What is the correct theory for avalanches?

Kay Wiese

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with Cathelijne ter Burg, Gauthier Mukerjee

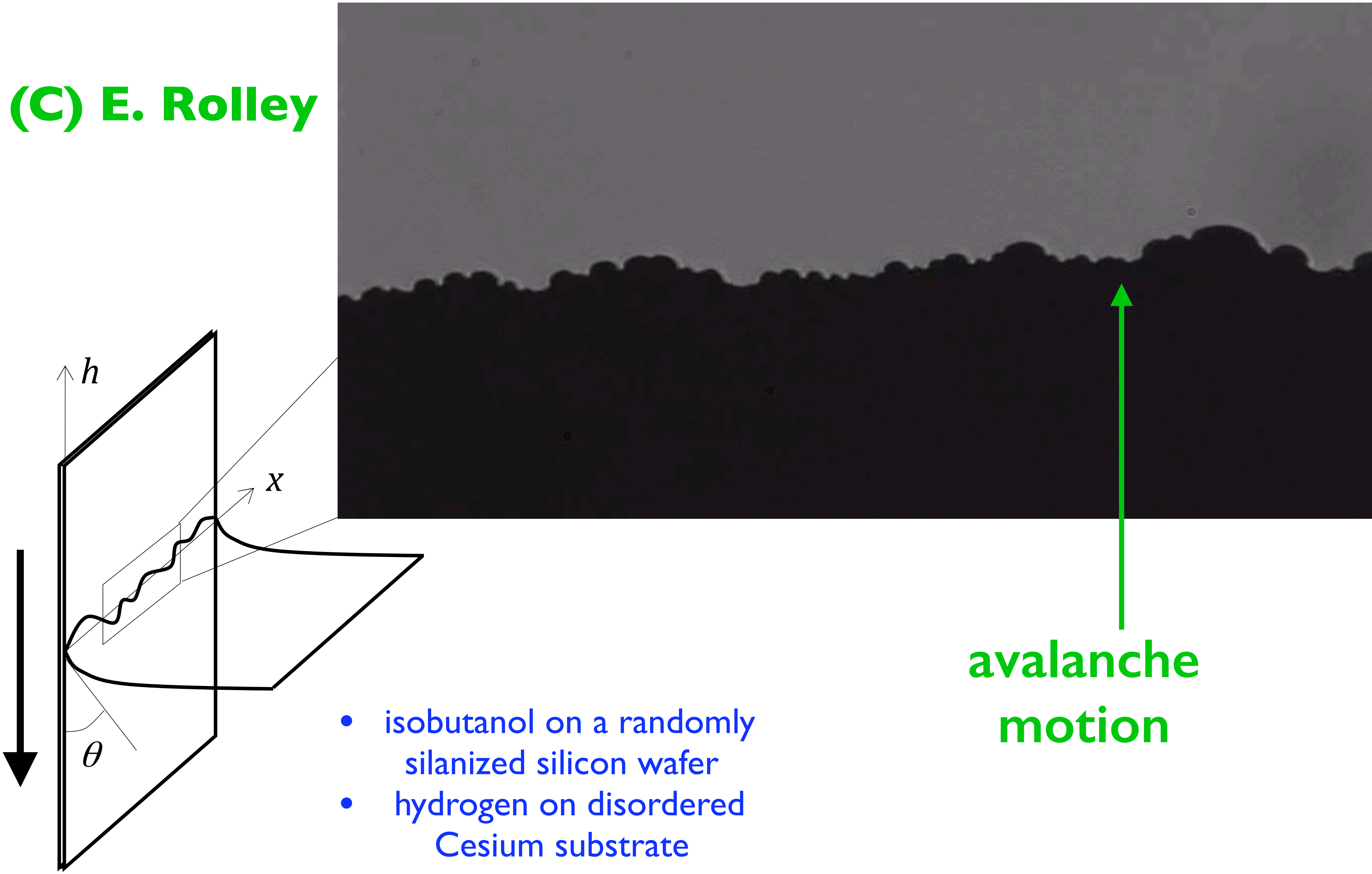
Debrecen, August 2022

http://www.phys.ens.fr/~wiese/

Review: arXiv:2102.01215

Contact line wetting

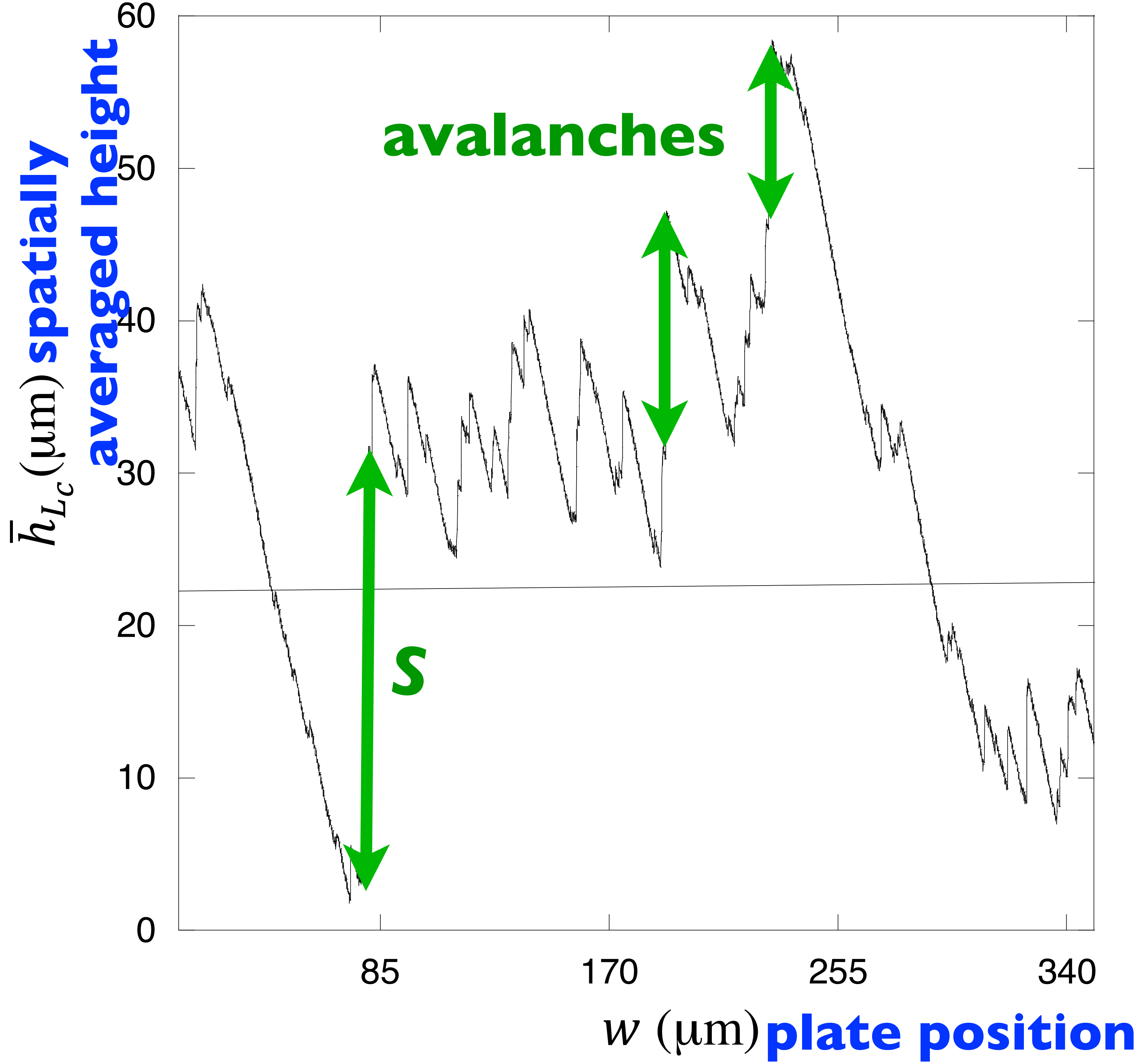
(C) E. Rolley



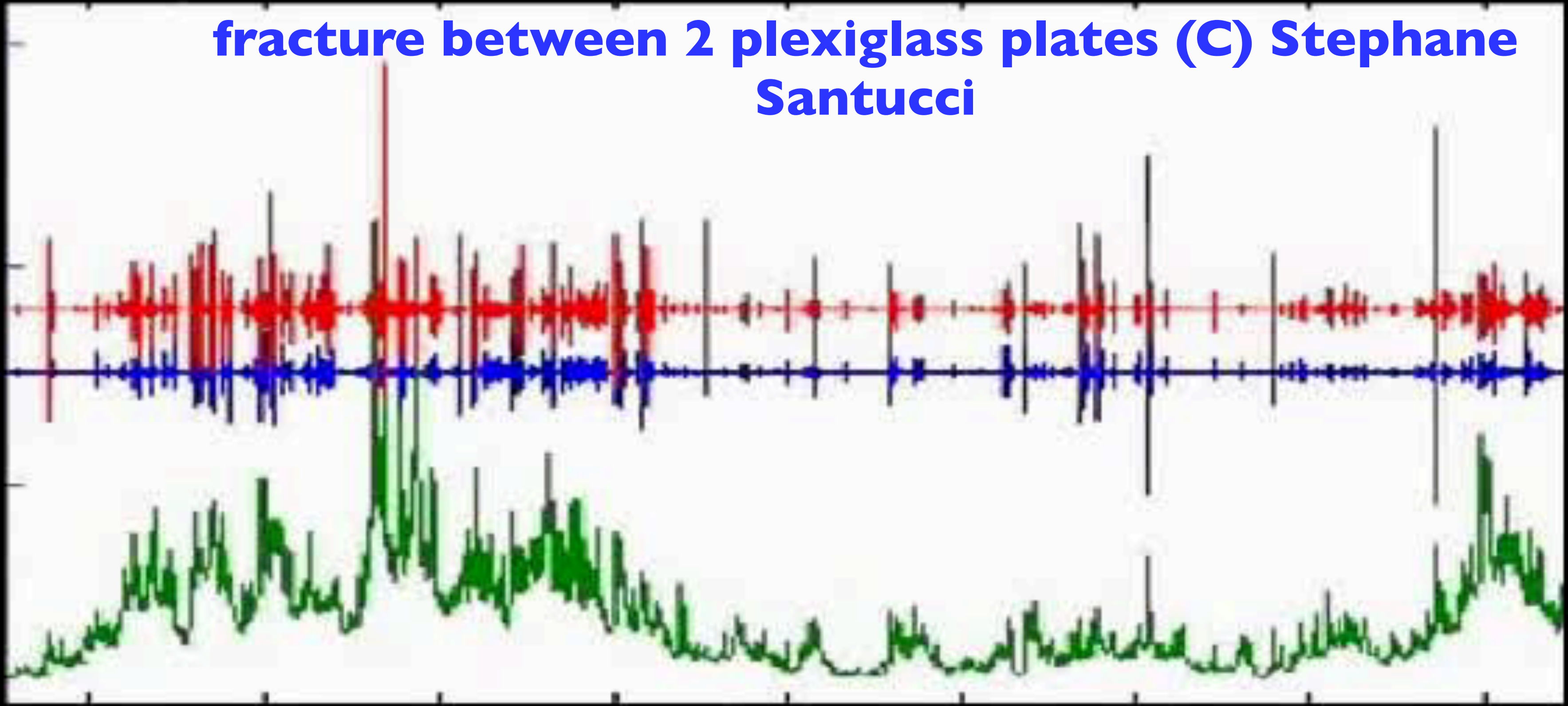
avalanche motion

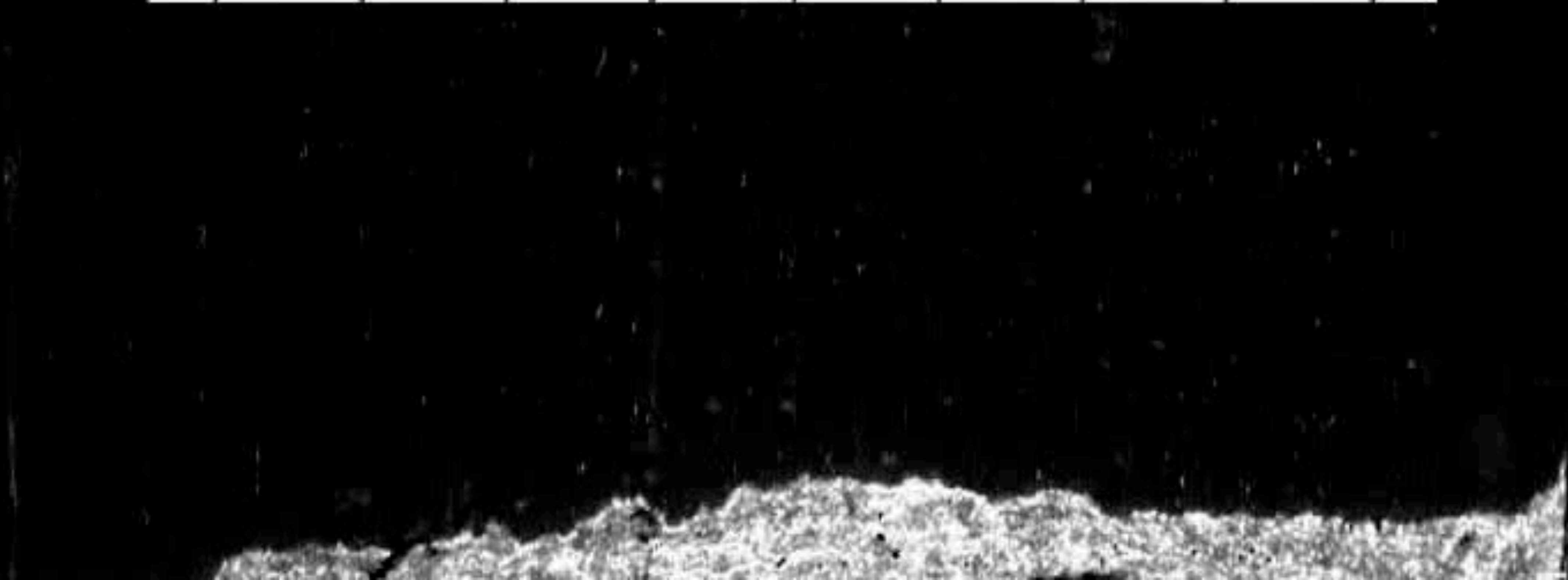
- isobutanol on a randomly silanized silicon wafer
- hydrogen on disordered Cesium substrate

