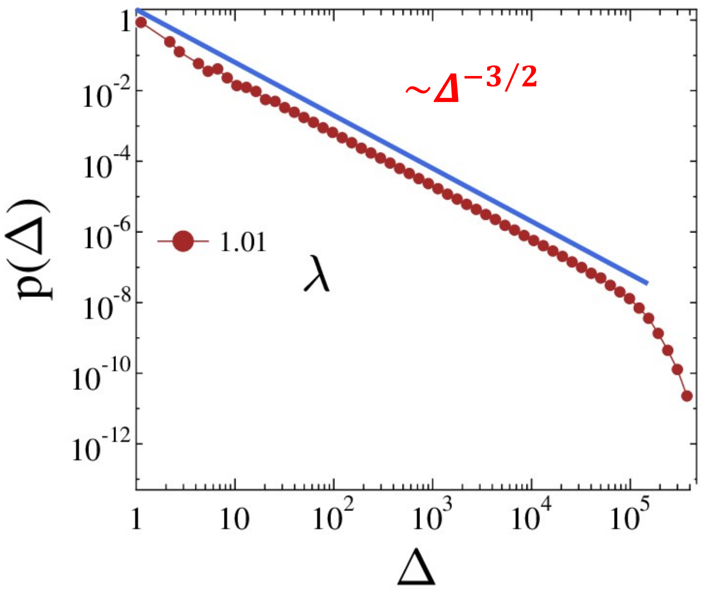
V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Burst size distribution – finite cutoffs

$N = 10^7, \varepsilon_{max} = 10$



$N = 10^7, \mu = 0.85$



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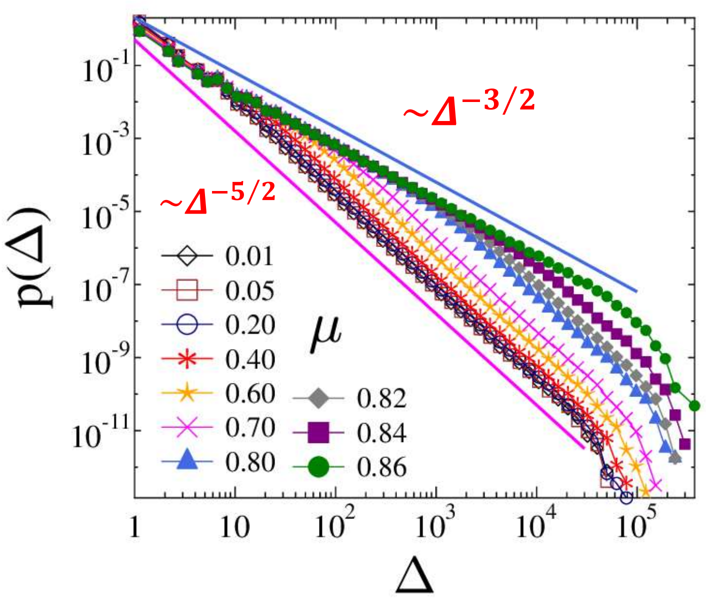
$\lambda \to \infty$

τ: $3/2$

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Burst size distribution - finite cutoffs

$N = 10^7, \varepsilon_{max} = 10$



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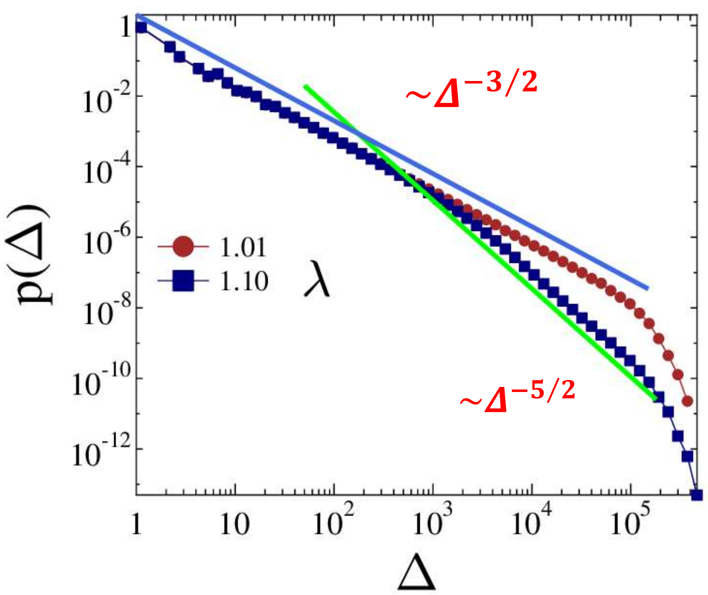

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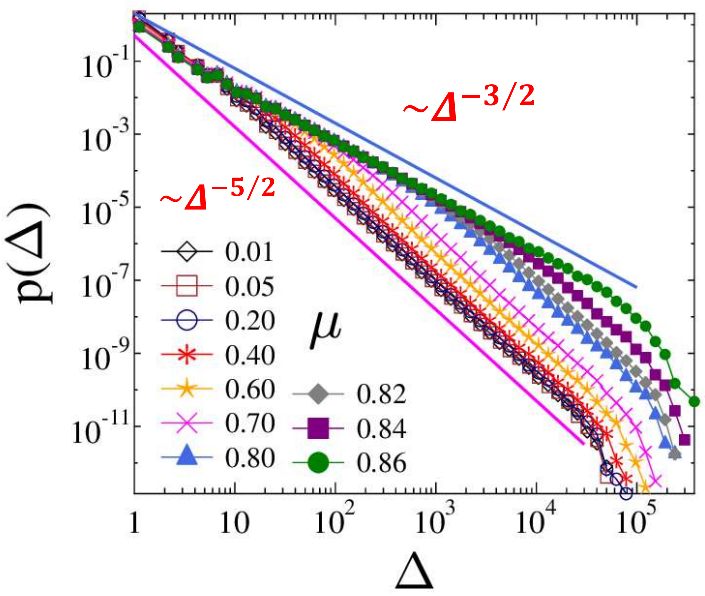

