Depinning transition and Barkhausen avalanches of thin film domain walls with internal degrees of freedom

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Avalanche 2022
Avalanche dynamics and precursors of catastrophic events

Introduction

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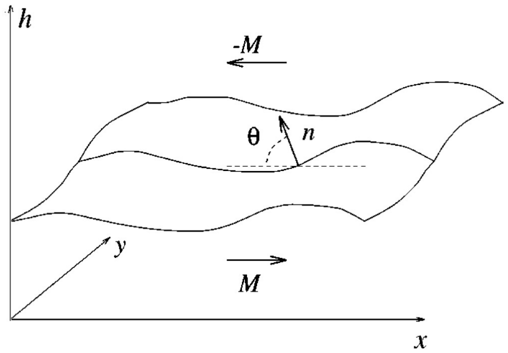
$$\frac{\partial h(\vec{r},t)}{\partial t} = H - k\bar{h} + \nu_0 \nabla^2 h(\vec{r},t) + \int d^2r' K(\vec{r}-\vec{r}')[h(\vec{r}') - h(\vec{r})] + \eta(\vec{r},h)$$



S. Zapperi *et al.*, Phys. Rev. B 58, 6353 (1998).

In the "1D model" of point-like DWs in nanowires, DW mid-point magnetization (an *internal degree of freedom*) is described by the variable ϕ in addition to the DW position q:

$$\alpha \frac{\dot{q}}{D} + \dot{\phi} = \gamma H_a$$

$$\frac{\dot{q}}{D} - \alpha\dot{\phi} = \frac{2\gamma K}{\mu_0 M_s} \frac{\sin(2\phi)}{2}$$

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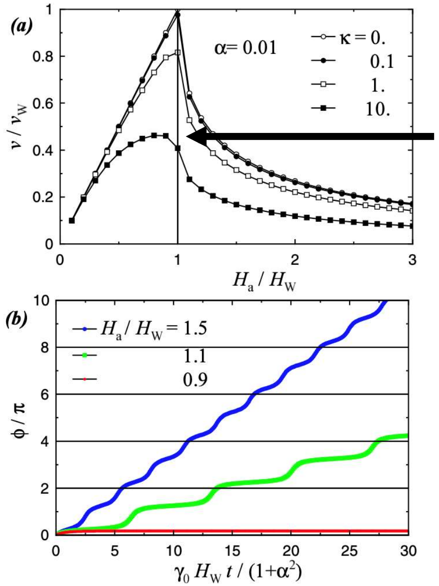

"Walker breakdown"

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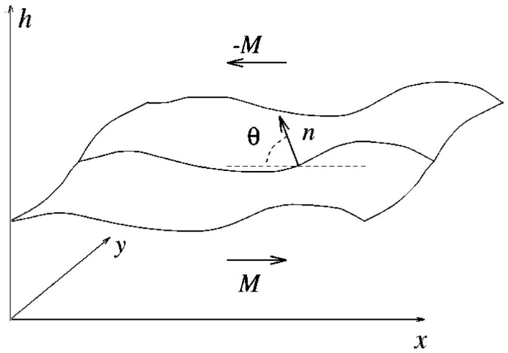

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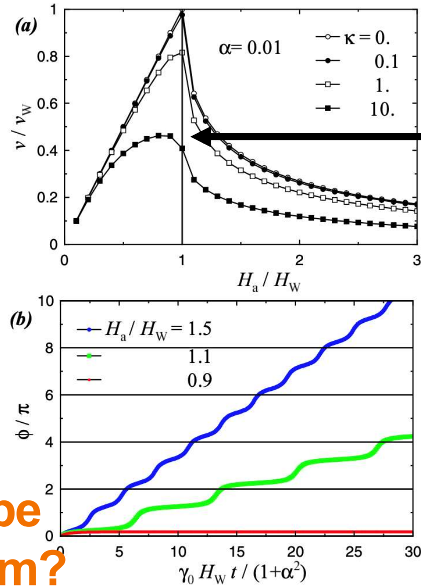

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How to combine these approaches to describe extended DWs with internal degrees of freedom?

Internal degrees of freedom in PMA thin films: Bloch lines

Line-like domain walls (DWs) in thin films/strips with perpendicular magnetic anisotropy (PMA) may contain *Bloch lines* (BLs).

The motion of BLs mediates large-scale precession of the DW magnetization.

Different BL configurations within a Bloch DW in a PMA thin film:

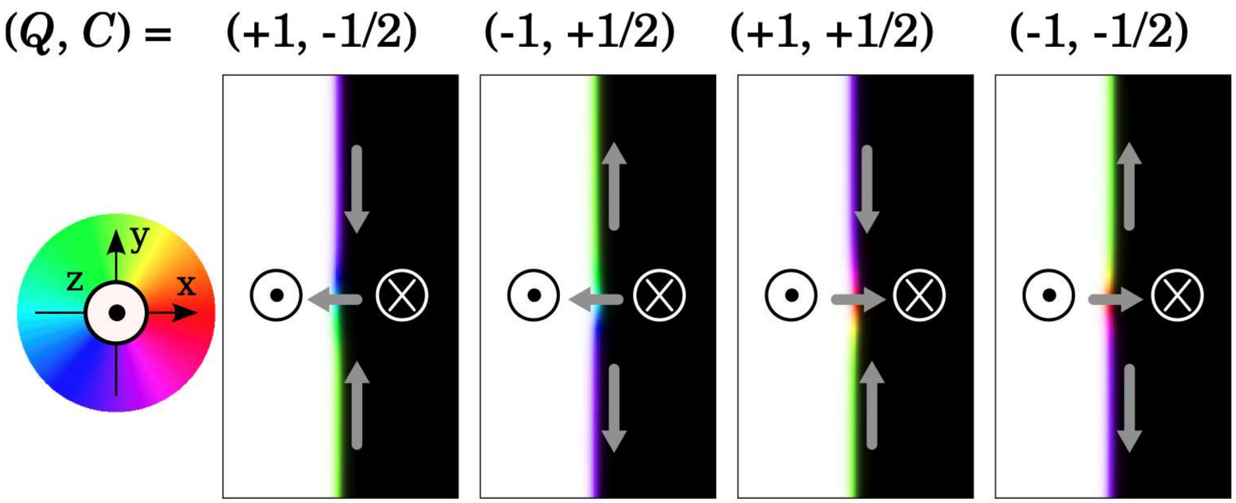

Figure: PhD thesis of Touko Herranen

Internal degrees of freedom in PMA thin films: Bloch lines

Line-like domain walls (DWs) in thin films/strips with perpendicular magnetic anisotropy (PMA) may contain *Bloch lines* (BLs).