height jumps = avalanches

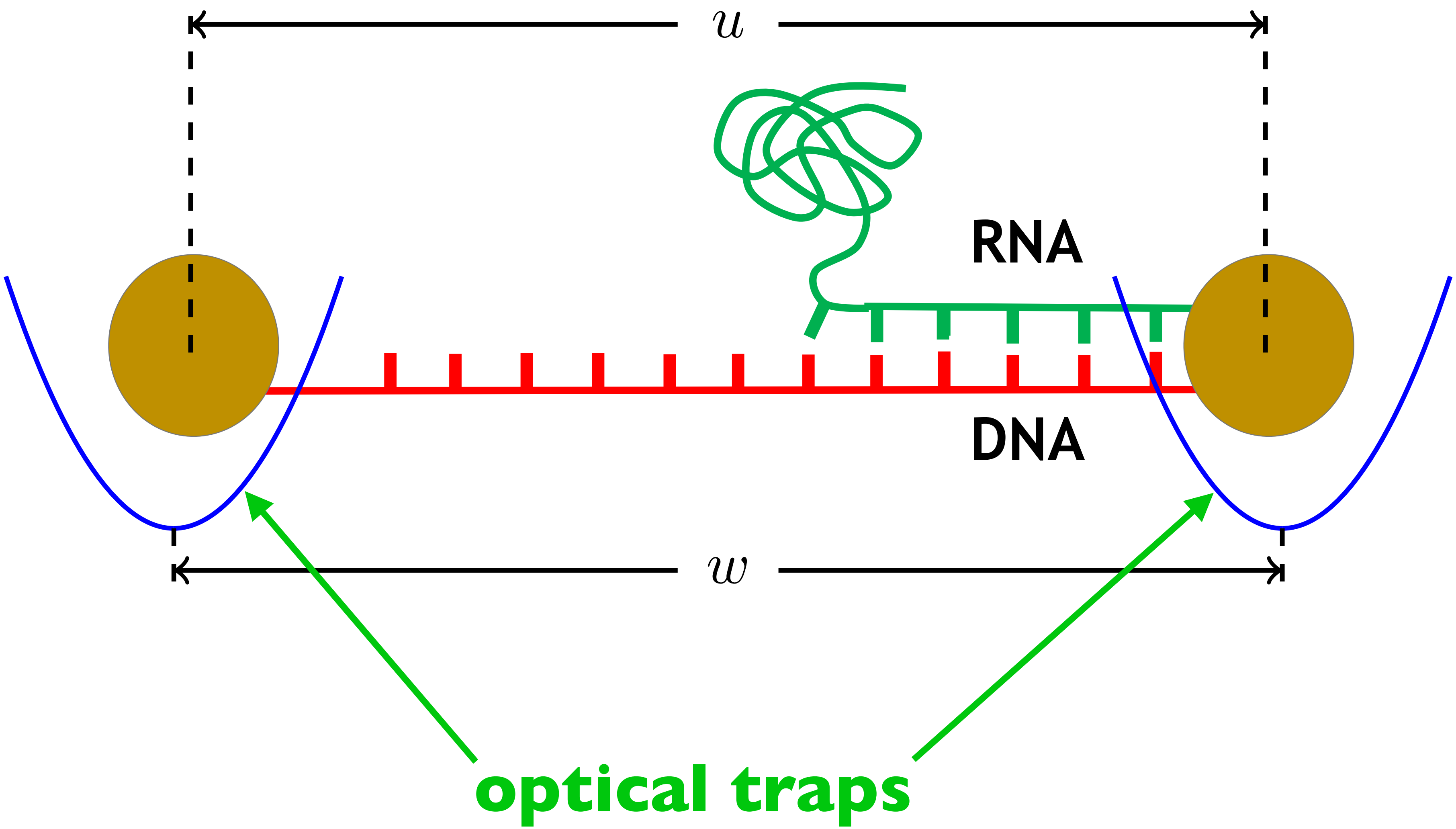


fracture between 2 plexiglass plates (C) Stephane Santucci

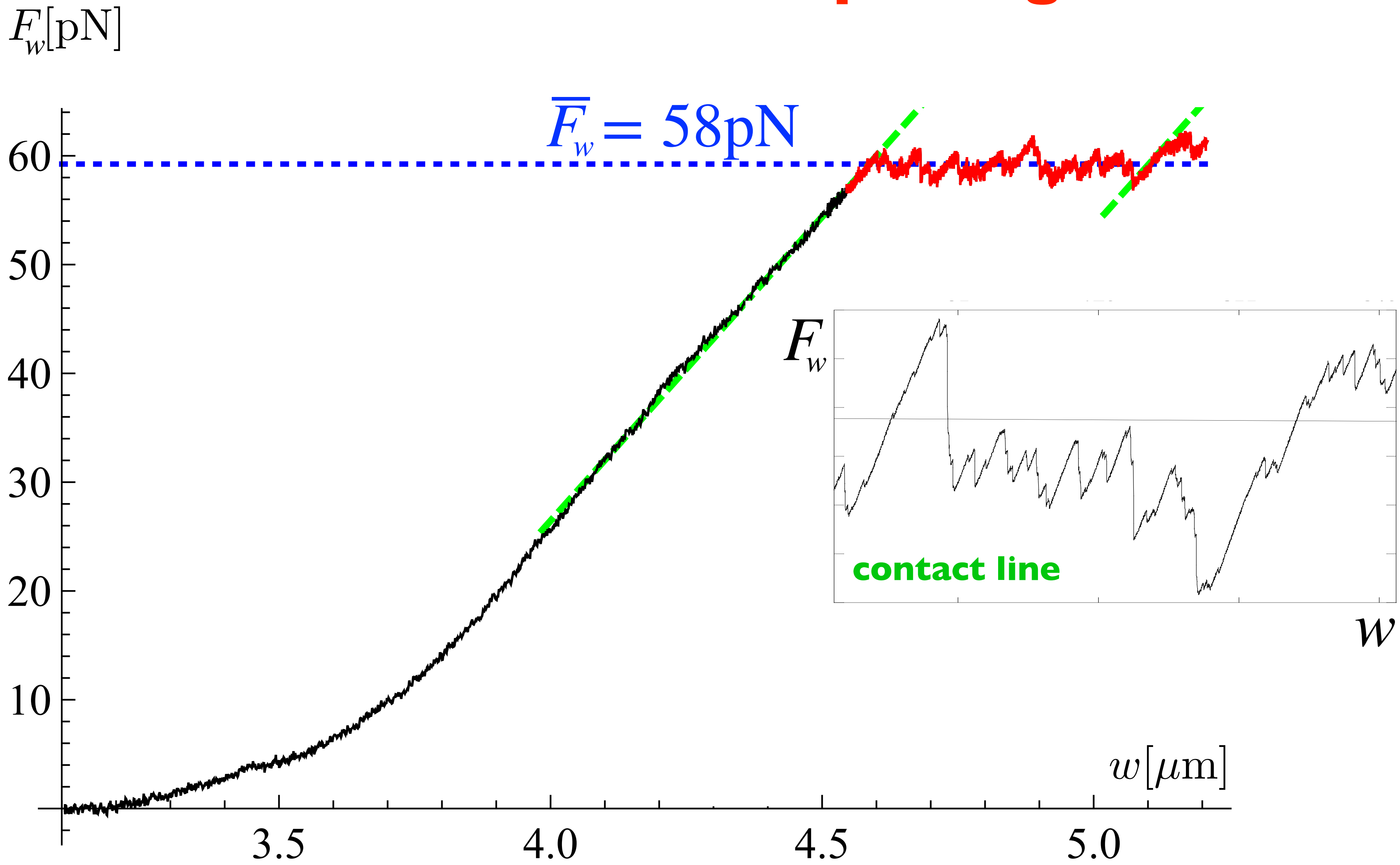




Peeling of an RNA/DNA double helix

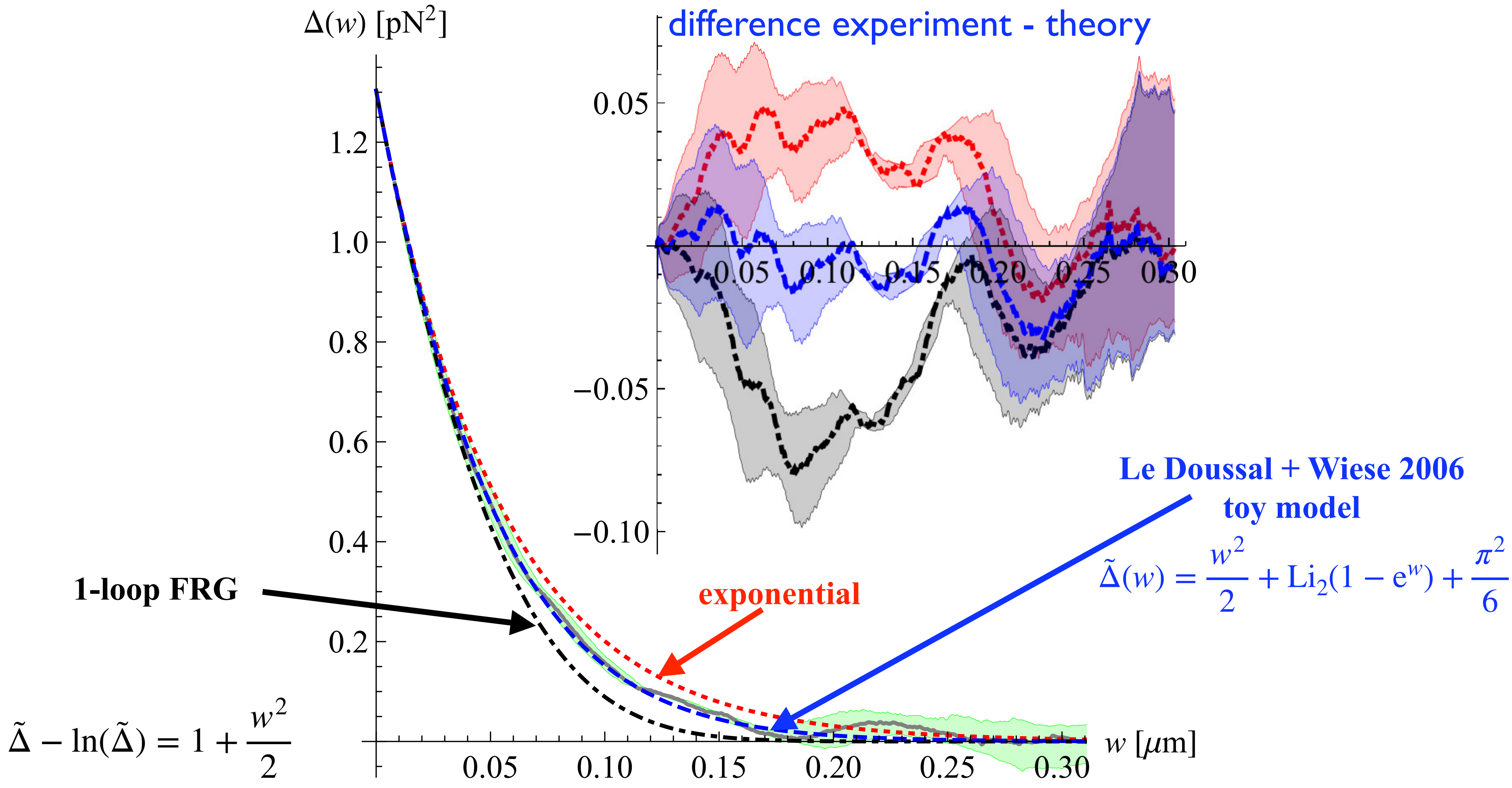


Force as a function of distance for RNA/DNA peeling



Force-force correlations

$$\Delta(w - w') := \overline{F_w F_{w'}}^{c} \equiv \overline{F_w F_{w'}} - \overline{F_w}\ \overline{F_{w'}}$$



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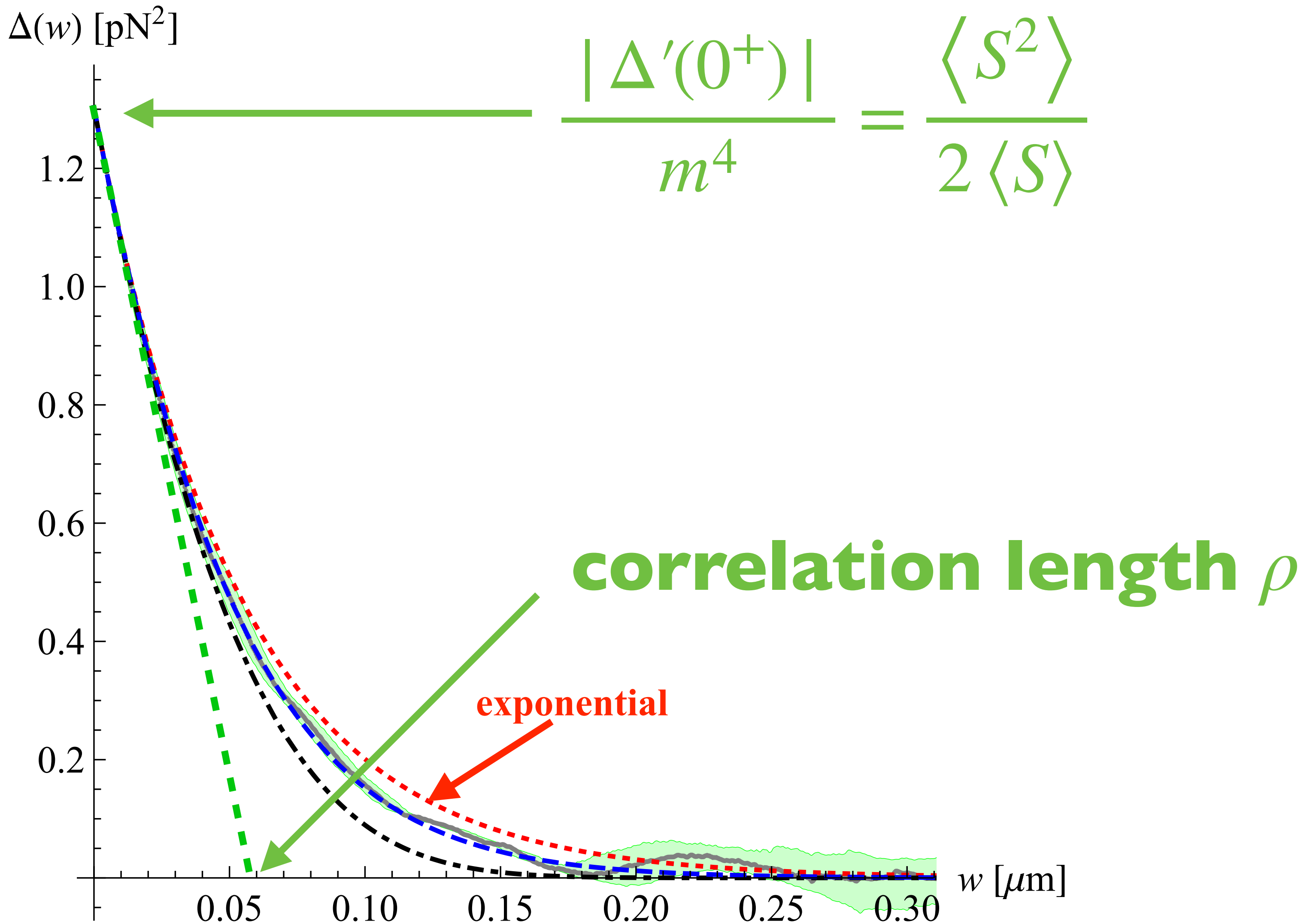
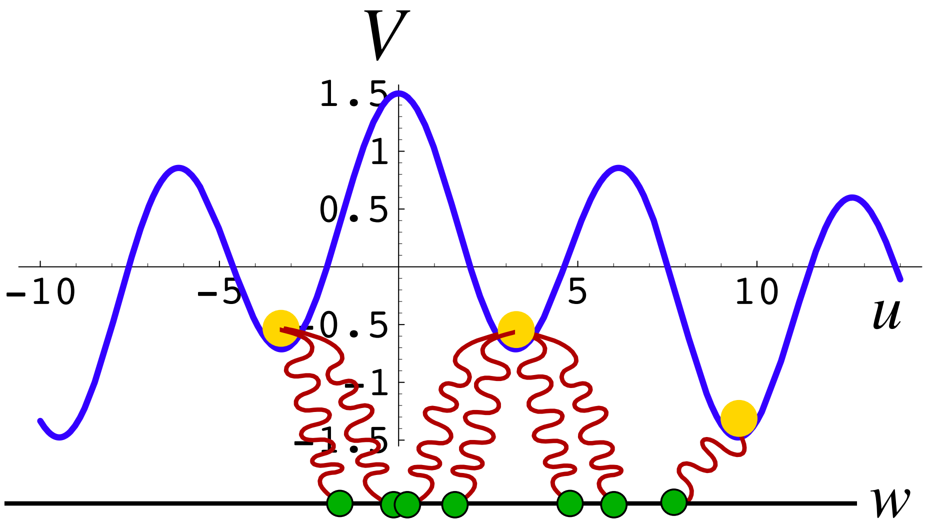
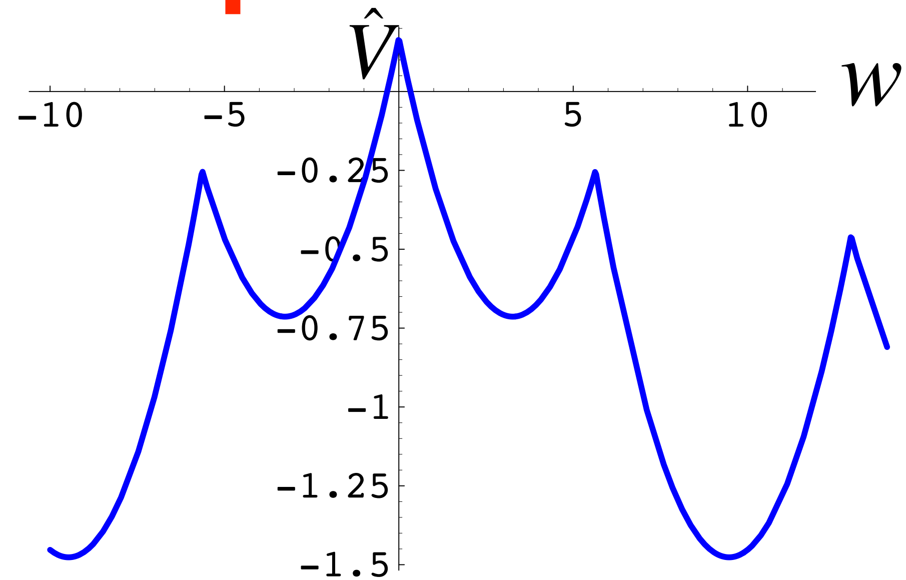
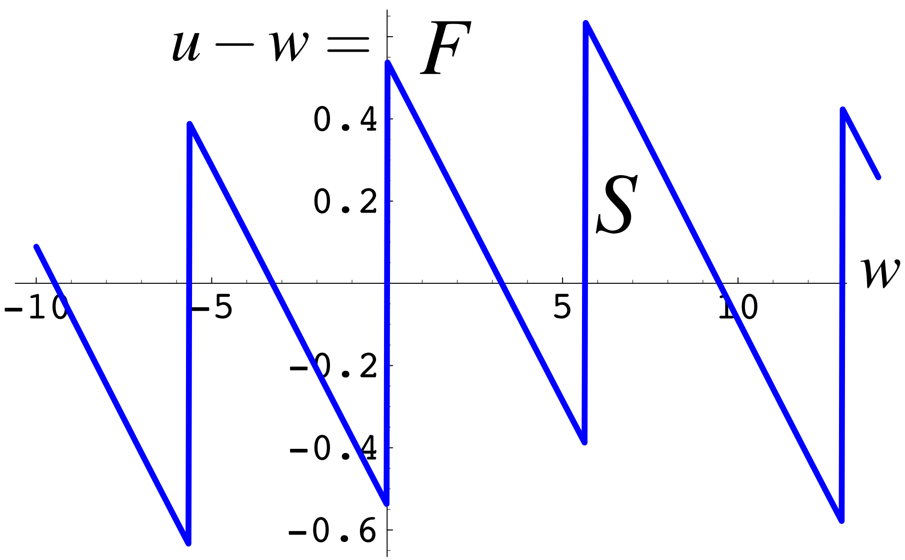