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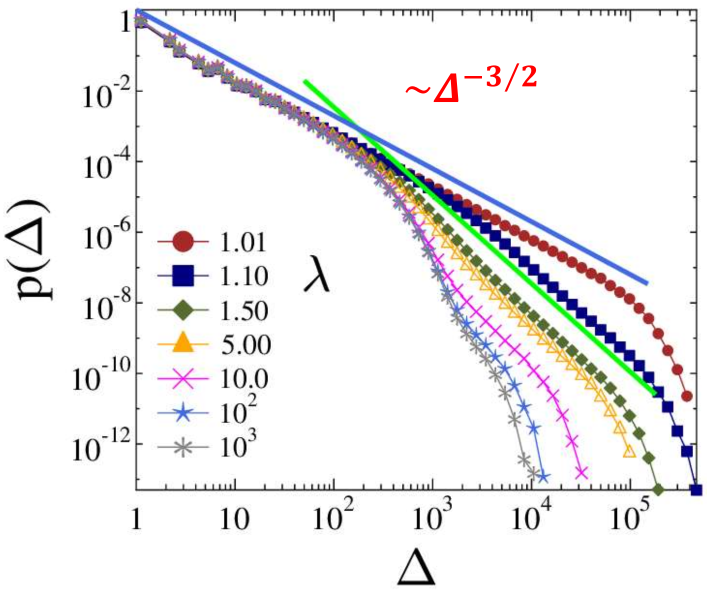

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$\lambda \to \infty$

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$\lambda \to 1 \qquad \Delta_0 \sim (\lambda - 1)^{-\gamma}, \qquad \gamma = 2$

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Burst size distribution – finite cutoffs

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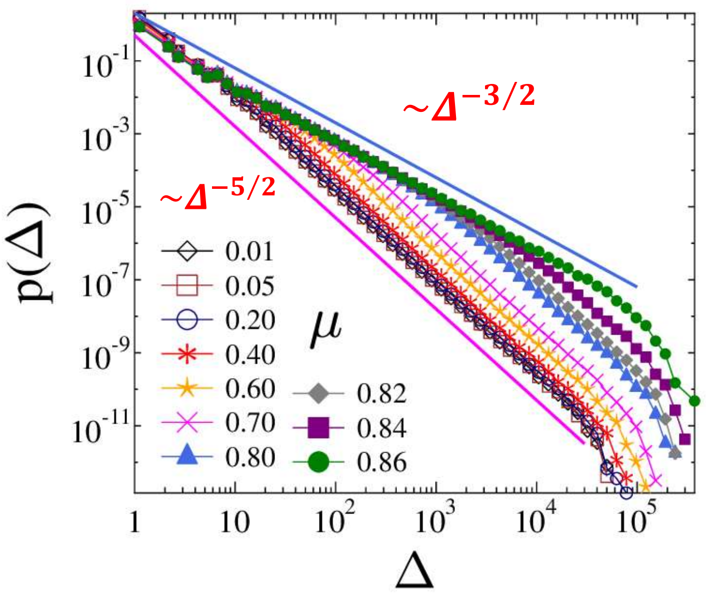

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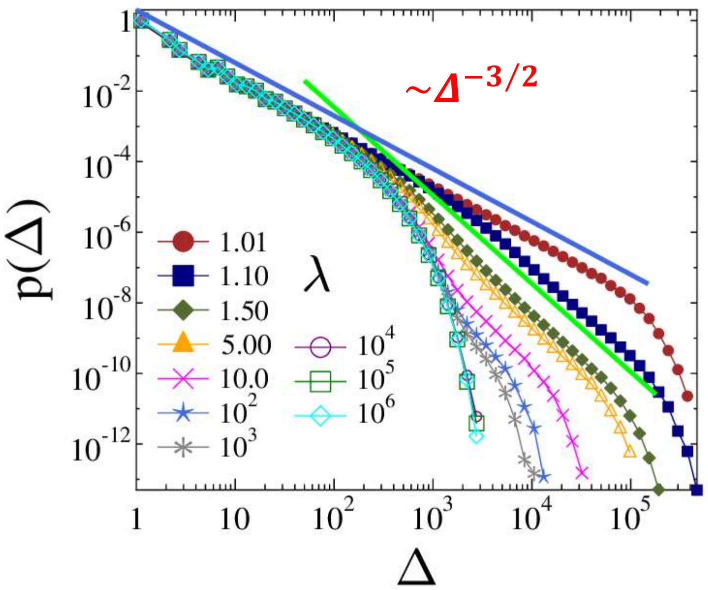

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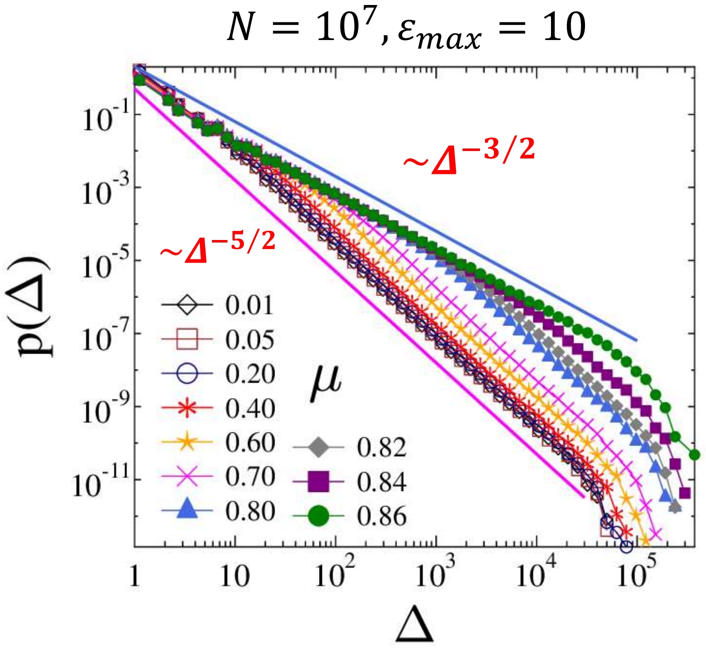