The motion of BLs mediates large-scale precession of the DW magnetization.

Our goal: Construct a model of line-like DWs in PMA films able to describe BL dynamics within the DW.

Different BL configurations within a Bloch DW in a PMA thin film:

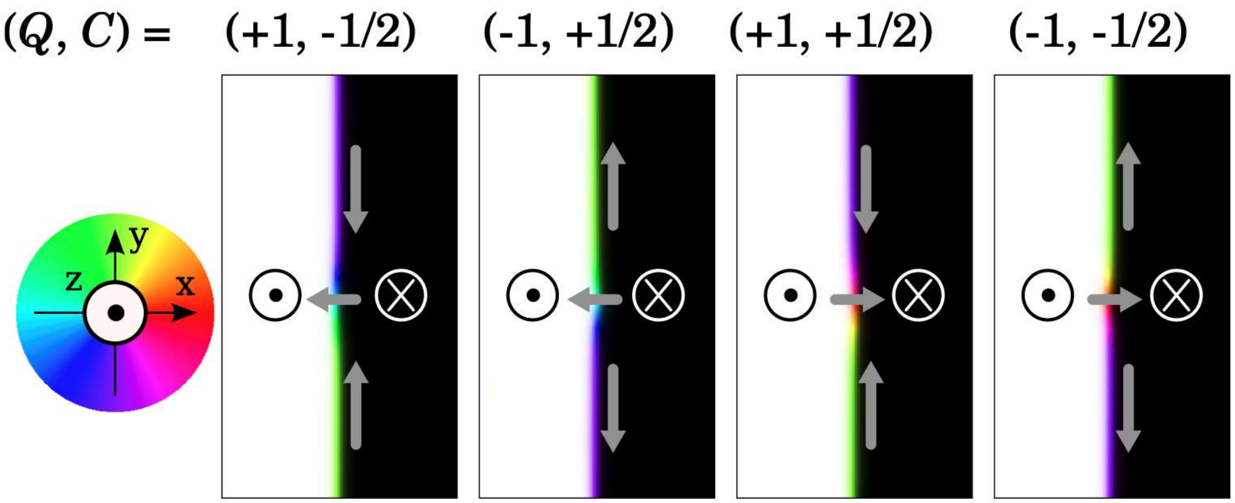

Figure: PhD thesis of Touko Herranen

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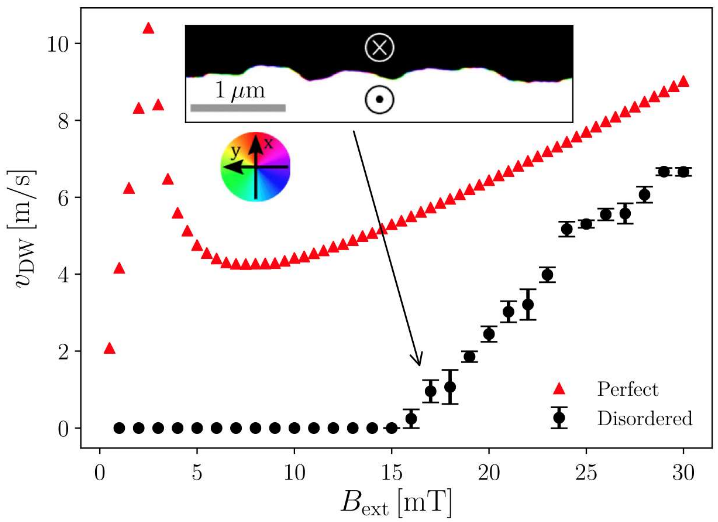
Micromagnetic simulations of domain walls with internal degrees of freedom

Micromagnetic simulations of field-driven motion of Bloch DWs in a 0.5-nm-thick Co film within a Pt/Co/Pt multilayer with PMA using the MuMax3 code, solving the Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{\partial \mathbf{m}}{\partial t} = -\frac{\gamma}{1+\alpha^2}\{\mathbf{m} \times \mathbf{B}_{\text{eff}} + \alpha[\mathbf{m} \times (\mathbf{m} \times \mathbf{B}_{\text{eff}})]\}$$

Disorder modelled following the procedure introduced by S. Moretti *et al*. [PRB 96, 054433 (2017)]:

- **Voronoi tessellation (grain size 20 nm).**
- **Normally distributed film thickness Δ, modelled by setting $K_u^G = K_u \Delta / t_G$ and $M_s^G = M_s t_G / \Delta$ in each grain.**
- **Adjust the disorder strength to reproduce the depinning field found by Metaxas *et al*. [PRL 99, 217208 (2007)].**
- **Study the Barkhausen effect.**



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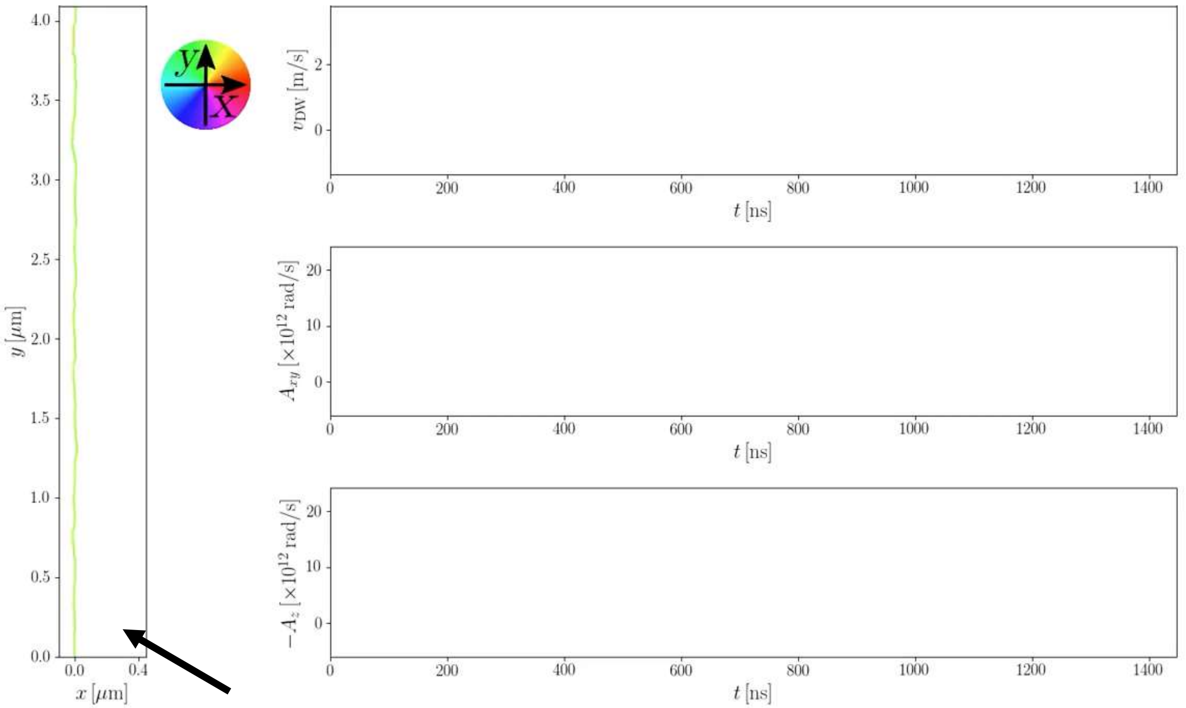
T. Herranen and LL, Phys. Rev. Lett. 122, 117205 (2019).