Reminder on the cusp



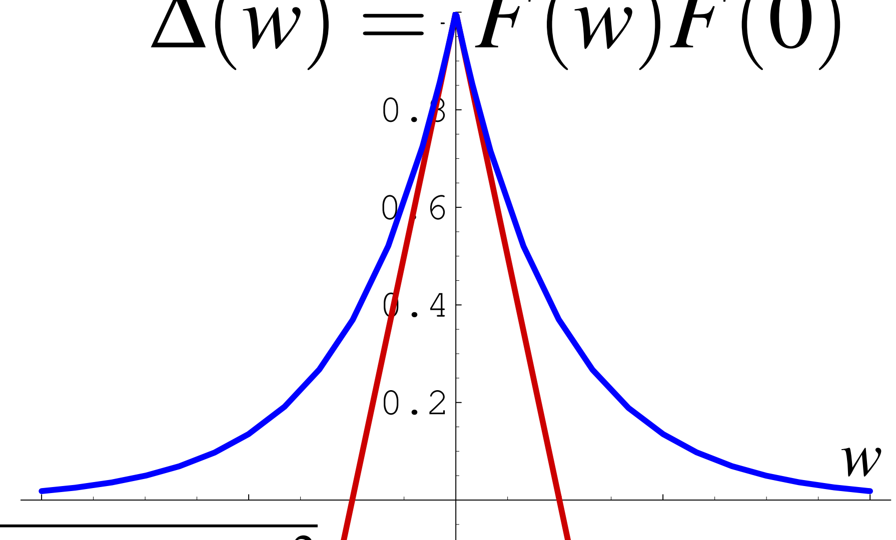
Min $\longrightarrow$





average $\longrightarrow$

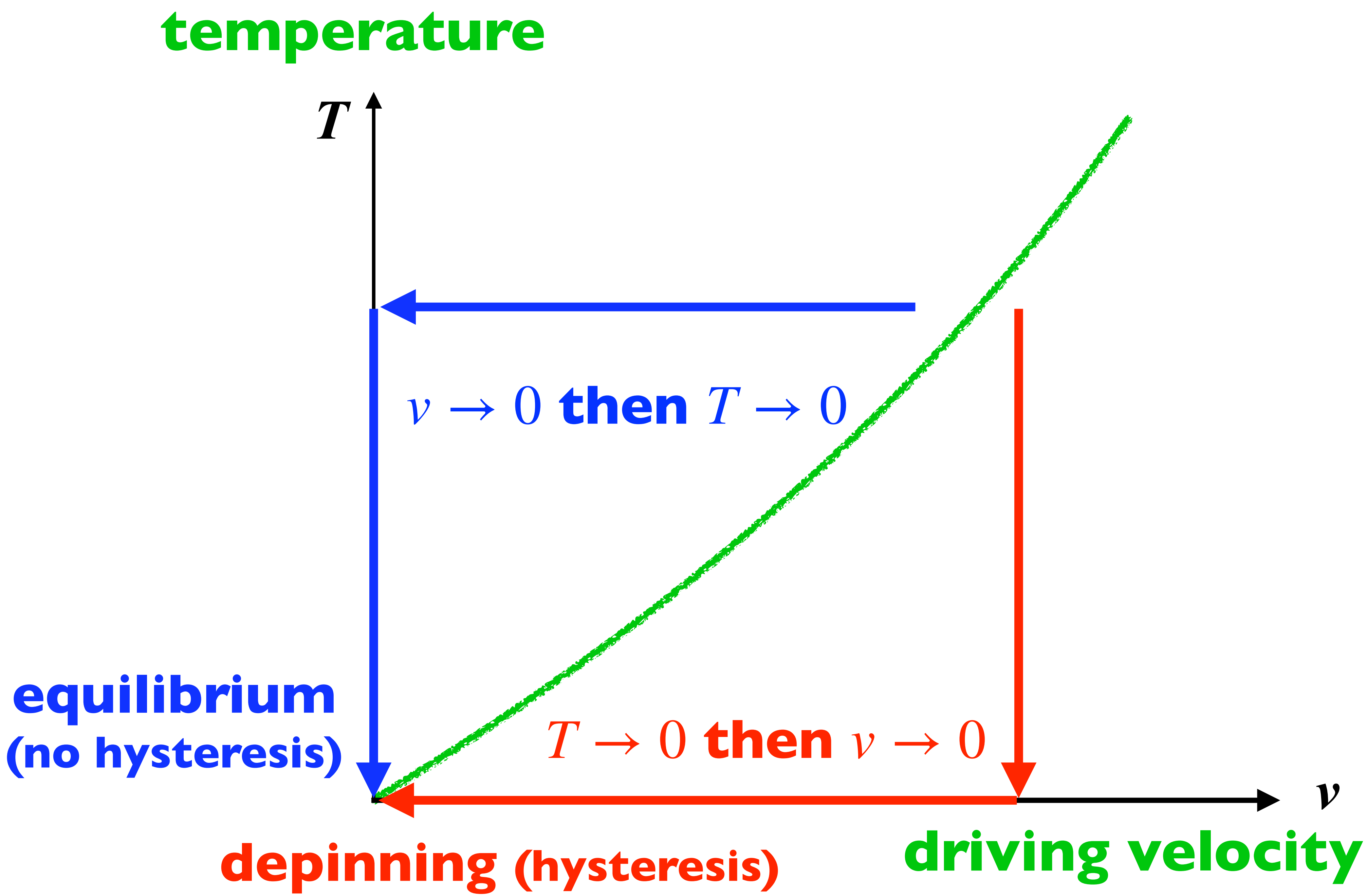
$$\Delta(w) = \overline{F(w)F(0)}^{\,c}$$



$$\overline{[F(w) - F(0)]^2} = \underbrace{\rho_{\rm shock}|w|}_{p_{\rm shock}}\langle S^2\rangle$$

$$-\Delta'(0^+) = \frac{\langle S^2\rangle}{2\langle S\rangle} \quad \Longleftarrow \quad \rho_{\rm shock} = \langle S\rangle^{-1}$$

$T > 0$ and $v > 0$: Equilibrium or Depinning ?



Field theory background

Equation of motion (for SR elasticity for simplicity)

height of the interface

$w = vt$

$$\partial_t u(x,t) = \nabla^2 u(x,t) + m^2[w - u(x,t)] + F(x, u(x,t))$$

Forces are drawn from a **Gaussian**, and have correlations

$$\overline{F(x,u)F(x',u')}^{\mathrm{c}} = \delta^d(x-x')\Delta(u-u')$$

Field theory (MSR=classical limit $\hbar \to 0$ of Keldysh)

$$\mathcal{S}[\tilde{u}, u] = \int_{x,t} \tilde{u}(x,t)\left[\partial_t u(x,t) - \nabla^2 u(x,t) + m^2\Big(u(x,t) - w\Big)\right]$$

$$-\frac{1}{2}\int_{x,t,t'} \tilde{u}(x,t)\tilde{u}(x,t')\Delta\Big(u(x,t) - u(x,t')\Big)$$

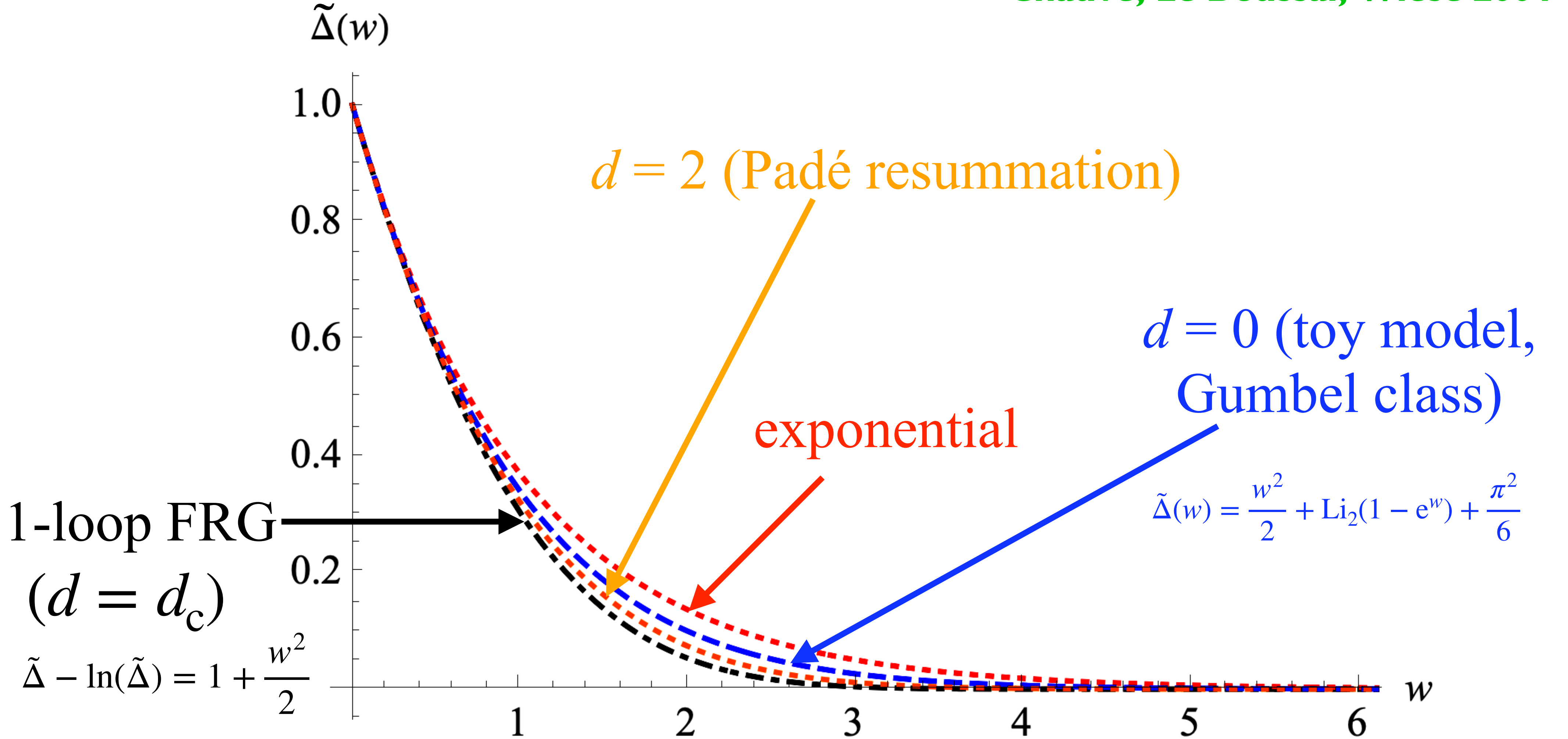
renormalize

Renormalization of disorder

$$-\frac{m\mathrm{d}}{\mathrm{d}m}\tilde{\Delta}(w) = (\epsilon - 2\zeta)\tilde{\Delta}(w) + \zeta w \tilde{\Delta}'(w) - \frac{1}{2}\partial_w^2\left[\tilde{\Delta}(w) - \tilde{\Delta}(0)\right]^2$$
$$+\frac{1}{2}\partial_w^2\left\{\left[\tilde{\Delta}(w) - \tilde{\Delta}(0)\right]\tilde{\Delta}'(w)^2 + \tilde{\Delta}'(0^+)^2\tilde{\Delta}(u)\right\}$$

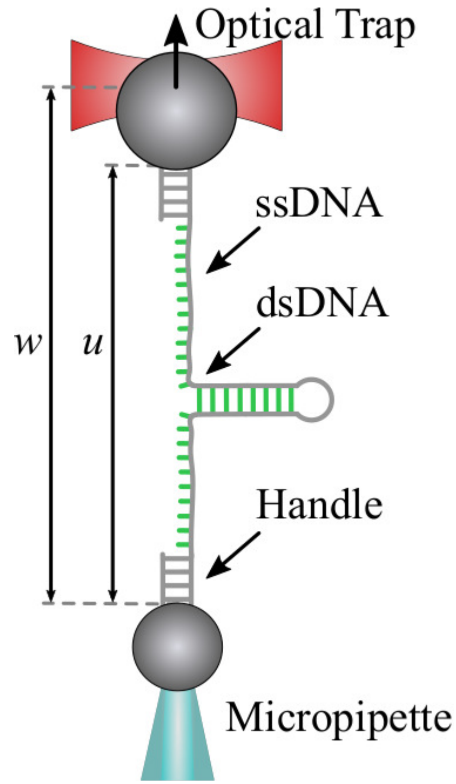
FRG

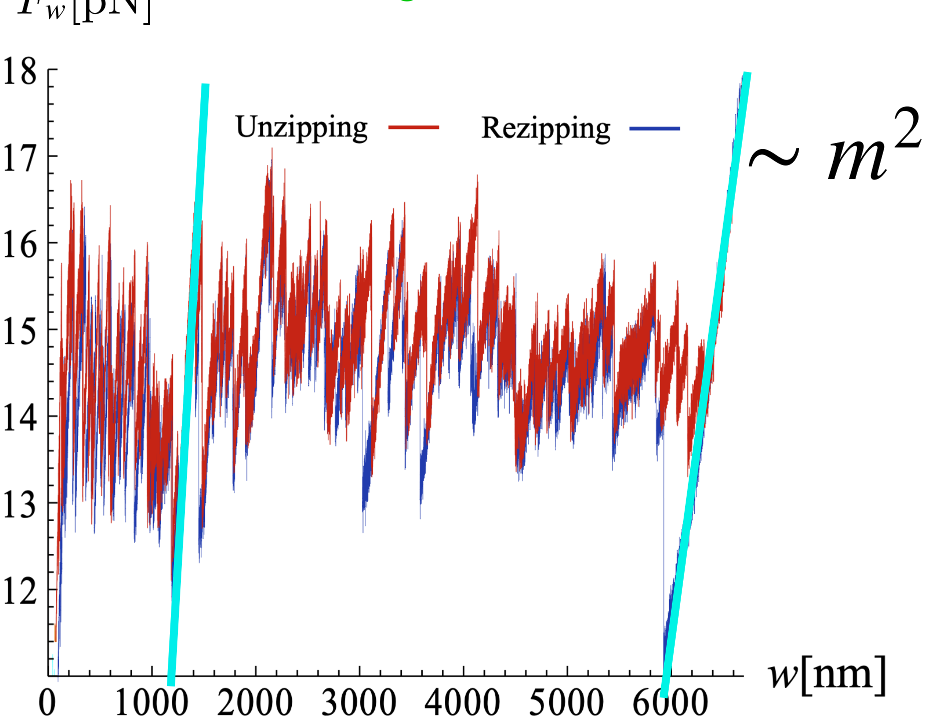
Chauve, Le Doussal, Wiese 2004

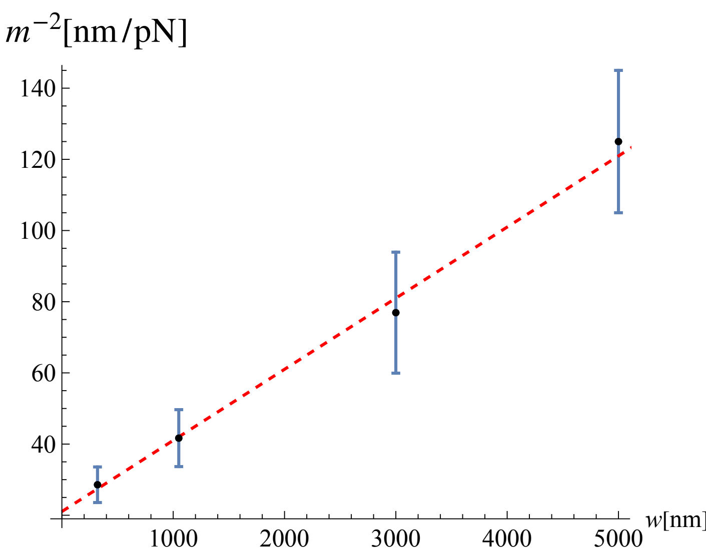


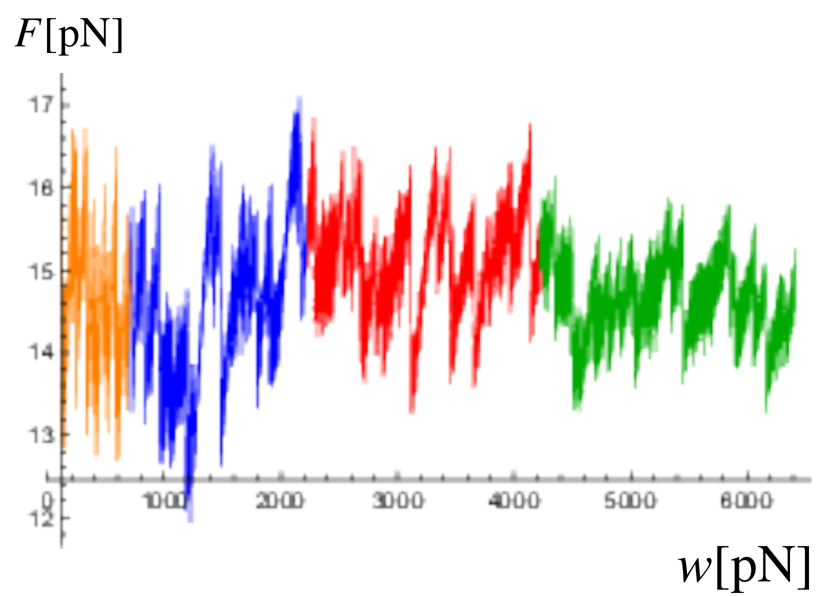
Renormalization in DNA-unzipping

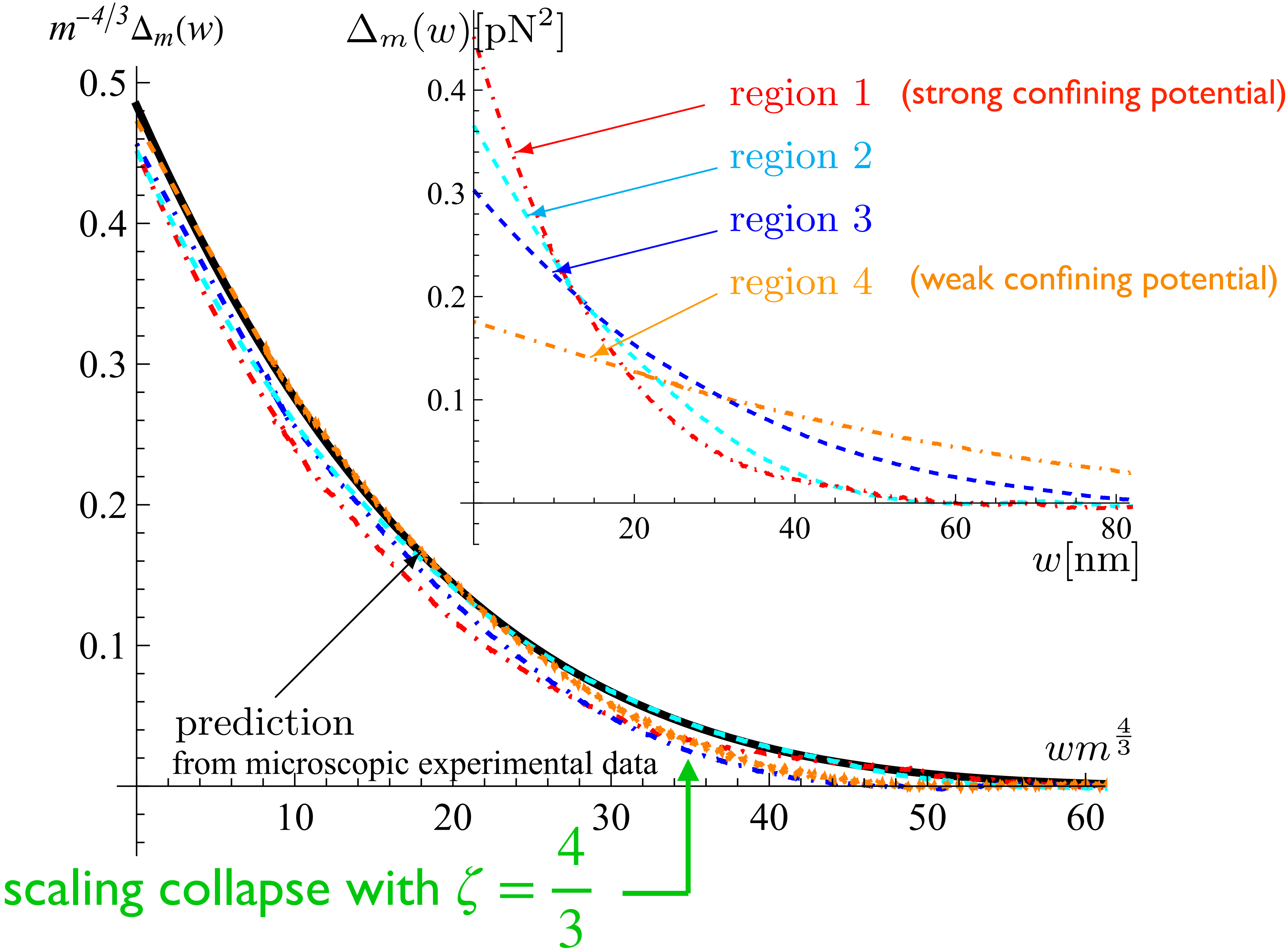
ter Burgh, Rissone, Ricco-Pastro, Ritort, Wiese 2022