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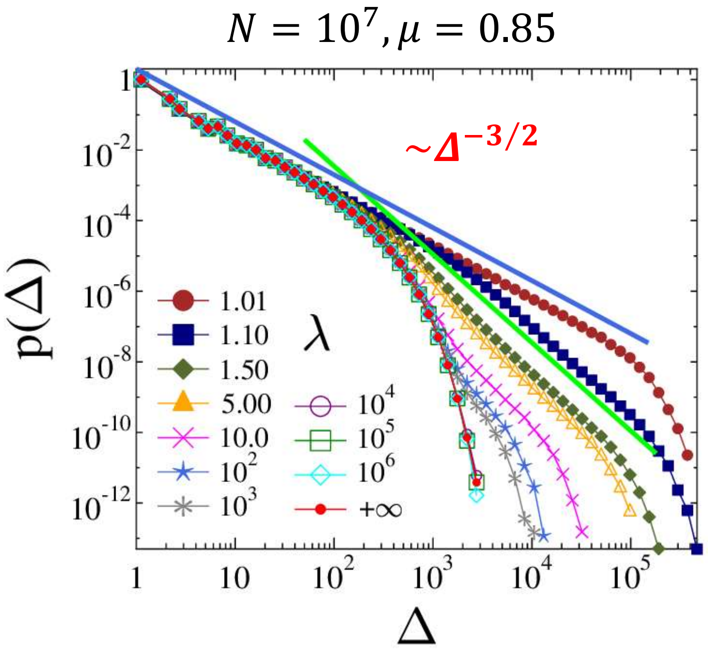

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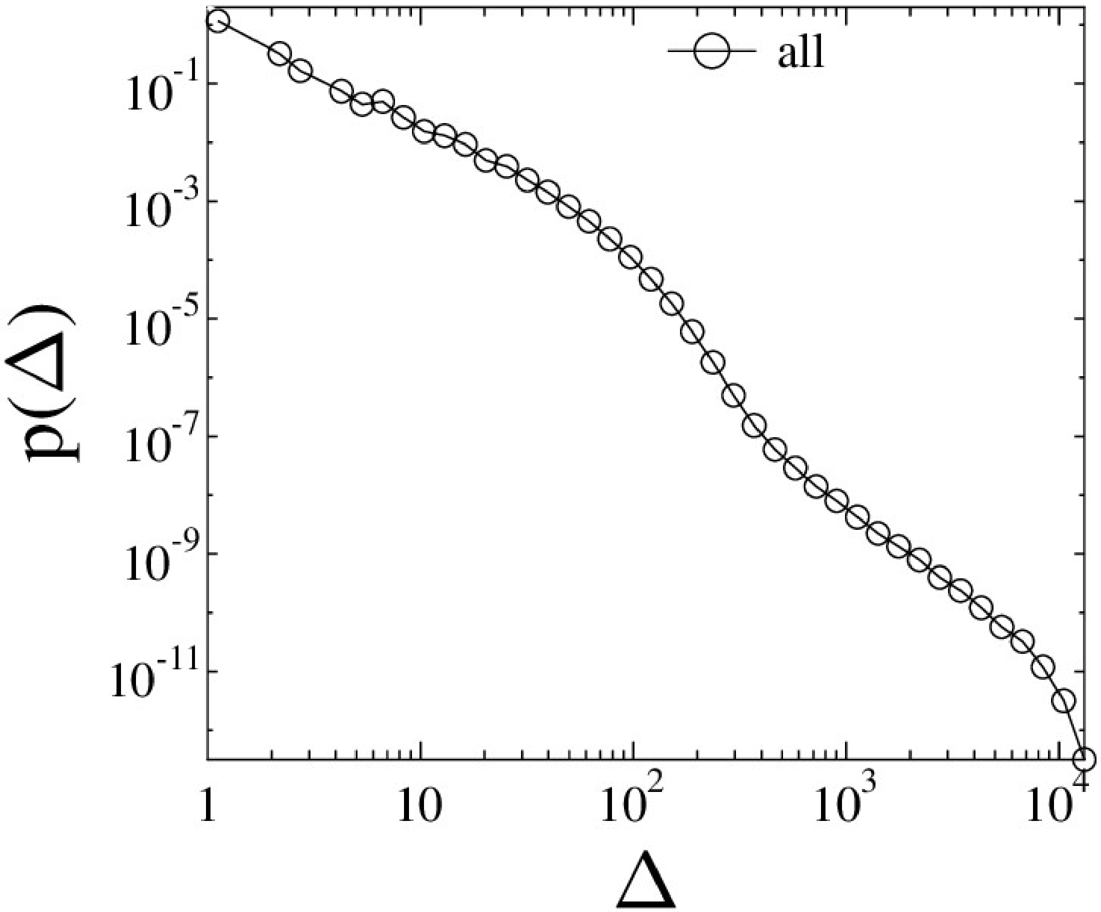
V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Burst size distribution – finite cutoffs