Micromagnetic simulations: Barkhausen noise from precessional DW motion

To simulate the Barkhausen effect, we use the following driving protocol:

- When $v_{\mathrm{DW}} > v_{\mathrm{DW}}^{\mathrm{th}} = 0.1\ \mathrm{m/s}$, reduce the applied field according to $\dot{B}_{\mathrm{ext}} = -k|v_{\mathrm{DW}}|$.
- In between Barkhausen jumps, increase the applied field at a constant rate, $\dot{B}_{\mathrm{ext}} = 0.037\ \mathrm{mT/ns}$.
- Measure the DW velocity, as well as the "activity signals" $A_{xy}(t) = \sum_{i \in B} \dot{\phi}_i \cdot |\mathbf{m}_{i,xy}|$ and $A_z(t) = \sum_{i \in B} \dot{\theta}_i$.
- Note that $A_{xy}(t)$ originates from inside the domain wall only (in-plane BL dynamics).



BLs nucleate, propagate and annihilate within the DW.

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Barkhausen noise from precessional DW motion

The bursts visible in the various signals $V(t)$ can be characterised by their sizes $S_V = \int_0^T [V(t) - V^{\mathrm{th}}]dt$ and durations T.

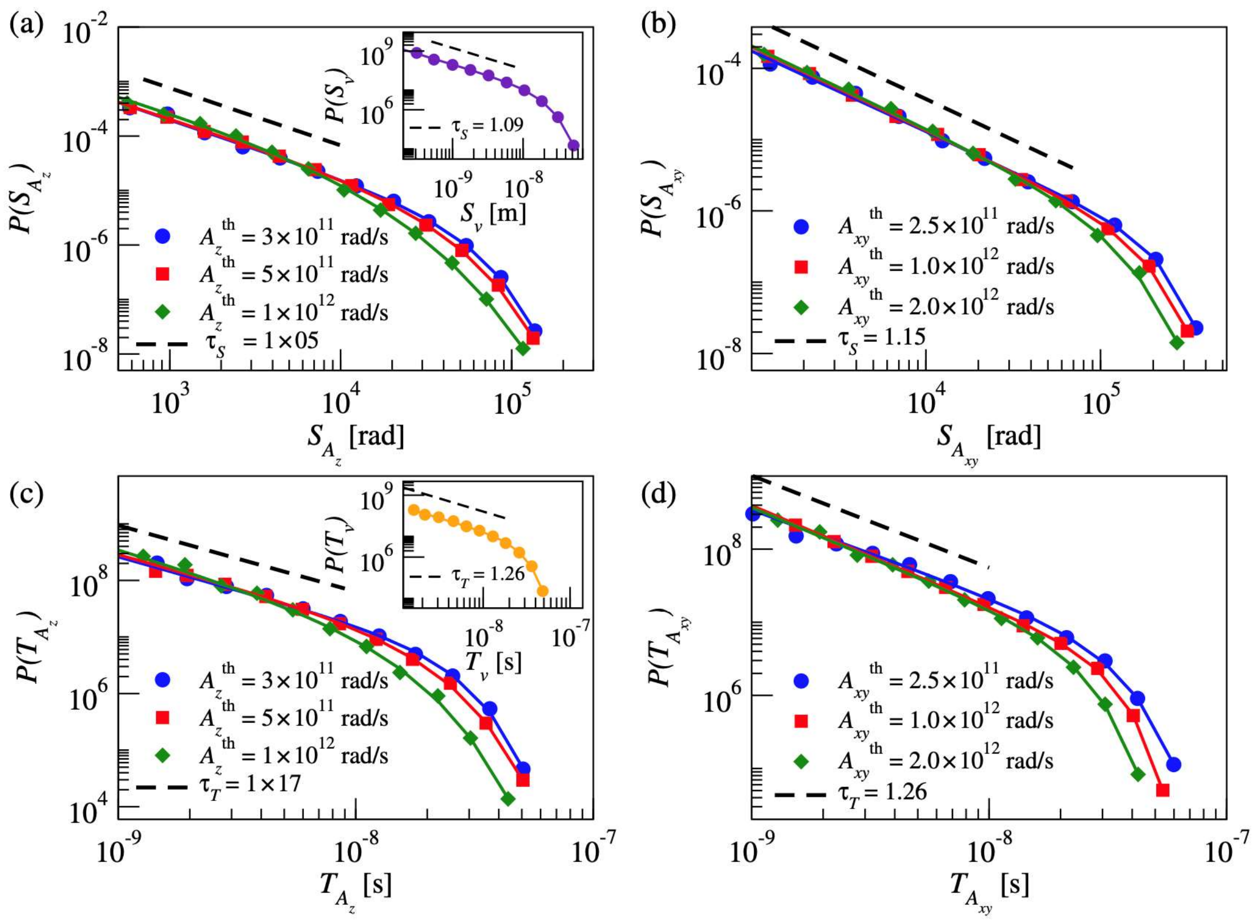
These follow power law distributions with cutoffs:

$$P(S_V) = S_V^{-\tau_S} \exp[-(S_V/S_V^*)^\beta]$$

The exponents for sizes and durations are found to be

$$\tau_S = 1.1 \pm 0.1 \qquad \tau_T = 1.2 \pm 0.1$$

Also internal crackling dynamics!