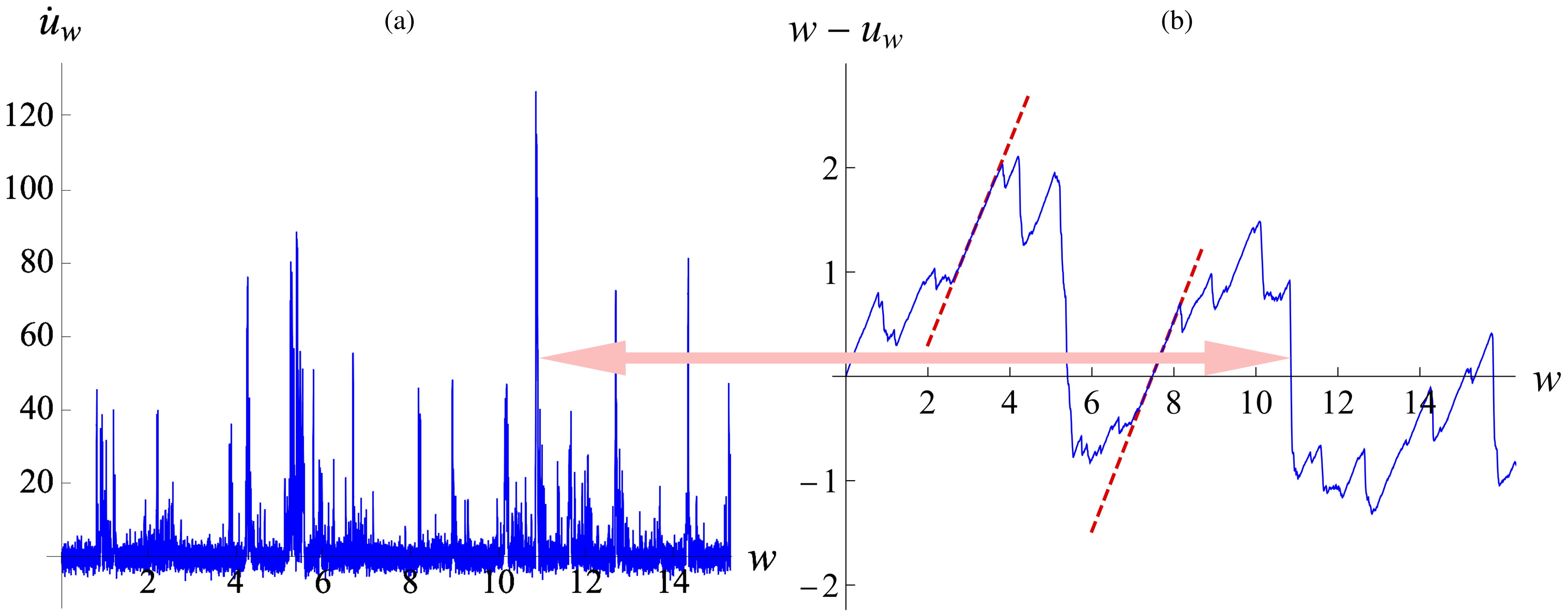


$m^{-4/3}\Delta_m(w)$
0.5
0.4
0.3
0.2
0.1
10
20
30
40
50
60
$wm^{\frac{4}{3}}$
prediction
from microscopic experimental data
$\Delta_m(w)[\mathrm{pN}^2]$
0.4
0.3
0.2
0.1
20
40
60
80
$w[\mathrm{nm}]$
region 1 (strong confining potential)
region 2
region 3
region 4 (weak confining potential)
scaling collapse with $\zeta = \frac{4}{3}$

Magnetic domain walls ($d = 2$)

(data by F. Bohn, G. Durin, R.L. Sommer)

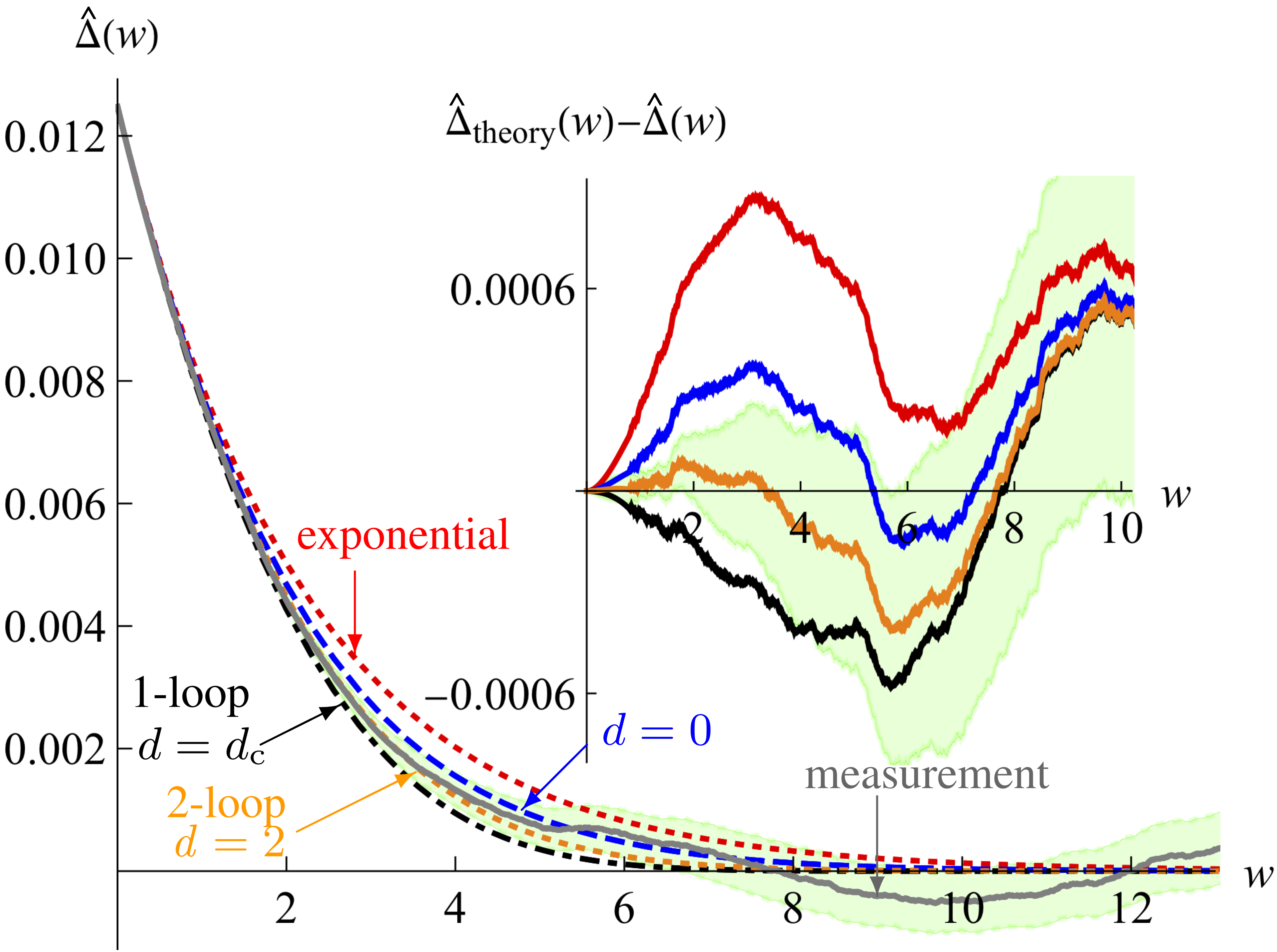
current in a pickup coil .... allows to construct :



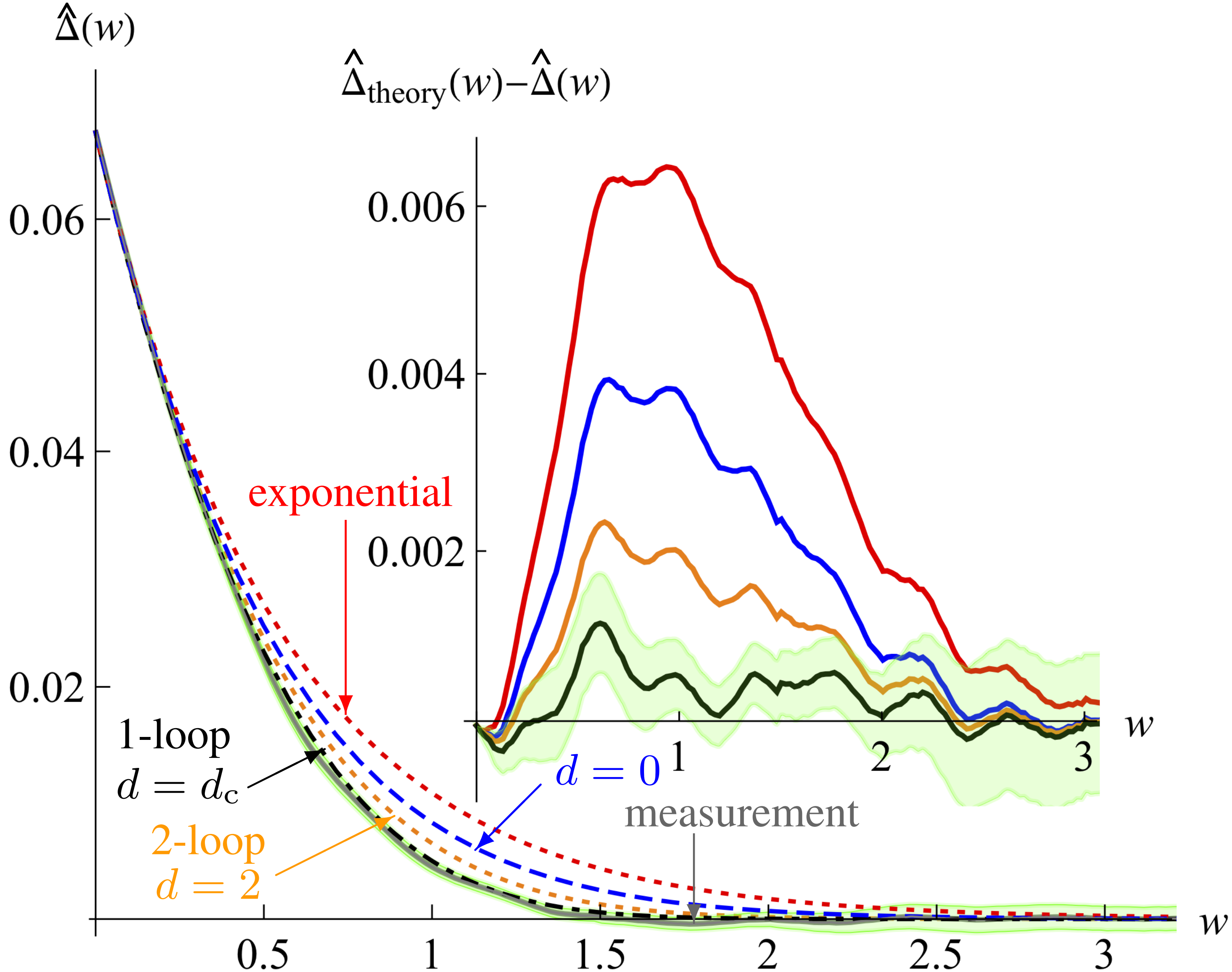
eliminate one unknown scale by the definition

$$\hat{\Delta}_v(w - w') := \overline{[w - u_w]\,[w' - u_{w'}]}^{\,c} = \frac{1}{m^4}\overline{F_w F_{w'}}^{\,c}$$

Magnetic domain walls SR elasticity $(d = 2)$



Magnetic domain walls ($d = 2$) with LR elasticity



- **1-loop FRG gives fixed point.**
- **this is not ABBM disorder:** $\Delta(0) - \Delta(w) \neq \sigma|w|$
- **ABBM only gives short-scale behavior correctly**

Correlations between avalanches

$$\frac{\langle S_{w_1} S_{w_2} \rangle}{\langle S \rangle^2} - 1 = -\hat{\Delta}''(w_1 - w_2)\,.$$

Thiery, Le Doussal, Wiese 2016

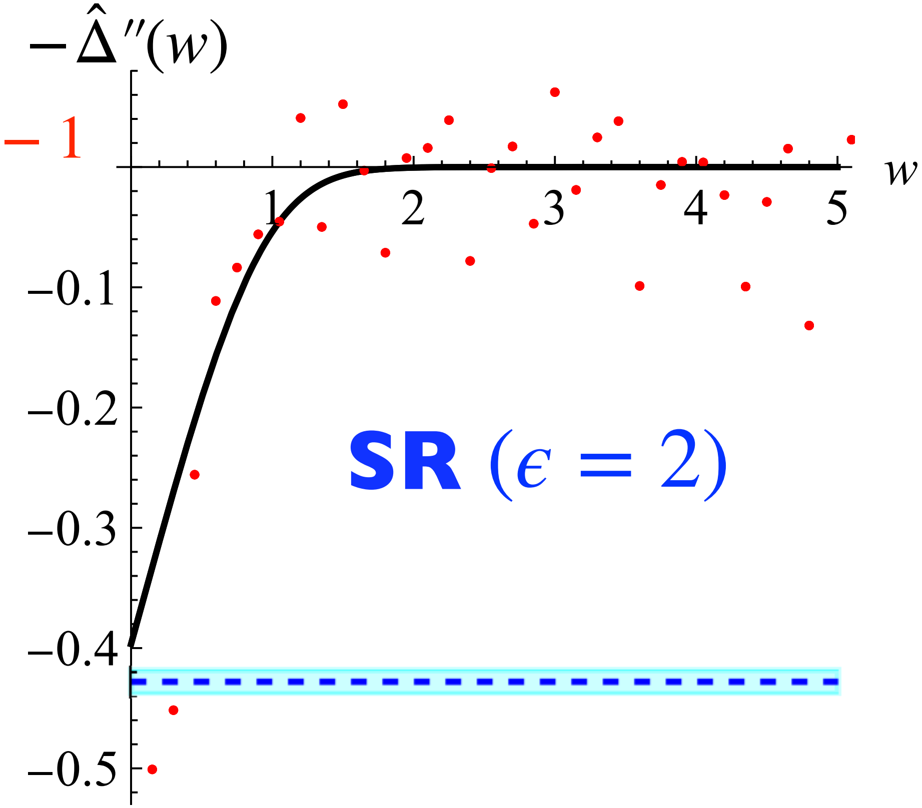
rescaled version of Δ

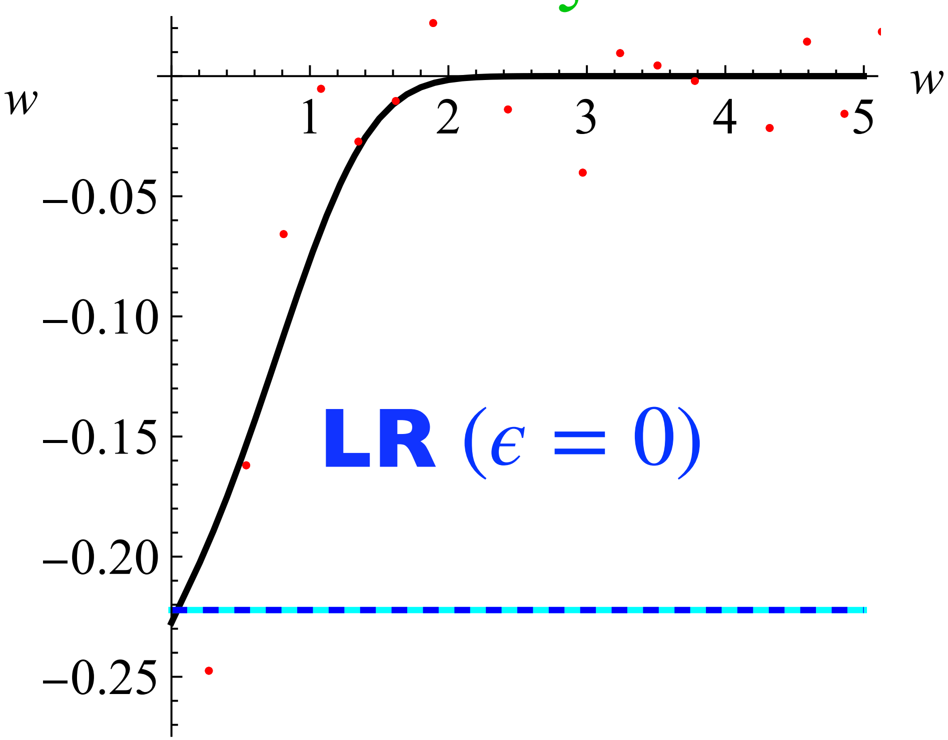
ϵ-expansion: $\hat{\Delta}''(0^+) \simeq \left(\frac{2}{9} + 0.107533\epsilon + \mathcal{O}(\epsilon^2) \right) \frac{1}{m^4 L^d I_1}$.

1-loop integral

Bound $m^4 L^d I_1 \geq 1$ is minimized in $d = 2$ for $mL \leq 2.4$.

ξ^{-1}





Experiments use optimal mL ! Effectively **one** domain wall!