$N = 10^7, \;\; \mu = 0.8, \;\; \lambda = 500$



Only the first L bursts are included in $p(\Delta)$

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

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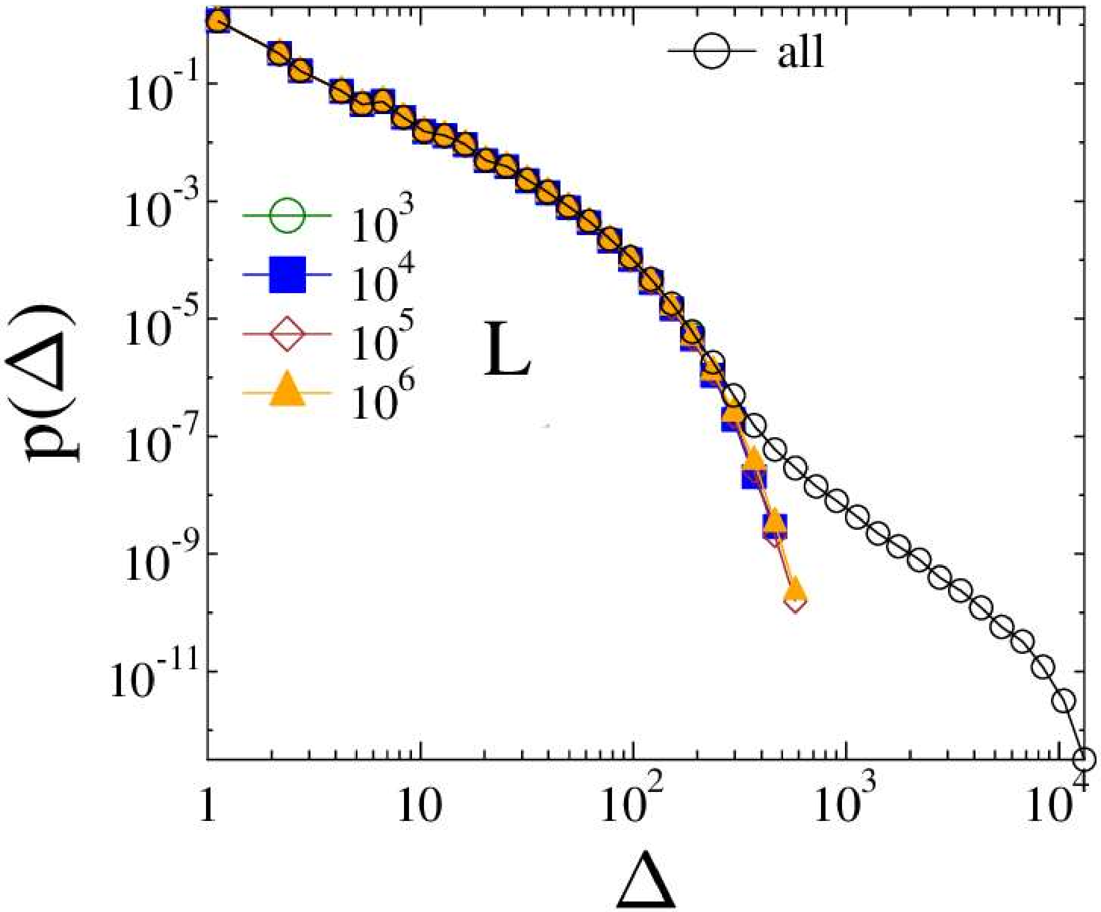

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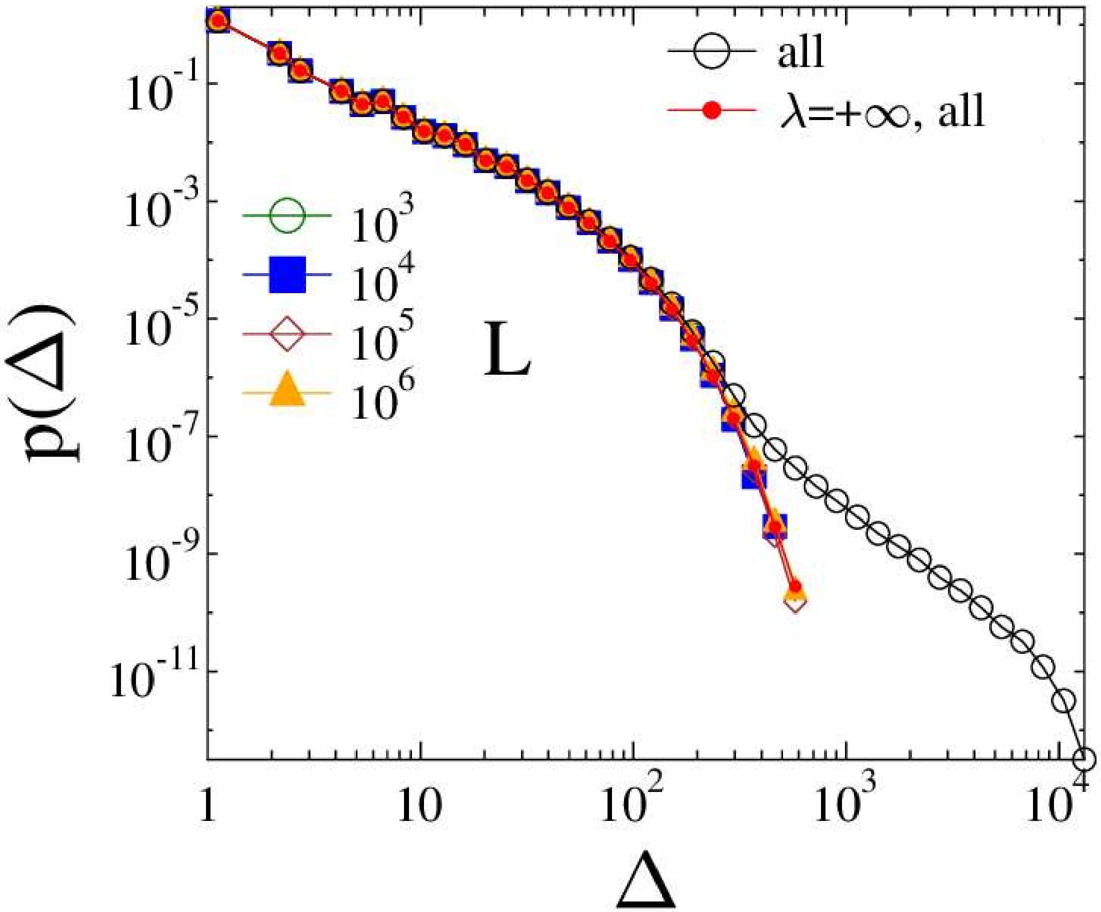