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Reduced model of a DW with BLs

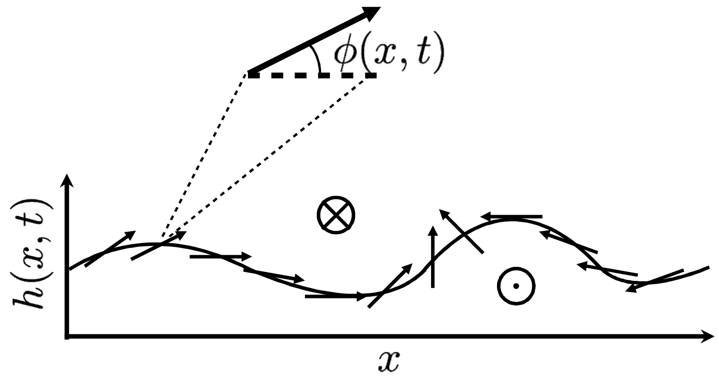
Micromagnetic simulations are limited in that only short (up to few μm in length) DWs can be simulated → need a reduced model to reach longer length scales (to minimize finite-size effects)!

Start by viewing the LLG equation as a dissipative Euler-Lagrange equation, and derive equations of motion for the collective coordinates $h(x,t)$ and $\phi(x,t)$, the latter describing the direction of the DW mid-point in-plane magnetization:

$$\dot{\phi} + \alpha\frac{\dot{h}}{D} = 2\frac{\gamma A_{\text{ex}}}{M_s D}h'' - \gamma B_a,$$

$$\alpha\dot{\phi} - \frac{\dot{h}}{D} = 2\frac{\gamma A_{\text{ex}}}{M_s}\phi'' - \frac{\gamma N_n}{2}\mu_0 M_s \sin[2(\phi - \chi)]$$

$\chi = \arctan h'$



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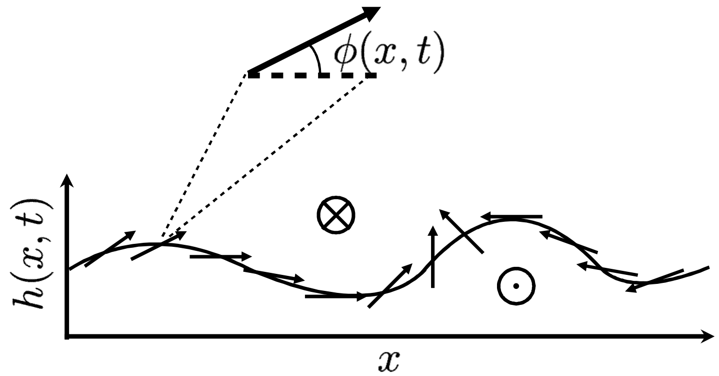

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Demagnetizing factor $N_n = \frac{\Delta}{\pi D}\ln 2$
[A. Skaugen *et al.*, PRB 100, 094440 (2019)]

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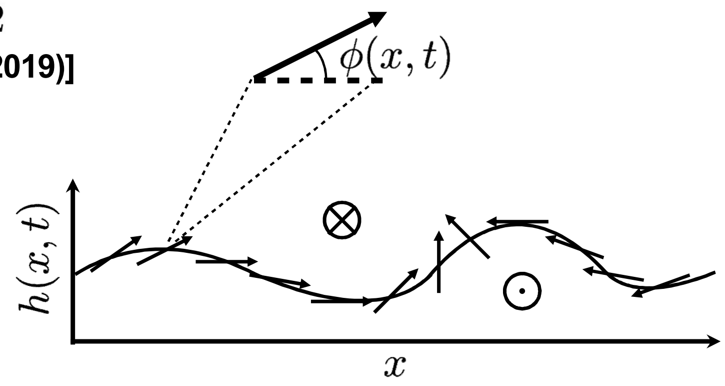

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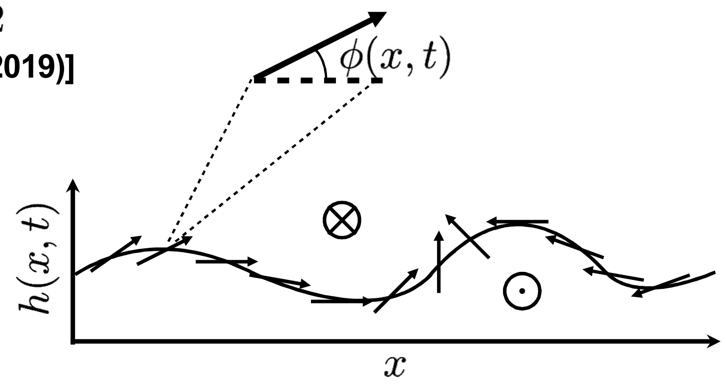

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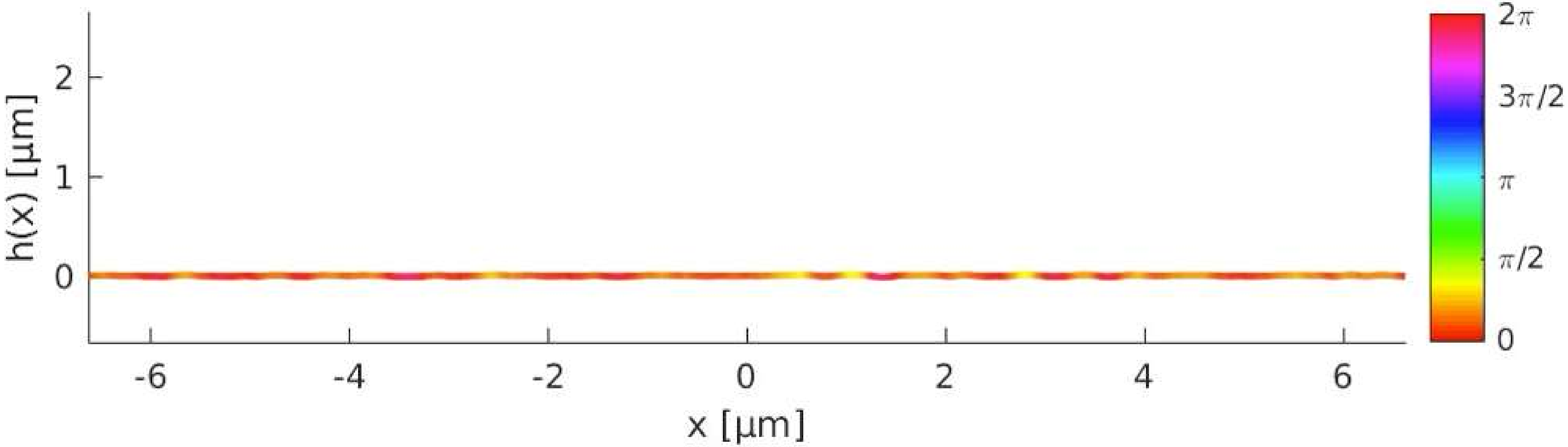
Reduced model of a DW with BLs