Correlations between avalanches

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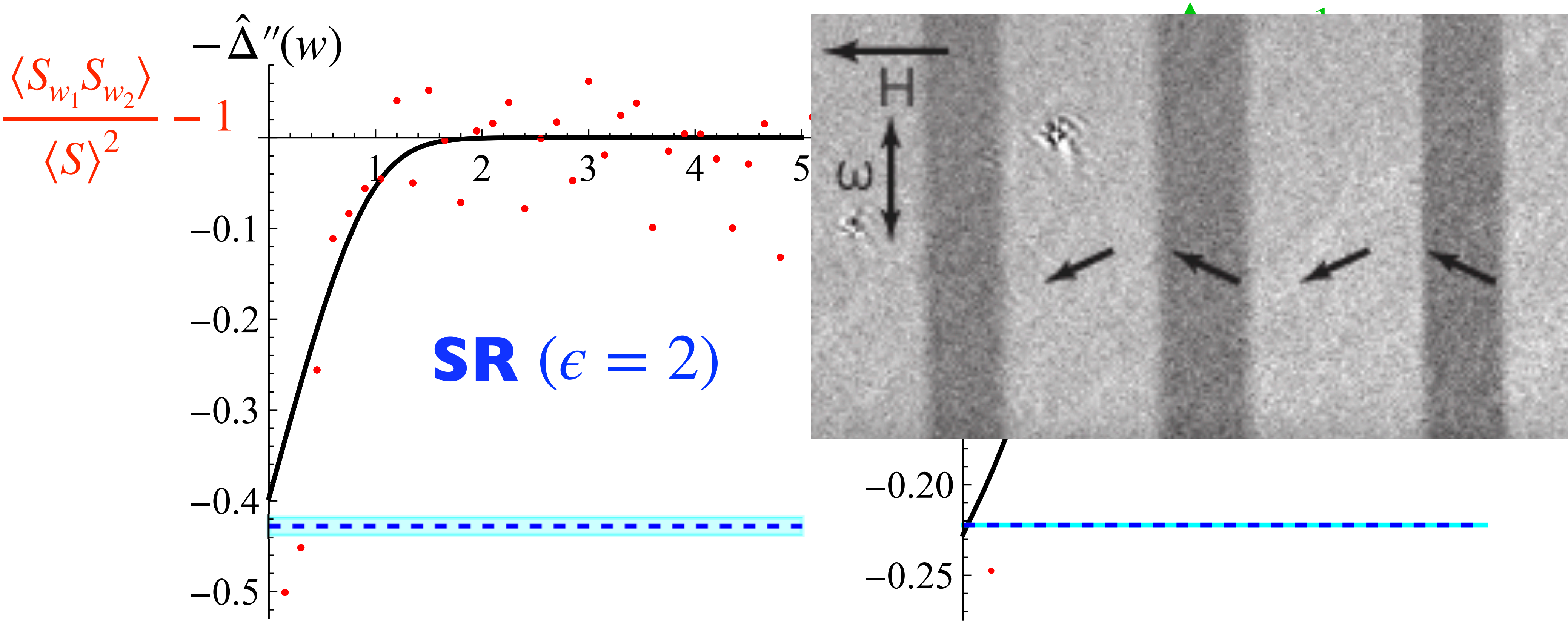

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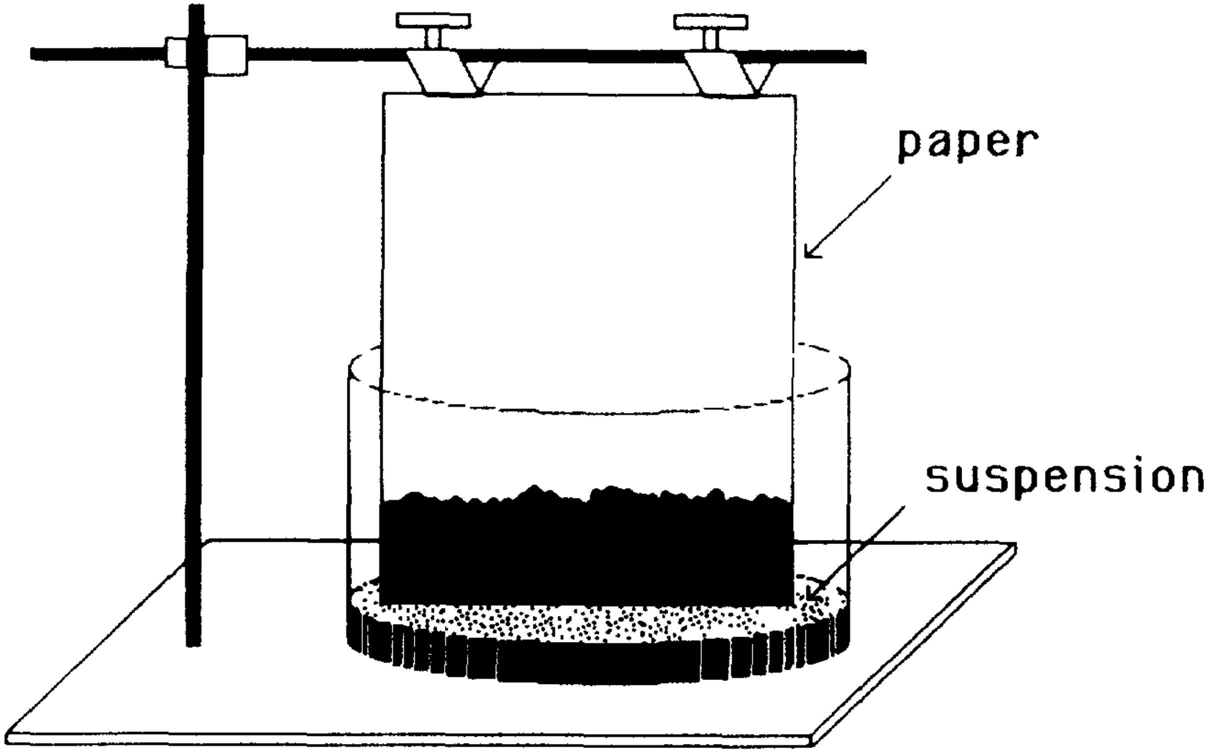
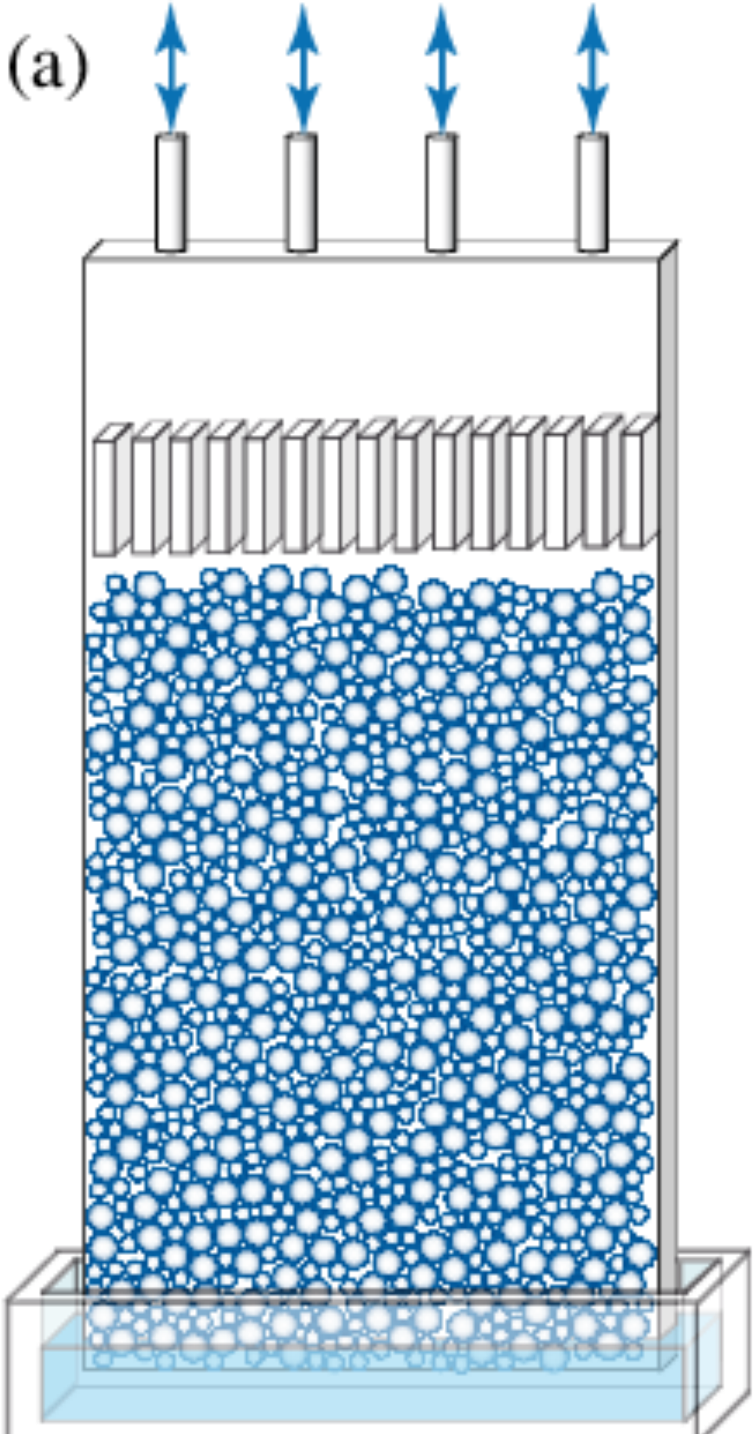
Imbibition (qKPZ)



S.V. Buldyrev, et al., Phys. Rev. A 45 (1992) R8313–16.



PRL 114, 234502 (2015) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2015

Experimental Evidence for Three Universality Classes for Reaction Fronts in Disordered Flows

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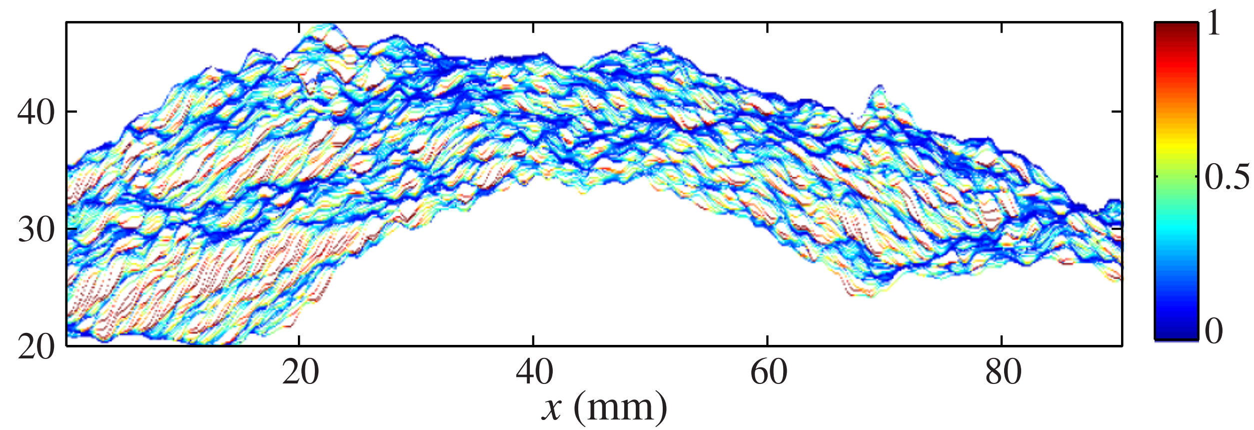
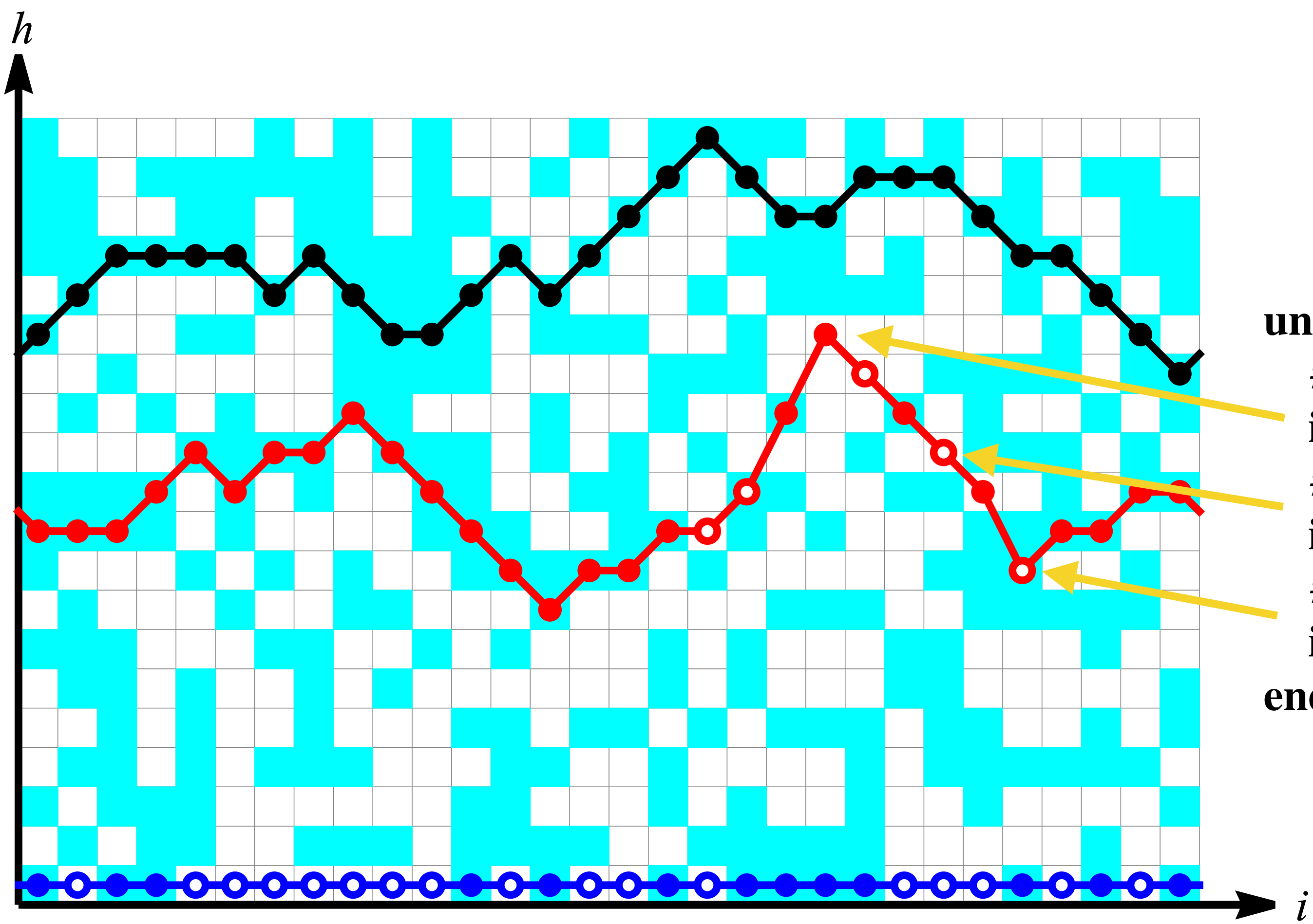
(Received 22 October 2014; published 11 June 2015)



Jordi Ortin's experiment

The Tang-Leschhorn cellular automaton of 1992
TL92



unstable(i)

 # links cannot be longer than 2

if $h(i) - h(\text{neighbor}) \geq 2$ **return** false

 # move forward if open

if $f\big(i, h(i)\big) > f_c$ **return** true

 # move forward if a neighbor is 2 ahead

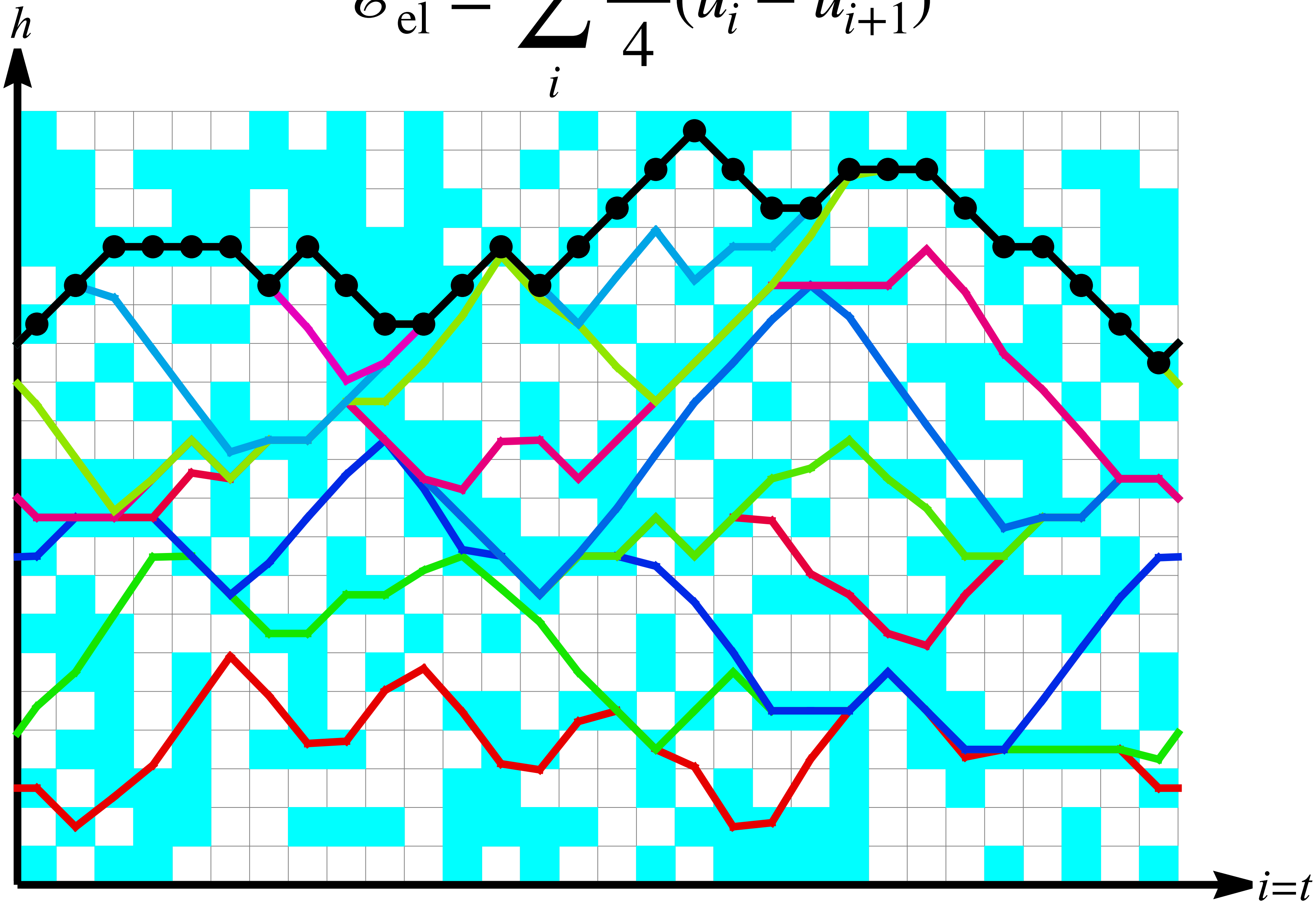
if $h(\text{neighbor}) - h(i) \geq 2$ **return** true