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Beginning of series is close to stationary

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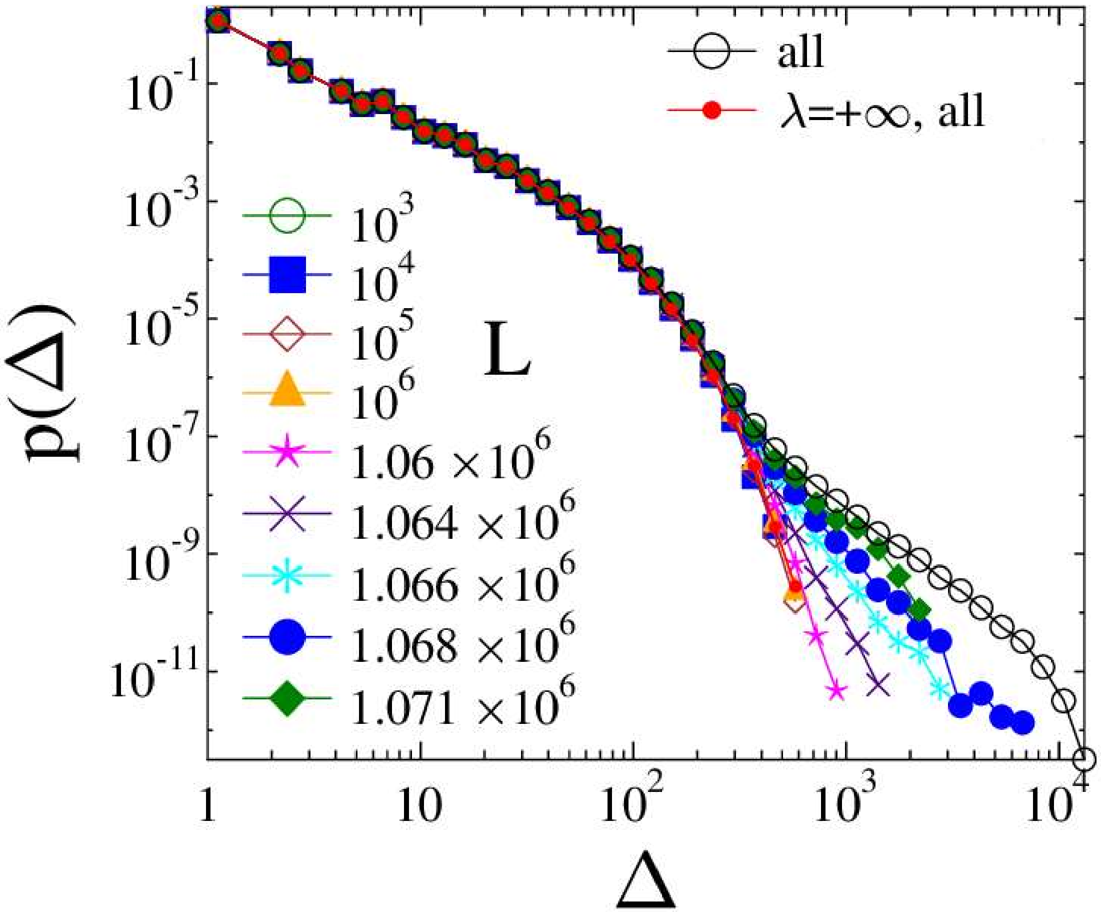