A small part of a much longer DW subject to $B_{\rm ext} = B_c = 2.22\ {\rm mT}$ for $\sigma = 7\ {\rm mT}$; the model allows to reach two orders of magnitude longer DW lengths (hundreds of μm's) than in micromagnetic simulations:



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Reduced model of a DW with BLs: $V(B_{\text{ext}})$

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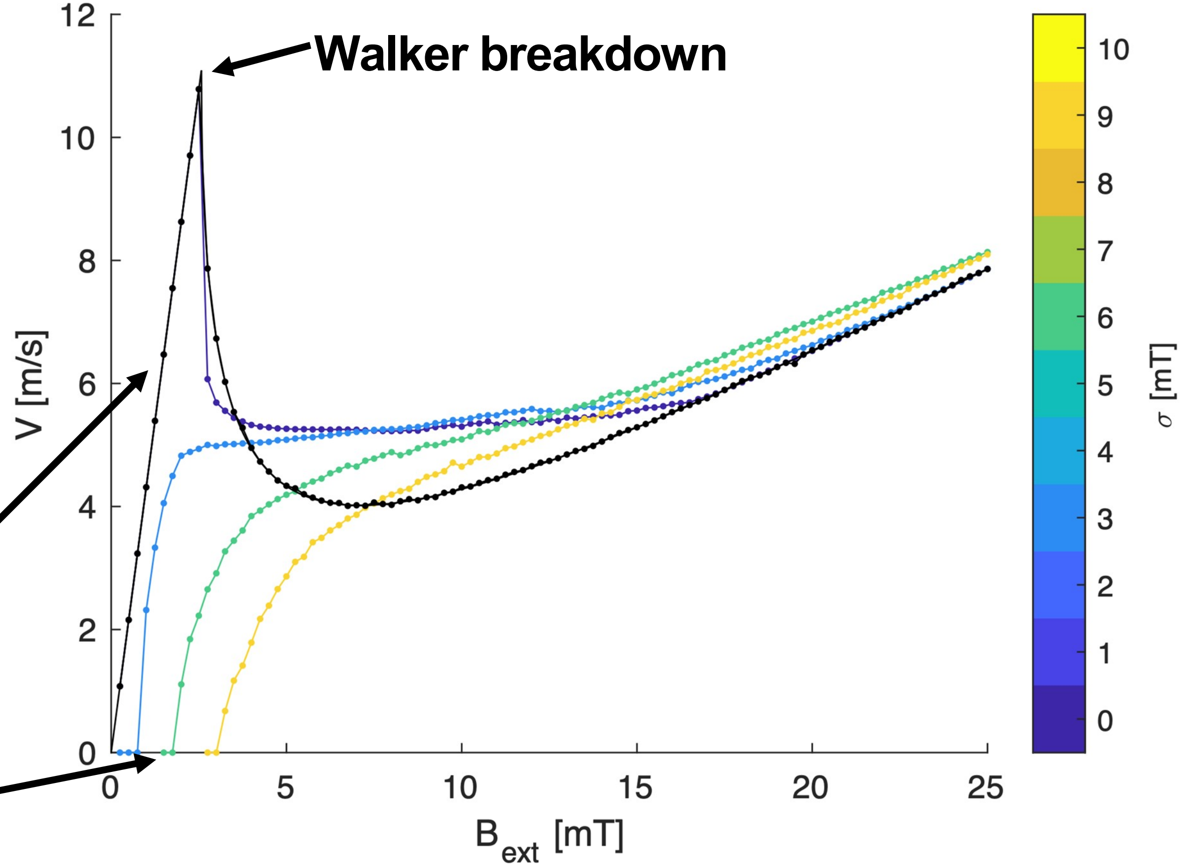
The model has two limiting behaviours:

- **Neglect spatial derivatives → 1D model.**
- **Neglect internal degrees of freedom (set ϕ to a constant) → quenched Edwards-Wilkinson (qEW) equation.**

Narrow systems with $\sigma = 0$ reproduce the 1D model result for $V(B_{\text{ext}})$.

Stronger disorder results in a disorder-dependent depinning field.



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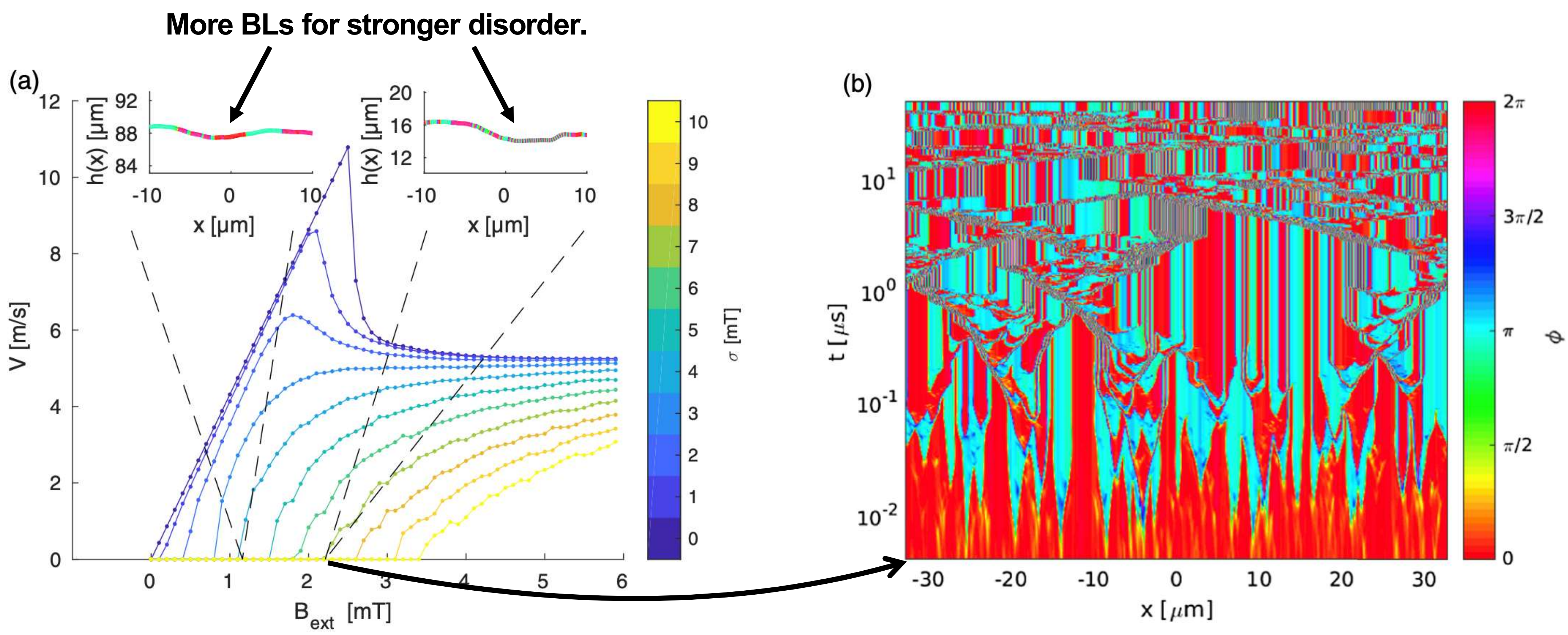