end

variants: Buldyrev, S. Havlin and H.E. Stanley1992

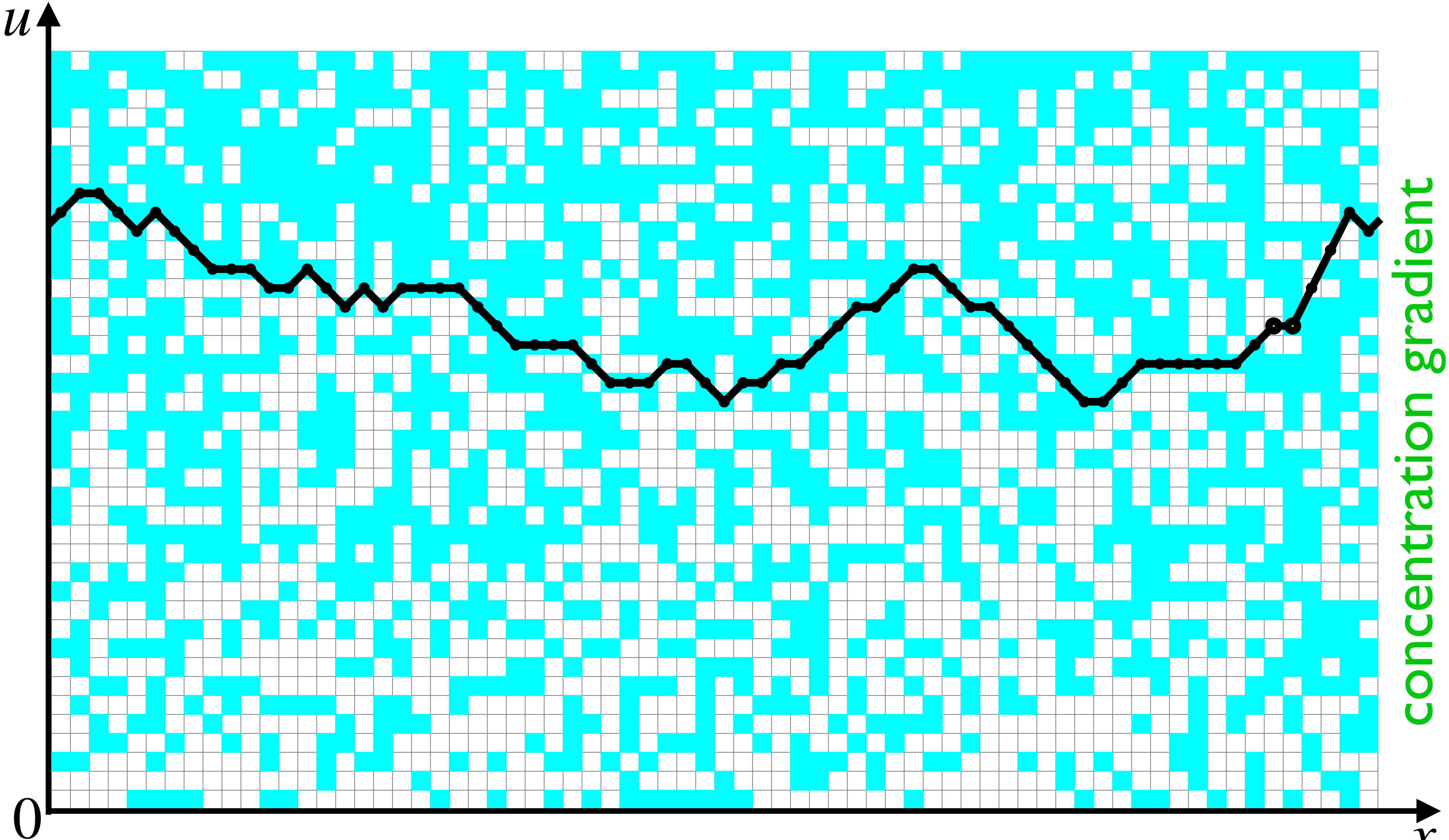
Anharmonic depinning = TL92

$$\mathcal{E}_{\rm el} = \sum_i \frac{c_4}{4}(u_i - u_{i+1})^4$$



anharmonic depinning respects the Middleton theorem
= return point memory (not guaranteed for qKPZ)

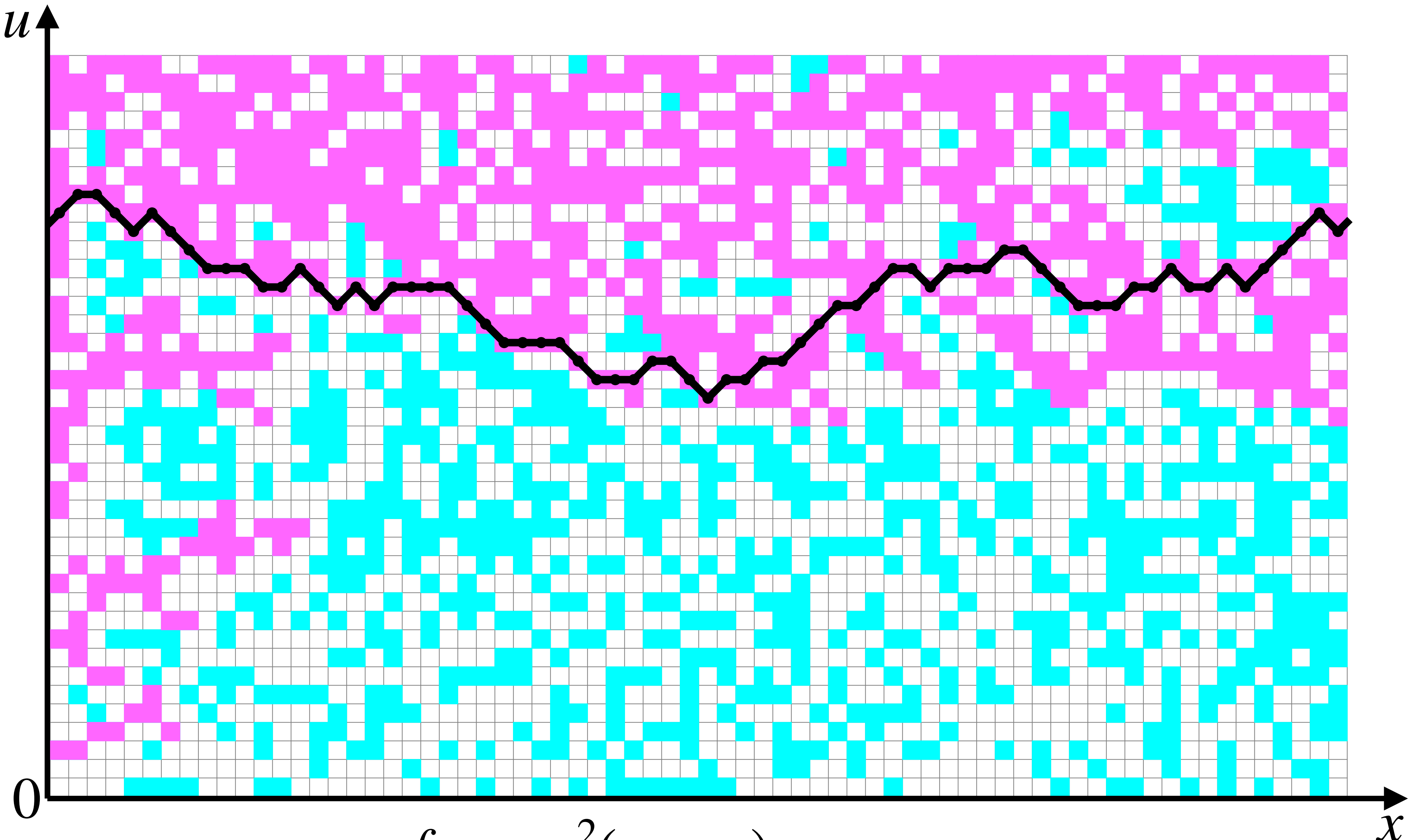
TL92 and directed percolation $(d = 1)$



$$f_c := m^2(u - w)$$

used for driving

TL92 and directed percolation $(d = 1)$

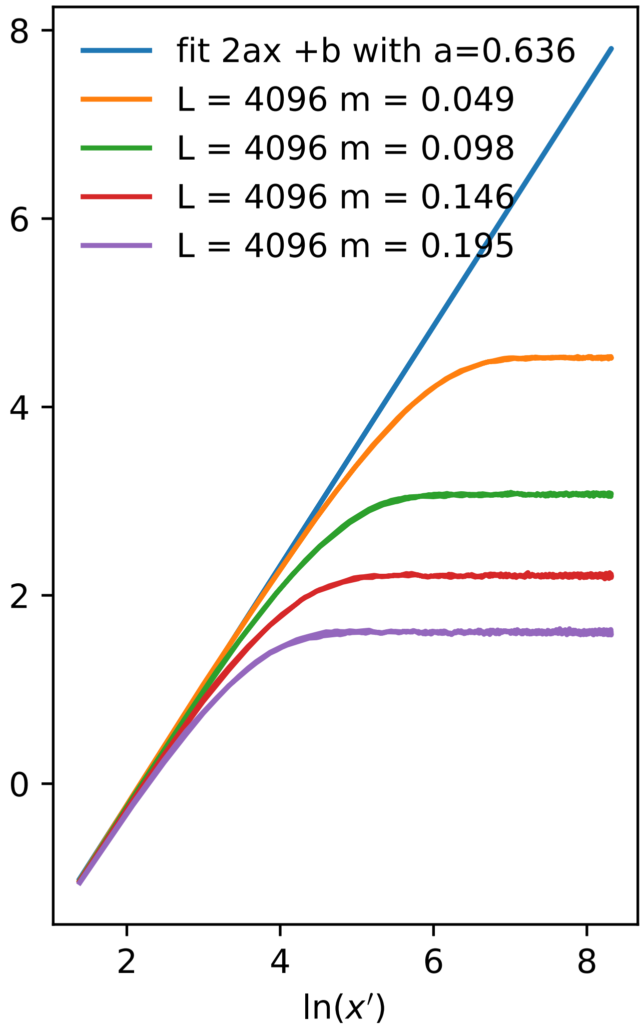


$$f_c := m^2(u - w)$$

used for driving

2-point function

$$\frac{1}{2}\overline{[u(x)-u(y)]^2} \sim \begin{cases} A\,|x-y|^{2\zeta}, & |x-y| < \xi \\ B\,m^{-2\zeta_m}, & |x-y| > \xi \end{cases}$$



from directed percolation

$$\zeta^{d=1} = \frac{\nu_\perp}{\nu_\parallel} = 0.632613(3)$$

$$\zeta_m^{d=1} = \frac{2\nu_\perp}{1+\nu_\perp} = 1.046190(4)$$

two distinct exponents in all d

$$\zeta_m > \zeta$$

Consequences (an example)

avalanche-size exponent different from qEW

$$P(S) \sim S^{-\tau} \quad \Longrightarrow \quad \tau = 2 - \frac{2}{d\frac{\zeta_m}{\zeta} + \zeta_m}$$

$$= 2 - \frac{2}{d + \zeta}\frac{\zeta}{\zeta_m}$$

What is the appropriate long-distance theory?

Can we measure it?

standard elasticity

$c \to 0$

non-linear elasticity

$$\eta \partial_t u(x,t) = c\nabla^2 u(x,t) + c_4 \nabla [\nabla u(x,t)]^3 - m^2[u(x,t) - w]$$
$$+F\big(x, u(x,t)\big)$$

disorder force

confining potential

background field

What is the appropriate long-distance theory?

Can we measure it?

standard elasticity
$c > 0$

non-linear elasticity

$$\eta \partial_t u(x,t) = c \nabla^2 u(x,t) + c_4 \nabla [\nabla u(x,t)]^3 - m^2 [u(x,t) - w(x,t)]$$

$$+ \lambda [\nabla u(x,t)]^2 + F\big(x, u(x,t)\big)$$

KPZ term

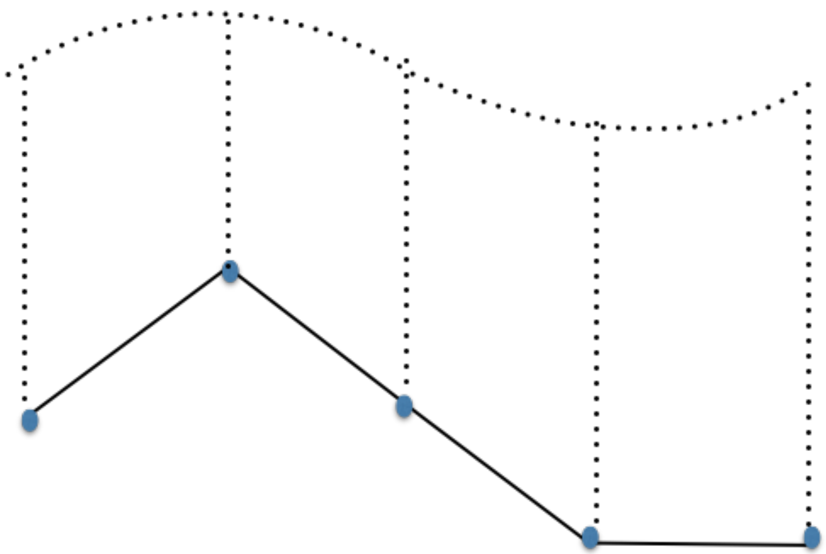
disorder force

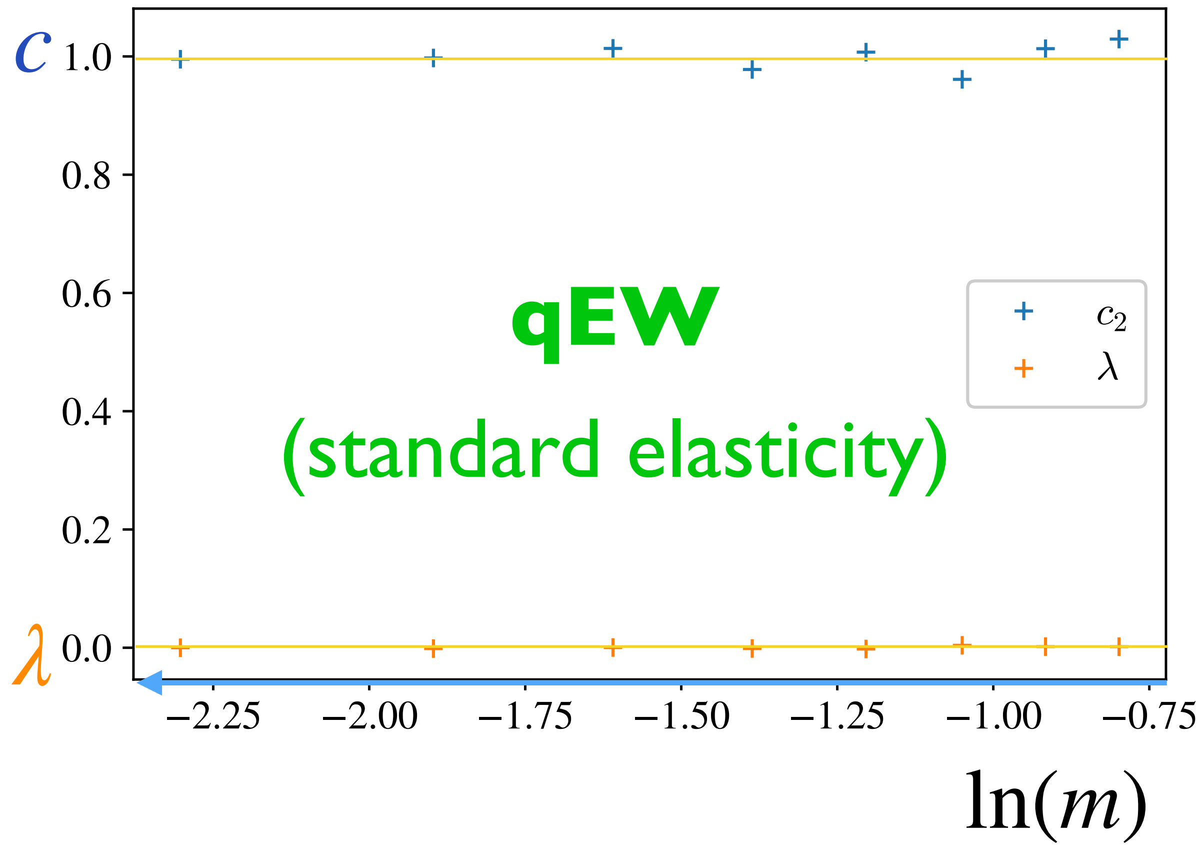
confining potential
(unchanged)

background field
(modulated)

Measuring the elastic constants for harmonic depinning (qEW)