Only the first L bursts are included in $p(\Delta)$

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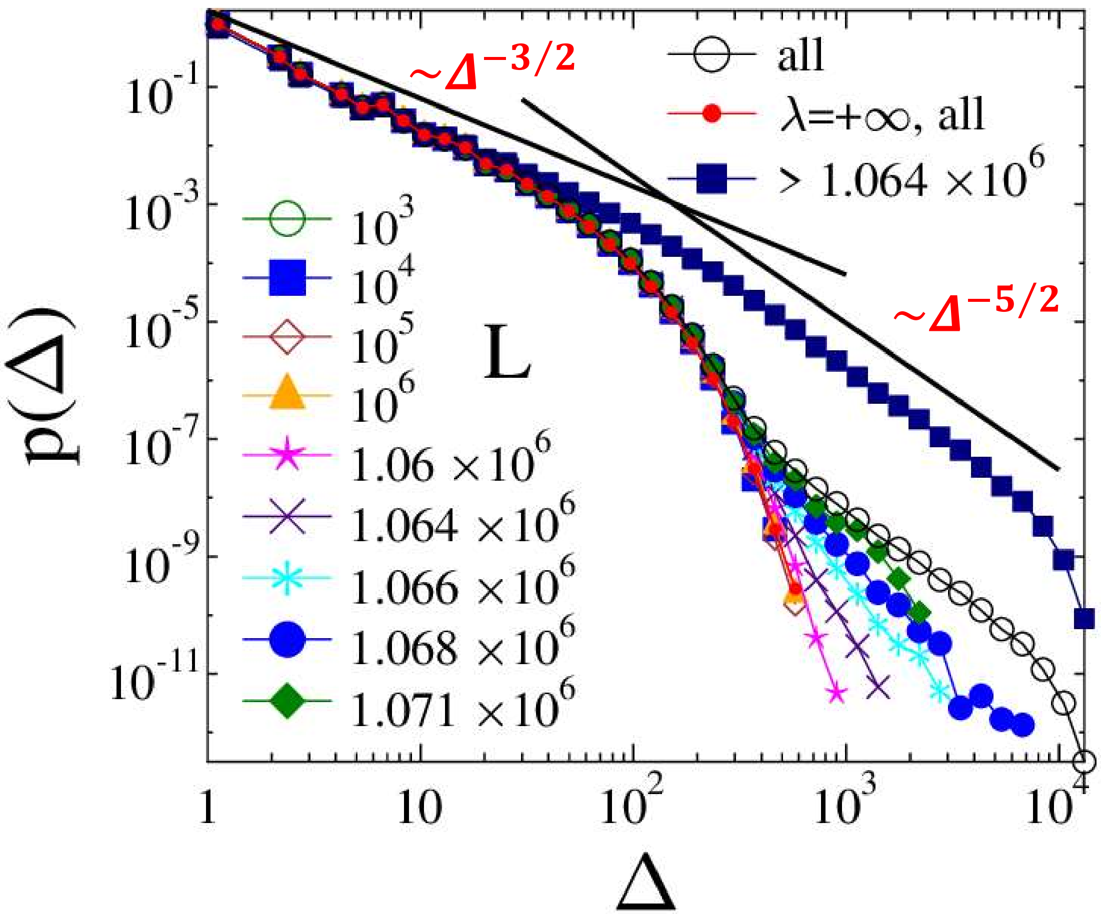
Sort accelerating regime

Diminishing contribution

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Burst size distribution – finite cutoffs

$N = 10^7, \ \ \mu = 0.8, \ \ \lambda = 500$



Only the first L bursts are included in $p(\Delta)$

Beginning of series is close to stationary

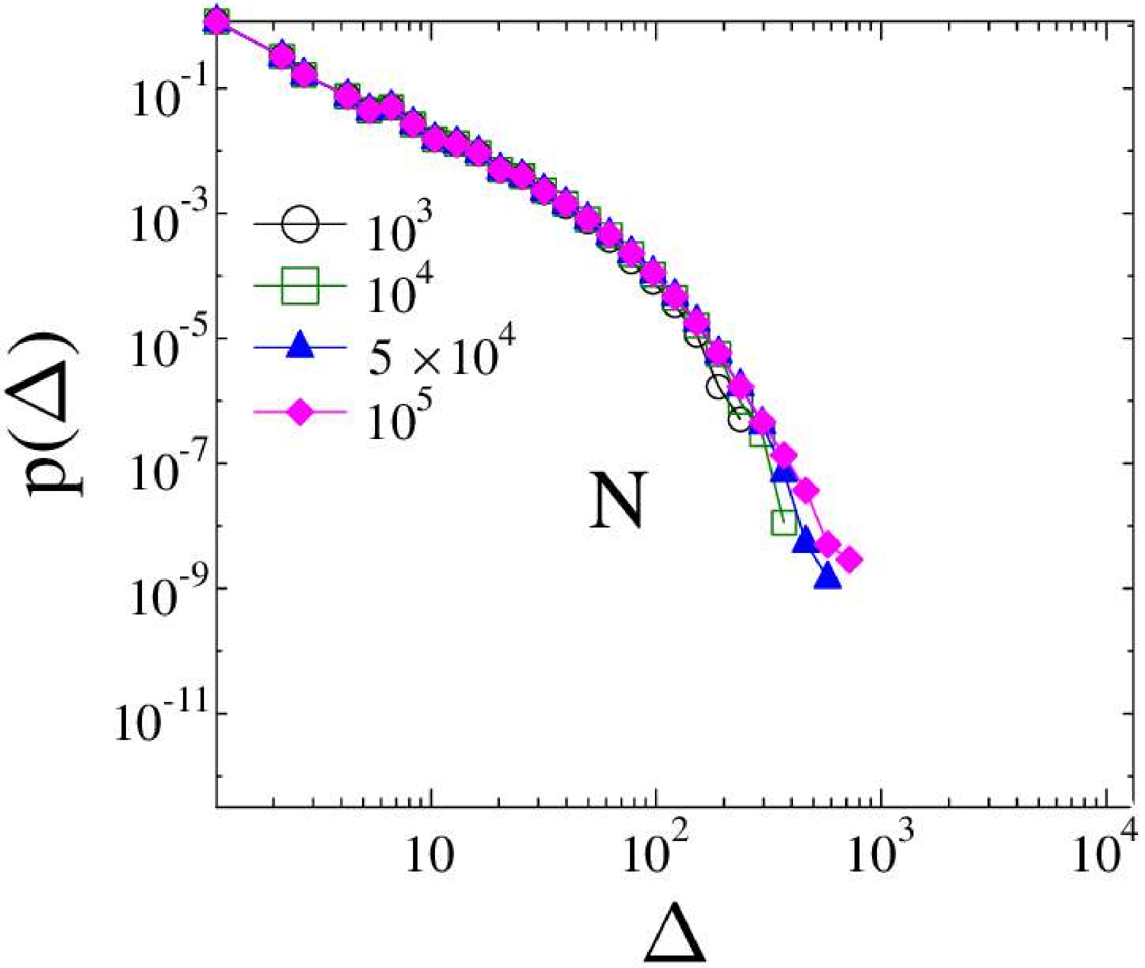
Sort accelerating regime

Diminishing contribution

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Size dependence of burst distributions