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DW depinning transition: disorder-dependent exponents

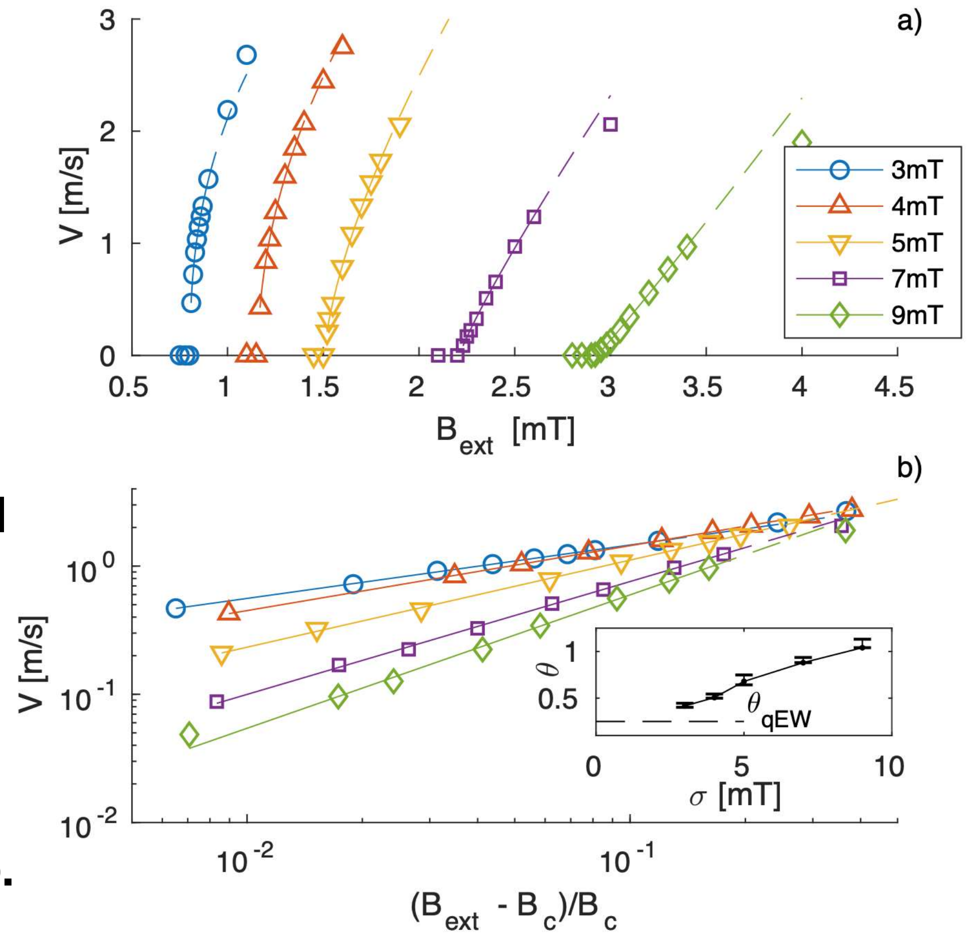
Focus on the proximity of the DW depinning transition for different disorder strengths σ.

Close to $B_{\text{ext}} = B_c$, the DW velocity obeys

$$V(B_{\text{ext}}) \propto (B_{\text{ext}} - B_c)^{\theta}$$

The exponent θ appears to depend on the disorder strength σ:

- Small σ results in qEW exponents and only a small number of BLs within the DW.
- Larger σ leads to a larger θ, and more BLs within the DW.
- Also other exponents (related to critical relaxation and roughness) exhibit dependence on σ (up next).