$$w(x) = w_0 + A sin(\frac{\pi x}{L})$$

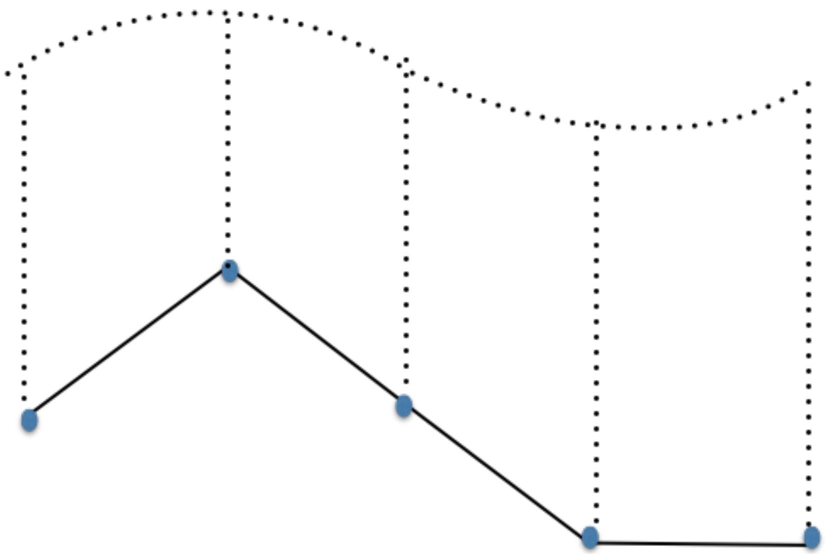


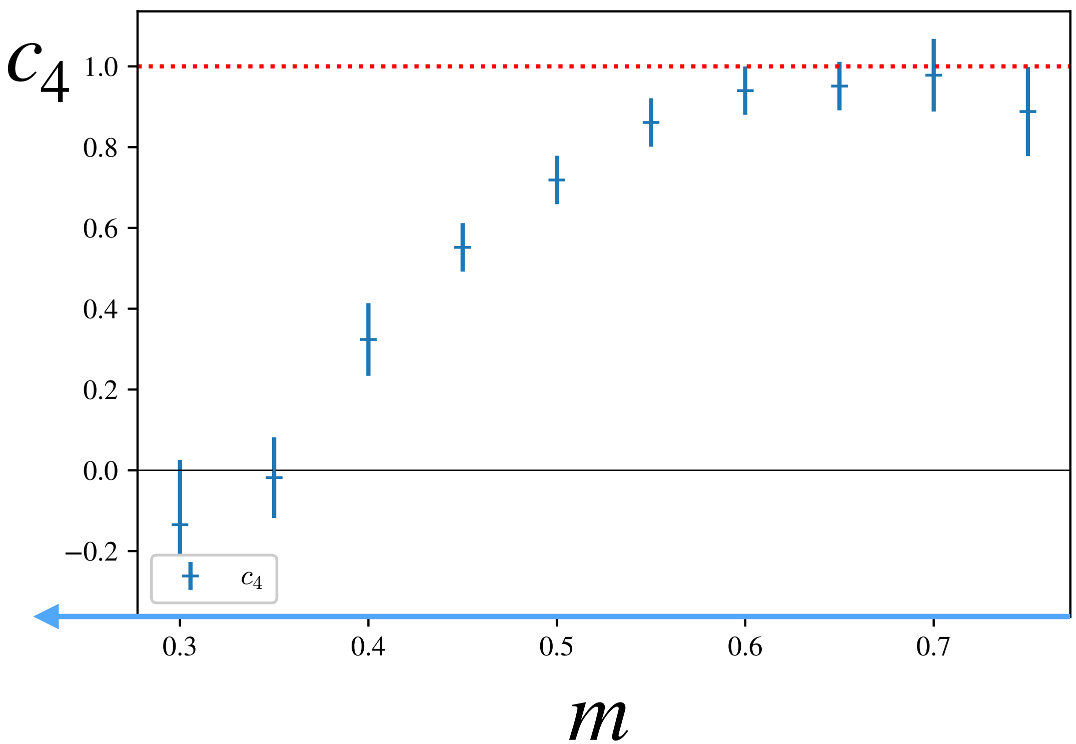


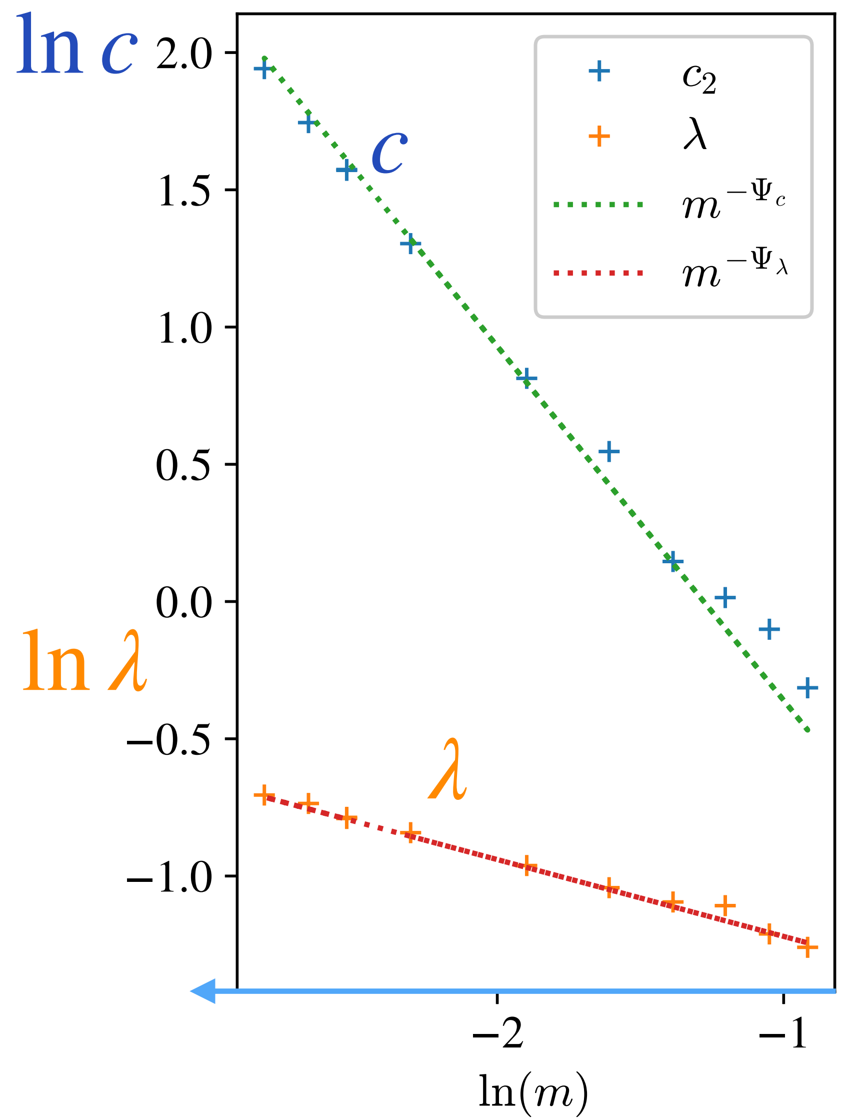
Measuring the elastic constants

anharmonic depinning ($c_4 > 0$)

$$w(x) = w_0 + A sin(\frac{\pi x}{L})$$





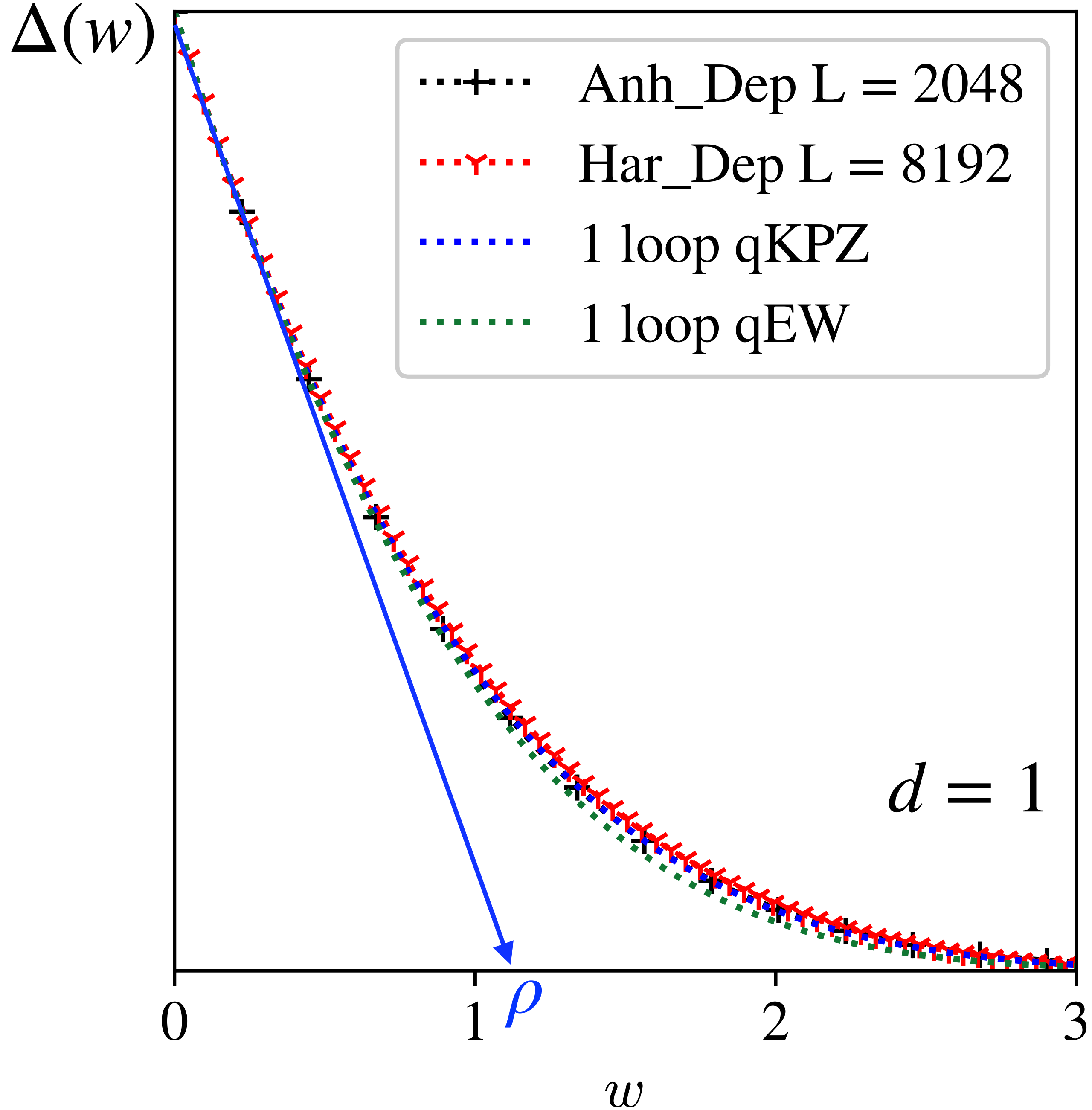


Measuring the effective force correlator

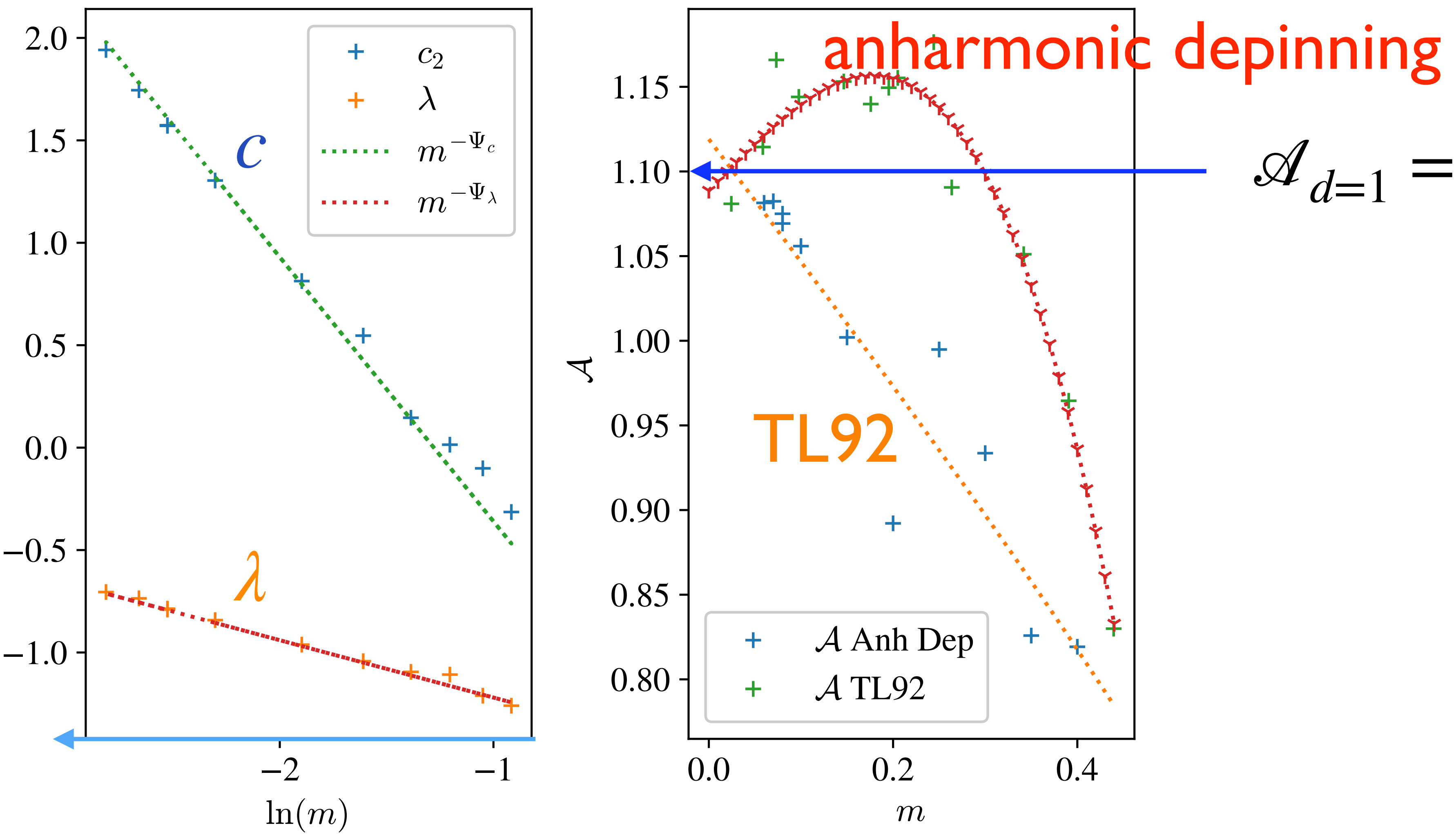
$$\Delta(w - w') = m^4 L^d \overline{(u_w - w)(u_{w'} - w}$$

$$u_w = \frac{1}{L^d} \int_x u_w(x)$$

centre-of-mass position given w



Coupling constant for qKPZ



$$\mathscr{A}_{d=1} = 1.10(2)$$

scale-free universal KPZ amplitude

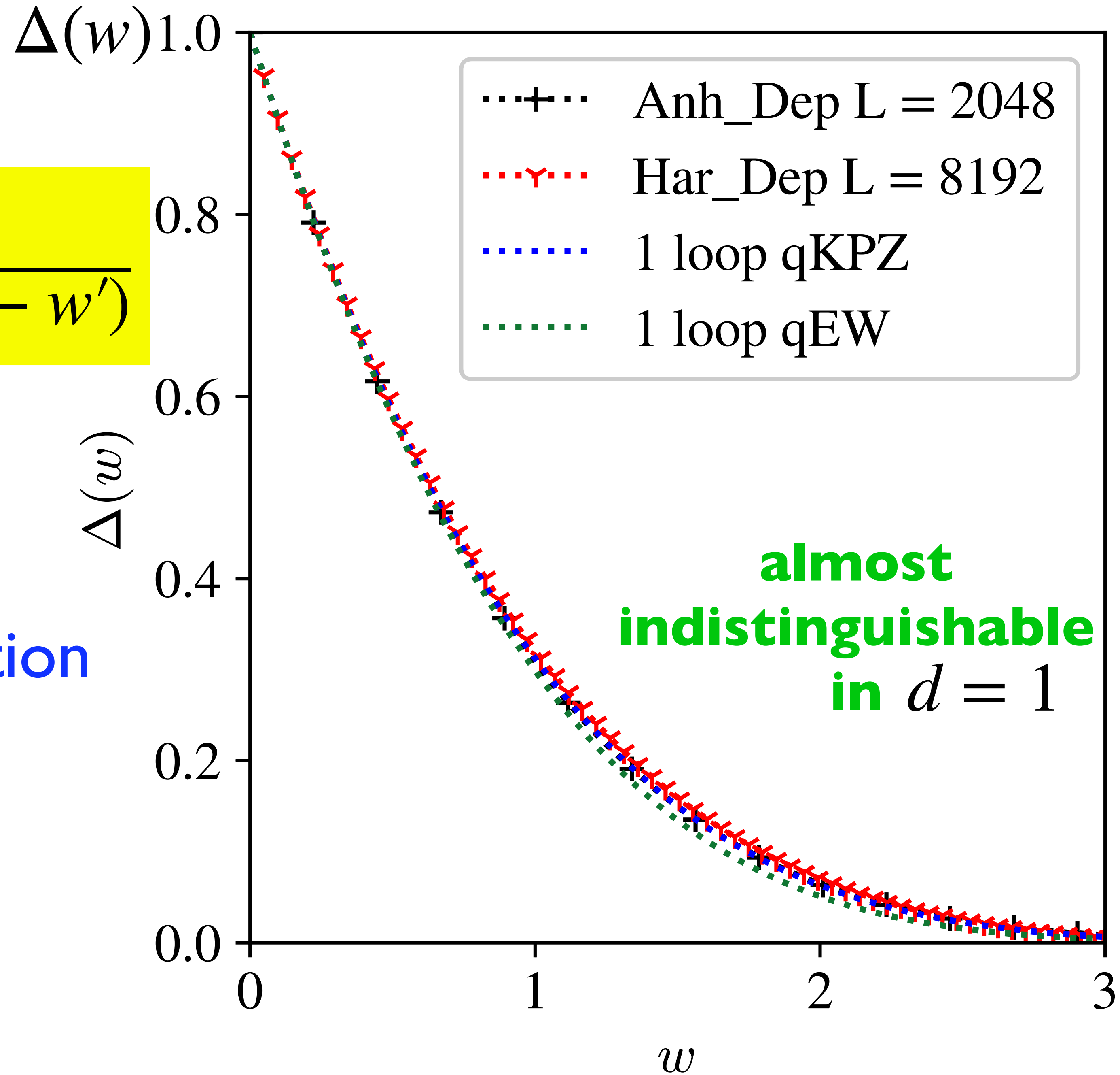
$$\mathscr{A} := \rho \frac{\lambda}{c} \equiv \frac{\Delta(0)}{|\Delta'(0^+)|} \frac{\lambda}{c}$$