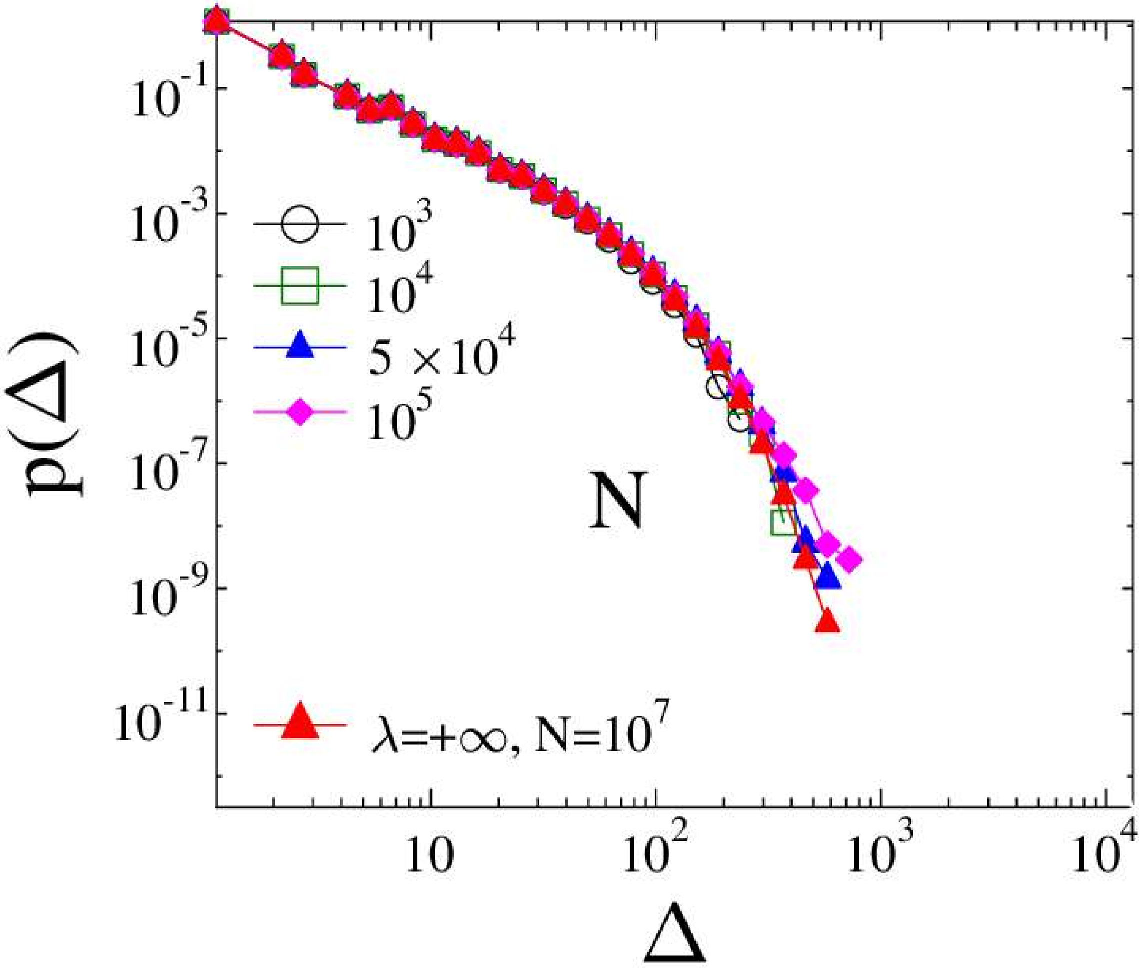
$\mu = 0.8, \quad \lambda = 500$



V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Size dependence of burst distributions