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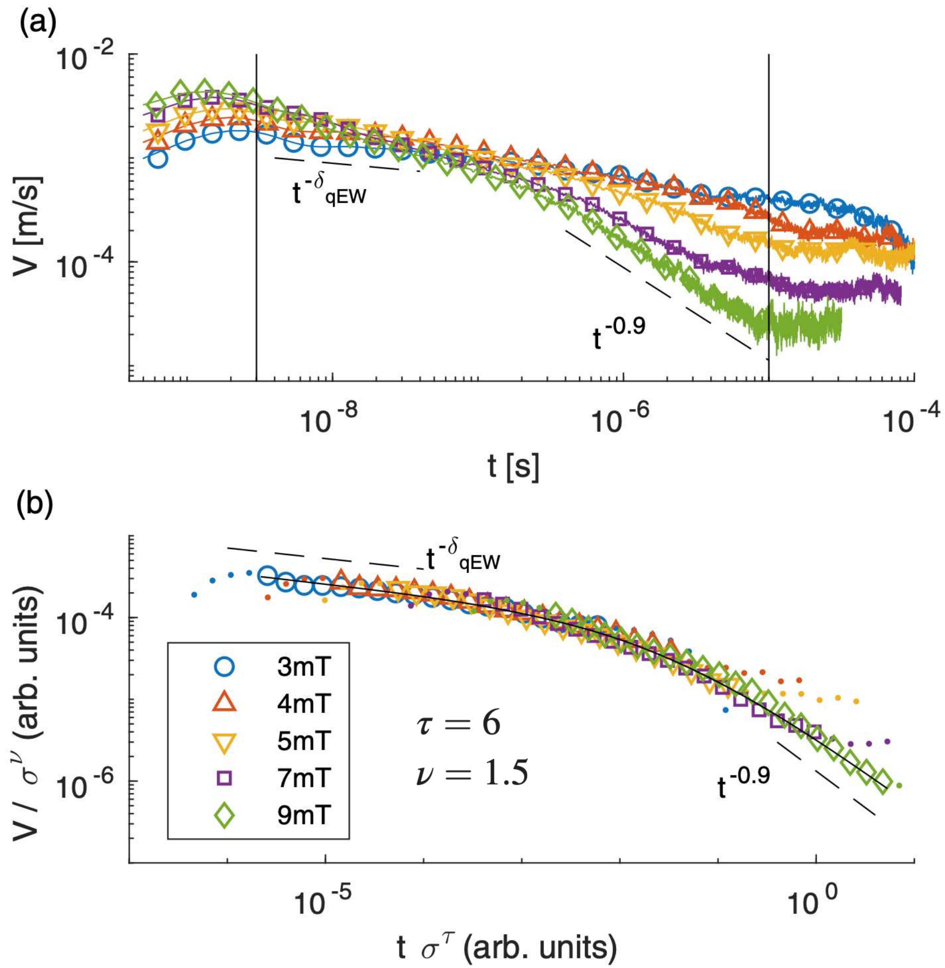
DW depinning transition: disorder-dependent exponents

Relaxation of $V(t)$ at $B_{\mathrm{ext}} = B_{\mathrm{c}}$:

- **Typically one expects $V(t) \propto t^{-\delta}$**
- **Here, a crossover between two asymptotic power laws, $V(t \ll t_c) \propto t^{-\delta_{\mathrm{qEW}}}$ and $V(t \gg t_c) \propto t^{-\delta_{\mathrm{sat}}}$, with a disorder-dependent crossover timescale $t_c(\sigma)$:**

$$V(t, B_{\mathrm{ext}} = B_c) = Ct^{-\delta_{\mathrm{qEW}}}\left[1 + \left(\frac{t}{t_c}\right)^{k(\delta_{\mathrm{sat}} - \delta_{\mathrm{qEW}})}\right]^{-1/k}$$

$$\delta_{\mathrm{qEW}} = 0.129 \qquad \delta_{\mathrm{sat}} \approx 0.9 \qquad k \approx 0.48$$



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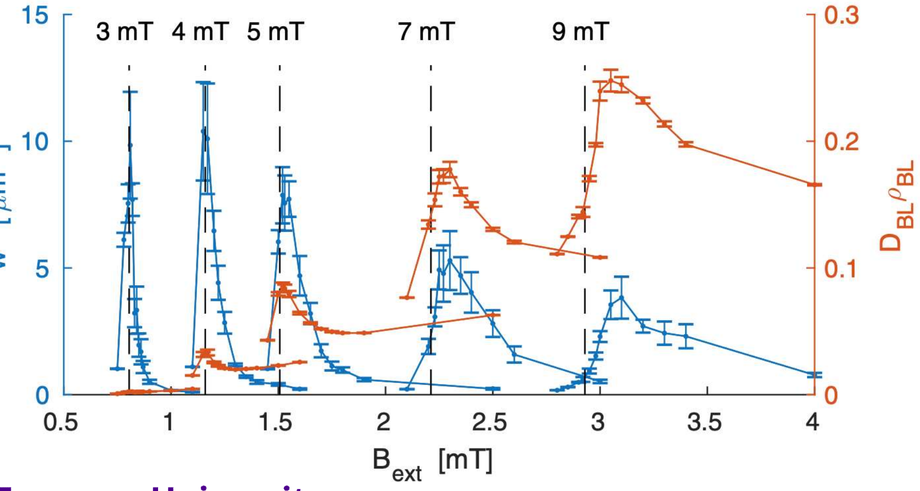
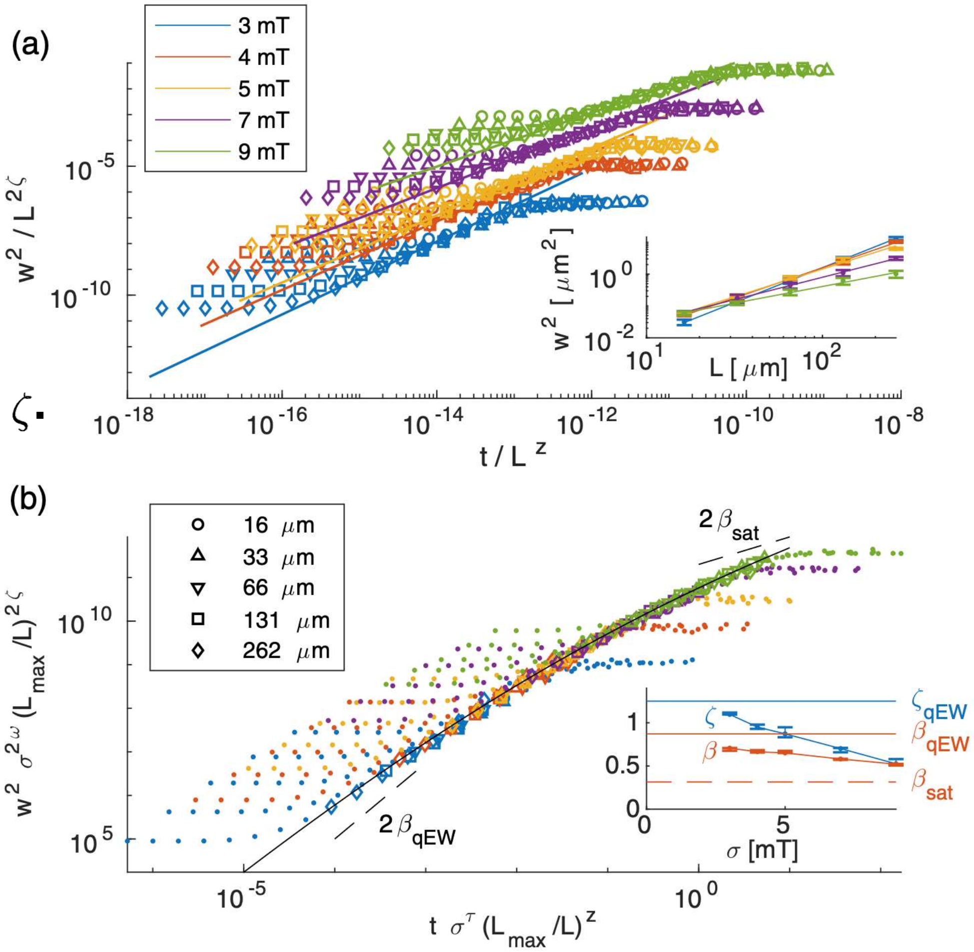
DW depinning transition: disorder-dependent exponents