Measuring the effective force correlator

$$\Delta(w - w') = m^4 L^d \overline{(u_w - w)(u_{w'} - w')}$$

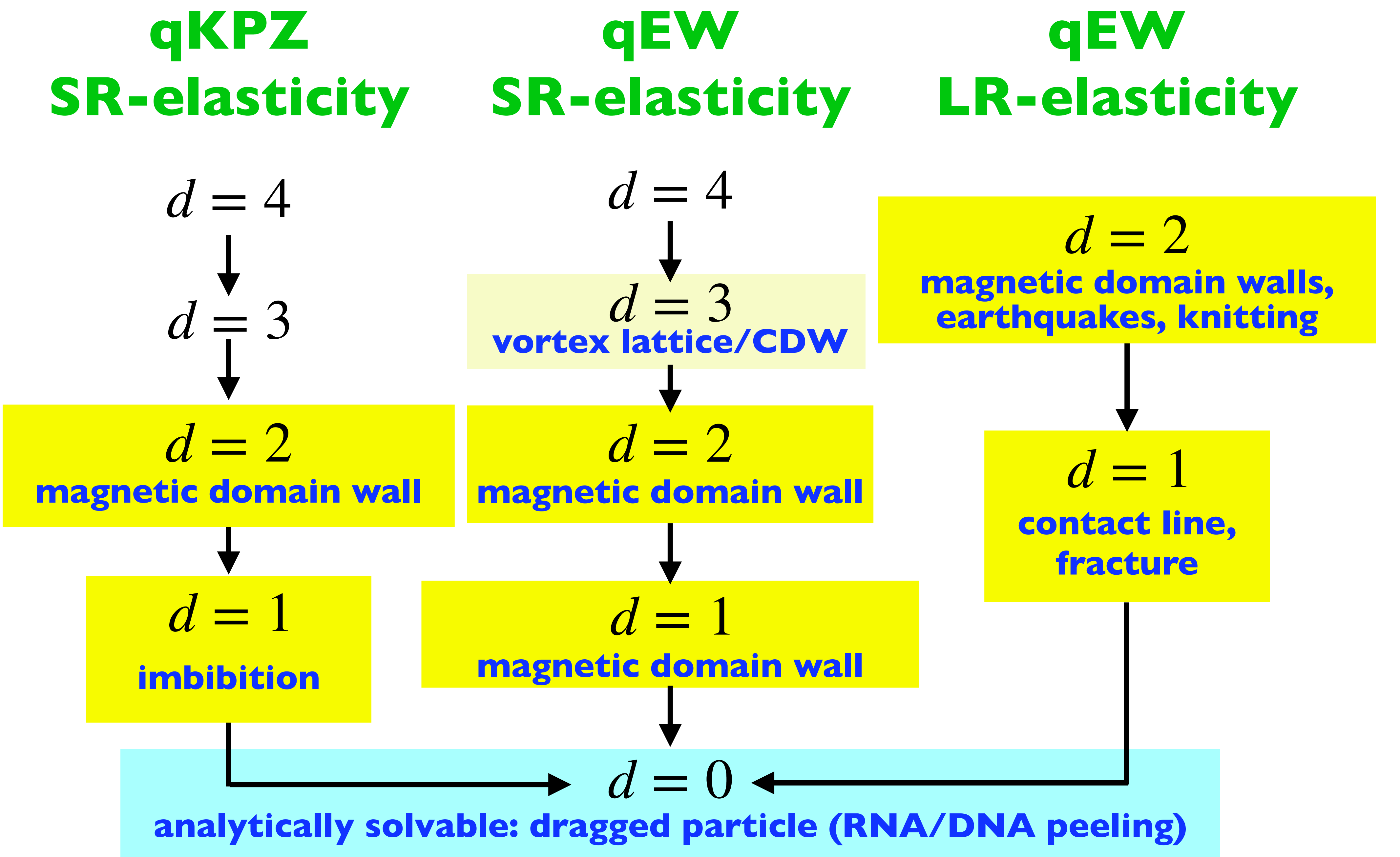
$$u_w = \frac{1}{L^d} \int_x u_w(x)$$

↑ centre-of-mass position given w



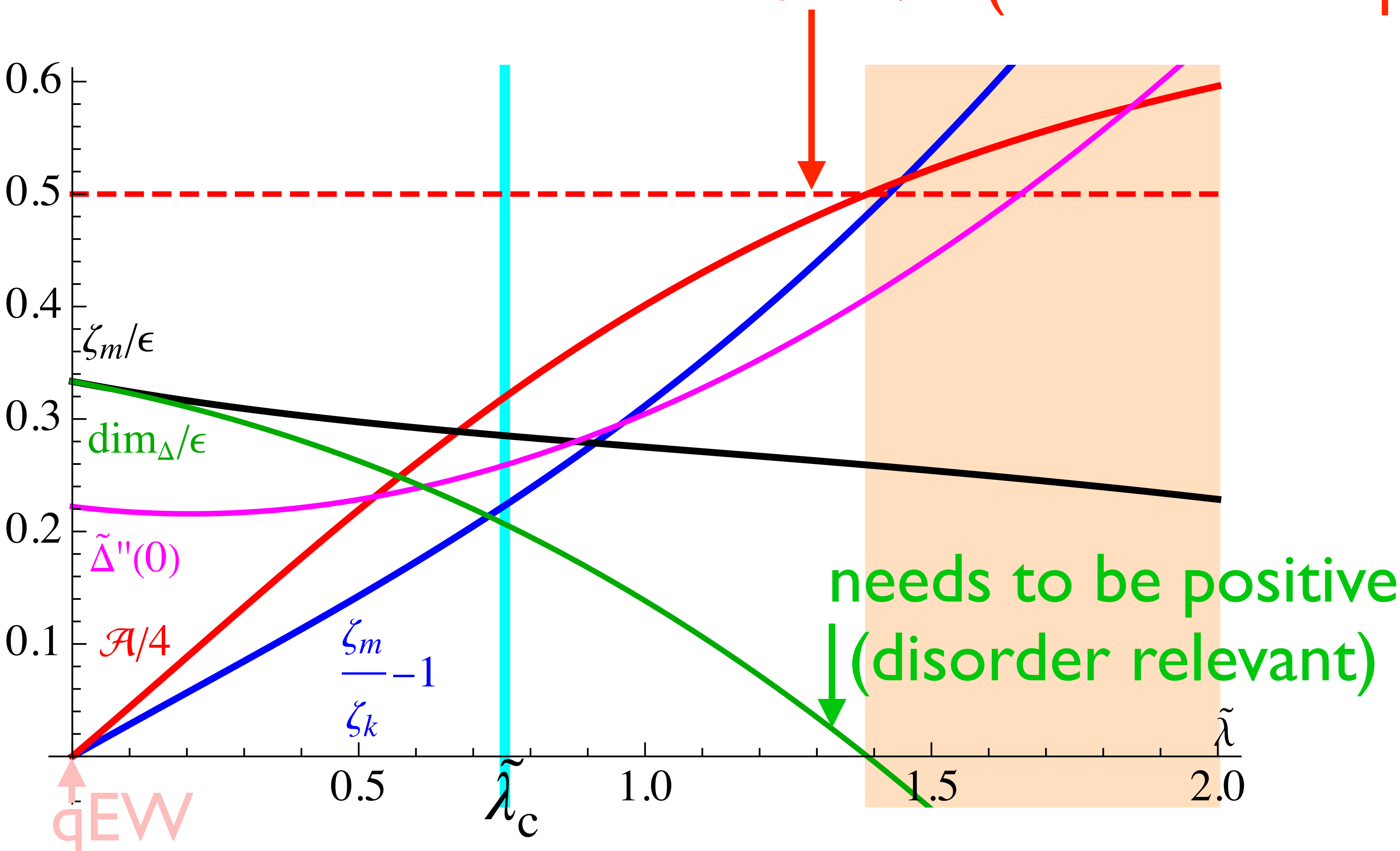
almost indistinguishable in $d = 1$

Universality classes for depinning



Solution in $d = 1$

$\mathscr{A} < 2$ (critical force positive)



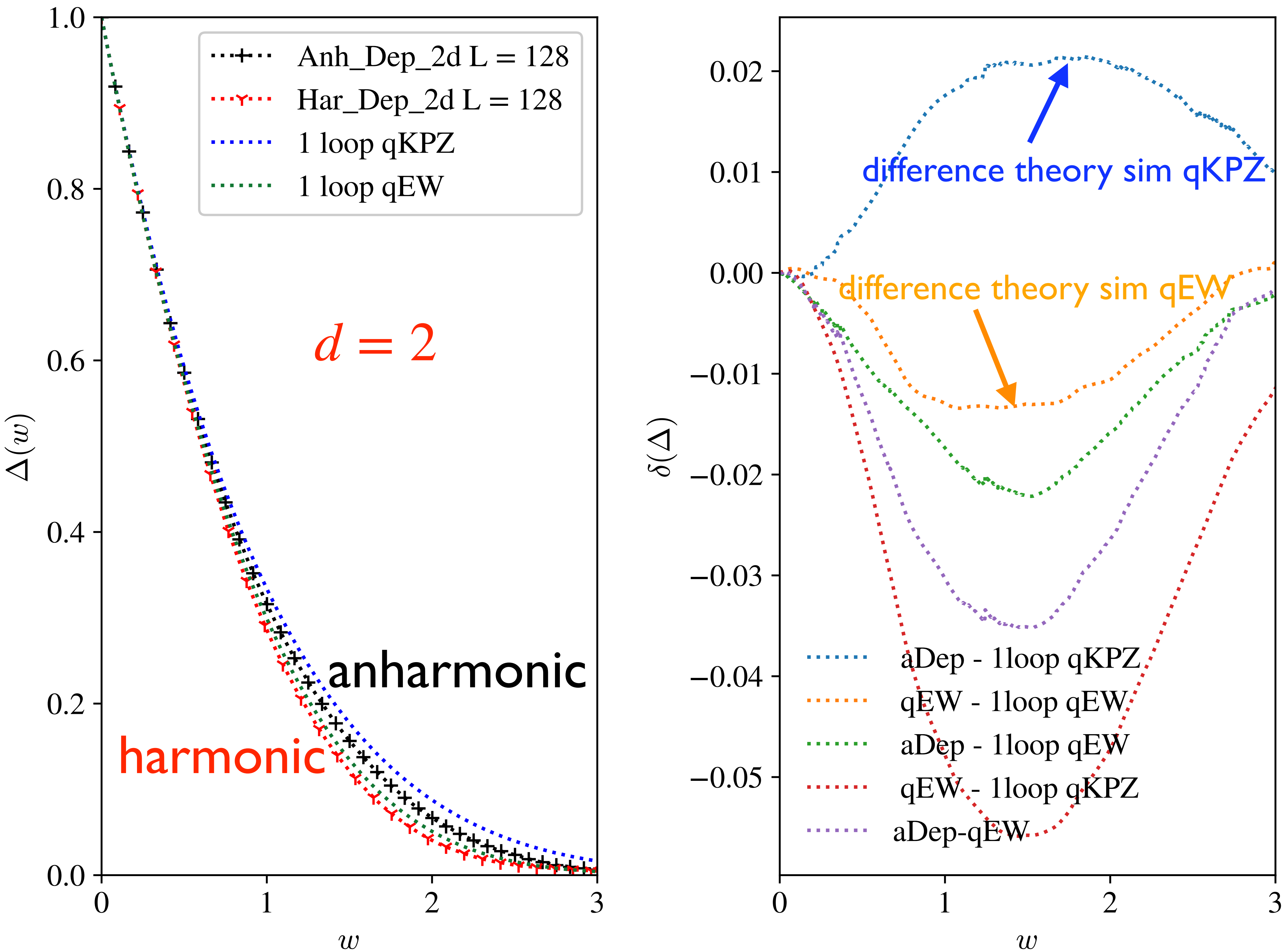
RG:

$$\zeta_m^{d=1} = 0.86$$
$$\zeta^{d=1} = 0.69$$
$$z^{d=1} = 1.27$$
$$\mathscr{A}^{d=1} = 1.27$$

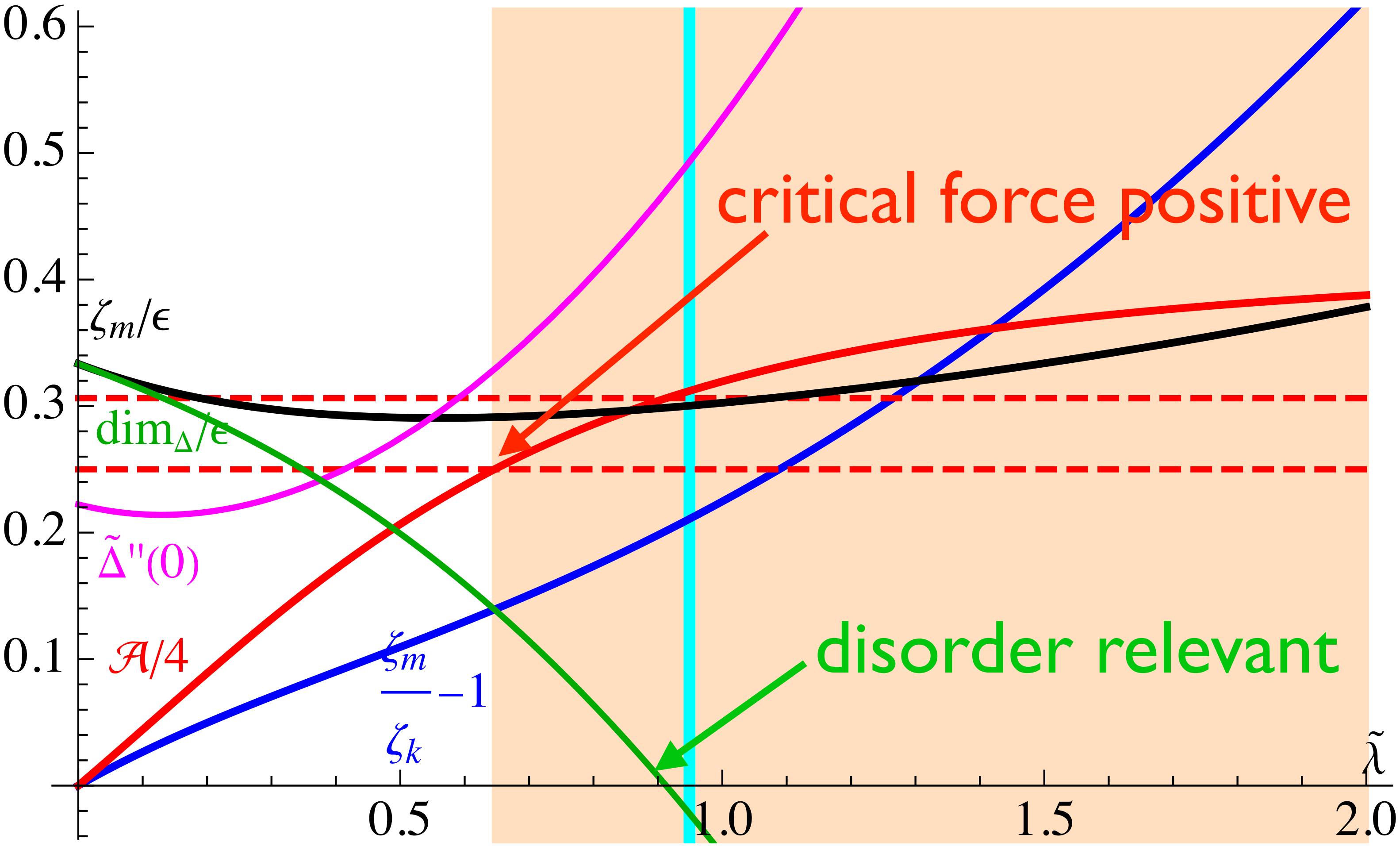
numerics:

$$\zeta_m^{d=1} = 1.05$$
$$\zeta^{d=1} = 0.63$$
$$z^{d=1} = 1.10(2)$$
$$\mathscr{A}^{d=1} = 1.10(2)$$

Shape of $\Delta(w)$ different in $d = 2$



Solution in $d = 2$



RG: $\zeta_m^{d=2} = 0.61$

$\zeta^{d=2} = 0.49$

$z^{d=2} = 1.41$

$\mathcal{A}^{d=2} = 1.25$

numerics (anh. dep): $\zeta_m^{d=2} = 0.61(2)$

$\zeta^{d=2} = 0.48(2)$

Theory and Experiments for Disordered Elastic Manifolds, Depinning, Avalanches, and Sandpiles

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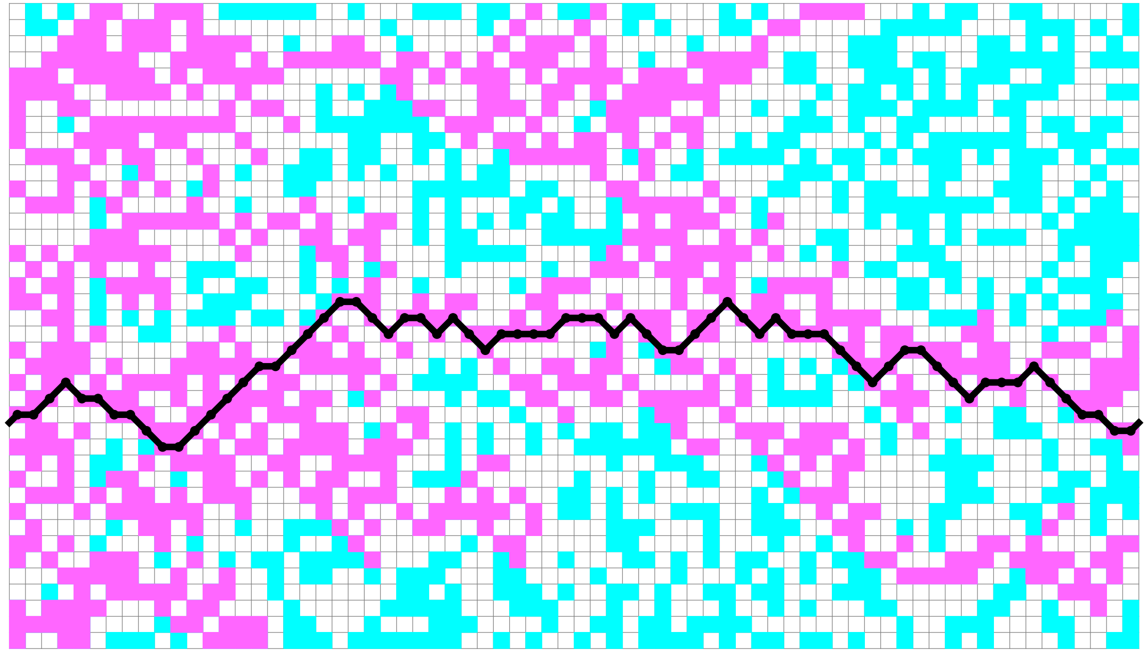
1 September 2021 – masterENS.tex – REVISION 1.1083

Abstract. Domain walls in magnets, vortex lattices in superconductors, contact lines at depinning, and many other systems can be modeled as an elastic system subject to quenched disorder. The ensuing field theory posesses a well-controlled perturbative expansion around its upper critical dimension. Contrary to standard field theory, the renormalization group flow involves a function, the disorder correlator $\Delta(w)$, and is therefore termed the functional renormalization group (FRG). $\Delta(w)$ is a physical observable, the auto-correlation function of the center of mass of the elastic manifold. In this review, we give a pedagogical introduction into its phenomenology and techniques. This allows us to treat both equilibrium (statics), and depinning (dynamics). Building on these techniques, avalanche observables are accessible: distributions of size, duration, and velocity, as well as the spatial and temporal shape. Various equivalences between disordered elastic manifolds, and sandpile models exist: an elastic string driven at a point and the Oslo model; disordered elastic manifolds and Manna sandpiles; charge density waves and Abelian sandpiles or loop-erased random walks. Each of the mappings between these systems requires specific techniques, which we develop, including modeling of discrete stochastic systems via coarse-grained stochastic equations of motion, super-symmetry techniques, and cellular automata. Stronger than quadratic nearest-neighbor interactions lead to directed percolation, and non-linear surface growth with additional KPZ terms. On the other hand, KPZ without disorder can be mapped back to disordered elastic manifolds, either on the directed polymer for its steady state, or a single particle for its decay. Other topics covered are the relation between functional RG and replica symmetry breaking, and random field magnets. Emphasis is given to numerical and experimental tests of the theory.

Review

arXiv:2102.01215

Rep. Prog. Phys. 85 (2022) 086502 (133pp)



Anisotropic depinning with its relation to directed percolation, explained in section 5.7.

pedagogic introduction in basic sections!

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Conclusions

- much can be learned by measuring the effective long-distance action (= theory/description)
- **qEW** (standard elastic theory) has non-trivial disorder correlator given by FRG
- imbibition (e.g. TL92), anharmonic depinning and qKPZ all belong to the same universality class: the effective long-wavelength theory is **qKPZ**
- you need to introduce a confining potential $m^2[w - u(x,t)]$ to measure disorder correlations
  $\Rightarrow$ give up the Cole-Hopf transform
  $\Rightarrow$ yields an RG fixed point
- a field theory can be build