$\mu = 0.8, \quad \lambda = 500$

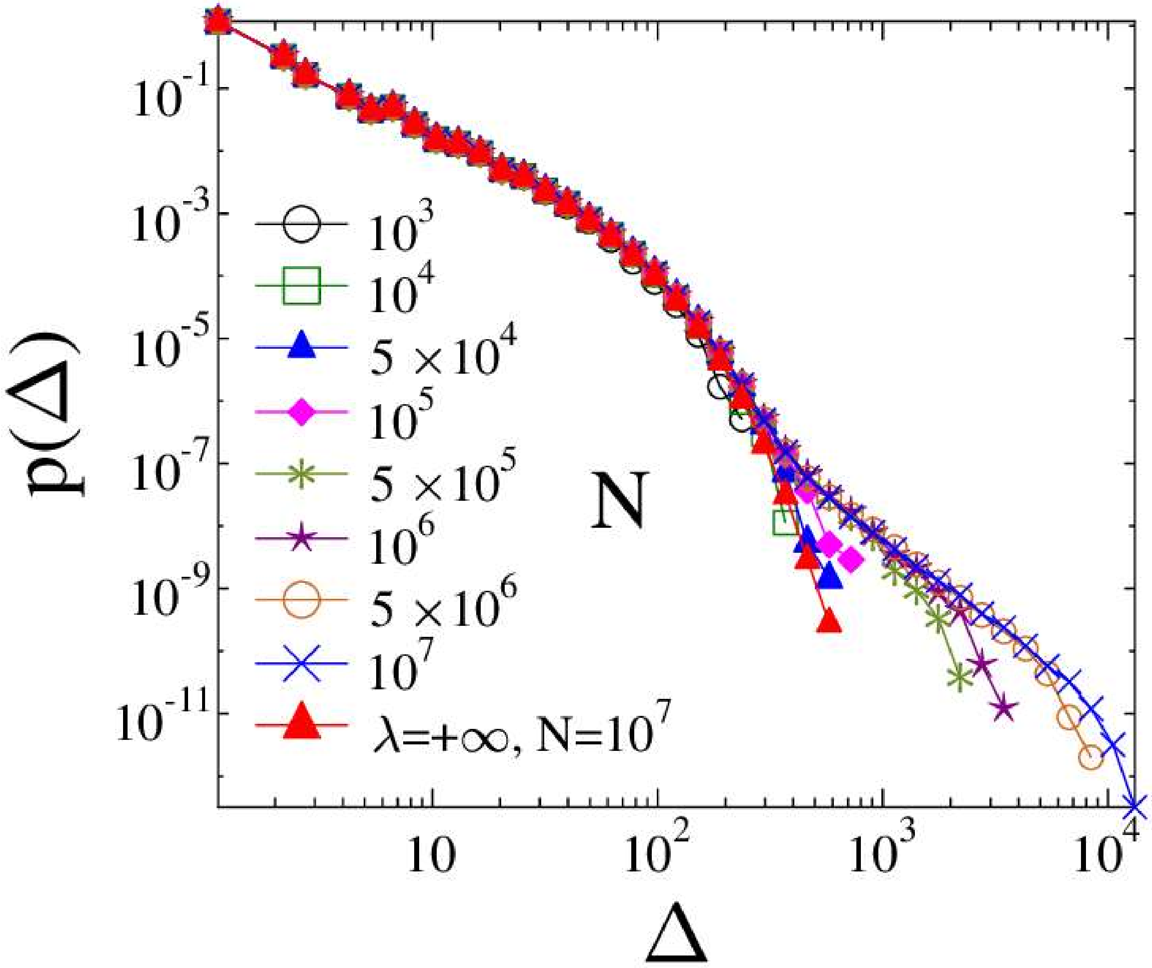


Small systems: stationarity

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Size dependence of burst distributions

$\mu = 0.8, \quad \lambda = 500$



Small systems: stationarity

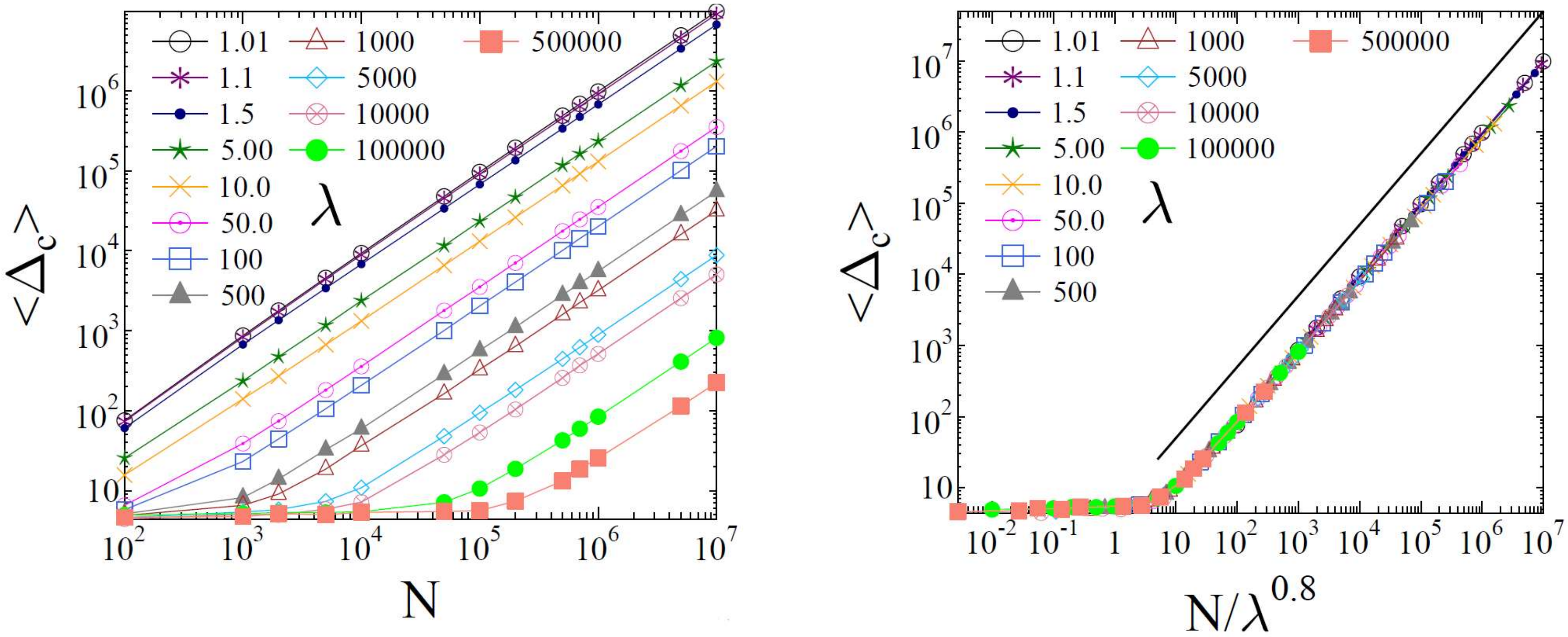
Second power law regime above a characteristic system size

System size has a strong effect on the functional form

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

System size dependence of catastrophic burst

$\mu = 0.8$



$N_c \sim \lambda^{\mu}$

Macroscopic failure may not be predictable

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019).

Thank you for your attention!

For further information see:

Zs. Danku, and F. Kun J. Stat. Mech. **2016**, 073211 (2016)

V. Kadar, Zs. Danku, F. Kun, Physical Review E **96**, 033001 (2017)

V. Kadar and F. Kun, Physical Review E **100**, 053001 (2019)

V. Kadar, G. Pal, and F. Kun, Scientific Reports **10**, 2508 (2020)

V. Kadar, Zs. Danku, G. Pal, and F. Kun, Physica A **594**, 127015 (2022)