DW roughness:

- **Saturated w^2 displays a peak close to $B_c(\sigma)$, with the peak height *decreasing* with σ.**
- **Roughness growth exhibits a crossover from $w \propto t^{\beta_{\rm qEW}}$ for early times to $w \propto t^{\beta_{\rm sat}}$ for late times.**
- **Saturated w^2 scales as $w^2 \propto L^{2\zeta}$ with a σ-dependent ζ.**





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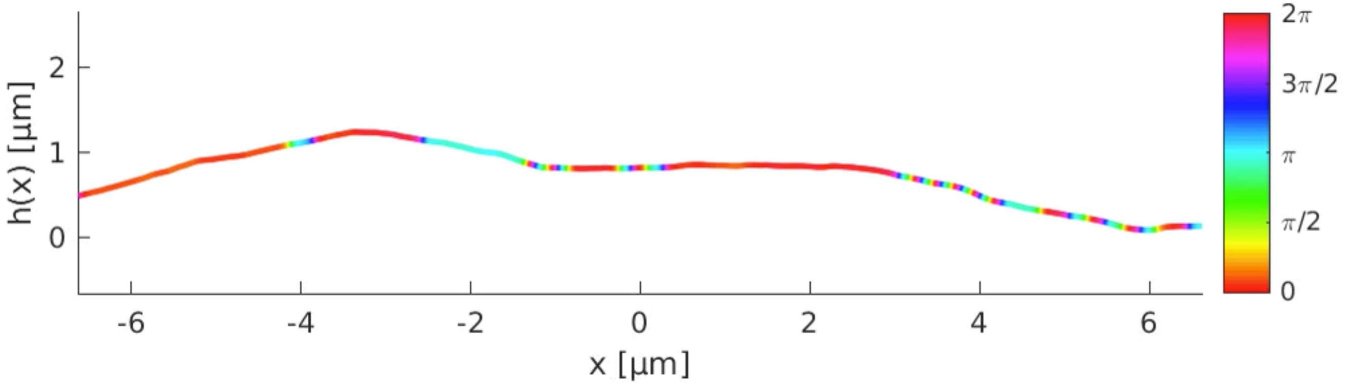
Summary & conclusions

- Barkhausen noise and DW depinning transition simulated in disordered PMA thin films.
- Bloch lines nucleate, propagate and annihilate during DW dynamics.
- For Barkhausen noise, we observe internal crackling dynamics within the DW due to intermittent Bloch line activity (micromagnetic simulations).
- The DW depinning transition seems to exhibit disorder-dependent exponents, and a disorder-dependent density of Bloch lines within the DW (simulations of the reduced DW model).
- Next: Barkhausen noise in the reduced model—disorder-dependent avalanche exponents?
- Possible extensions of the reduced DW model: Higher dimensional systems, effects due to Dzyaloshinskii-Moriya interaction, etc.



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Thank you for your attention!

Papers:

T. Herranen, and LL, *Barkhausen Noise from Precessional Domain Wall Motion*, Phys. Rev. Lett. 122, 117205 (2019).

A. Skaugen, and LL, *Depinning exponents of thin film domain walls depend on disorder strength*, A. Skaugen and LL, Phys. Rev. Lett. 128, 097202 (2022).

PHYSICAL REVIEW LETTERS **122,** 117205 (2019)

Barkhausen Noise from Precessional Domain Wall Motion

Touko Herranen[1] and Lasse Laurson[1,2,*]
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(Received 8 June 2018; revised manuscript received 28 January 2019; published 21 March 2019)

The jerky dynamics of domain walls driven by applied magnetic fields in disordered ferromagnets—the Barkhausen effect—is a paradigmatic example of crackling noise. We study Barkhausen noise in disordered Pt/Co/Pt thin films due to precessional motion of domain walls using full micromagnetic simulations, allowing for a detailed description of the domain wall internal structure. In this regime the domain walls contain topological defects known as Bloch lines which repeatedly nucleate, propagate, and annihilate within the domain wall during the Barkhausen jumps. In addition to bursts of domain wall propagation, the in-plane Bloch line dynamics within the domain wall exhibits crackling noise and constitutes the majority of the overall spin rotation activity.

DOI: 10.1103/PhysRevLett.122.117205

Understanding the bursty crackling noise response of elastic objects in random media—domain walls (DWs) [1], cracks [2], fluids fronts invading porous media [3], etc.—to since the 1970s [13]. Their role in the physics of the Barkhausen effect needs to be studied. The typical models of Barkhausen noise, such as elastic interfaces in random

PHYSICAL REVIEW LETTERS **128,** 097202 (2022)

Depinning Exponents of Thin Film Domain Walls Depend on Disorder Strength

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(Received 4 February 2021; accepted 8 February 2022; published 3 March 2022)

Domain wall dynamics in ferromagnets is complicated by internal degrees of freedom of the domain walls. We develop a model of domain walls in disordered thin films with perpendicular magnetic anisotropy capturing such features, and use it to study the depinning transition. For weak disorder, excitations of the internal magnetization are rare, and the depinning transition takes on exponent values of the quenched Edwards-Wilkinson equation. Stronger disorder results in disorder-dependent exponents concurrently with nucleation of an increasing density of Bloch lines within the domain wall.

DOI: 10.1103/PhysRevLett.128.097202

Domain walls (DWs) driven by applied magnetic fields in disordered ferromagnets constitute a paradigmatic system exhibiting a depinning transition between pinned and moving phases at vanishing temperatures T [1–4] as well as slow thermally activated creep motion for finite T [5]. Such effects were recently studied by full micromagnetic simulations of Barkhausen noise [21]. However, micromagnetic simulations describing the magnetization dynamics everywhere in the system are limited to small system sizes, resulting in significant finite size effects.