Funding:

The role of structural heterogeneity in avalanche statistics :

Deformability bridges universality classes in numerical granular assemblies under deviatoric loading.

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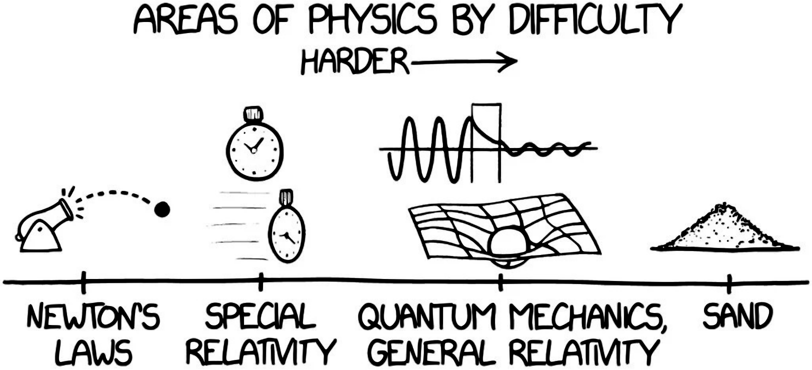
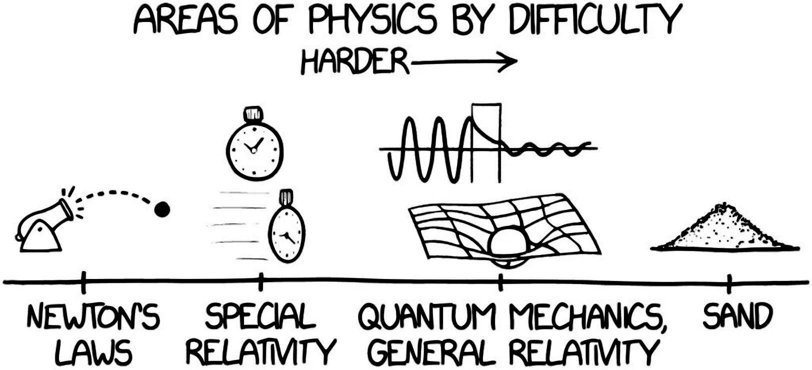


K. Daniels & R. Munroe *What Makes Sand Soft?*, The New York Times Nov 9, 2020



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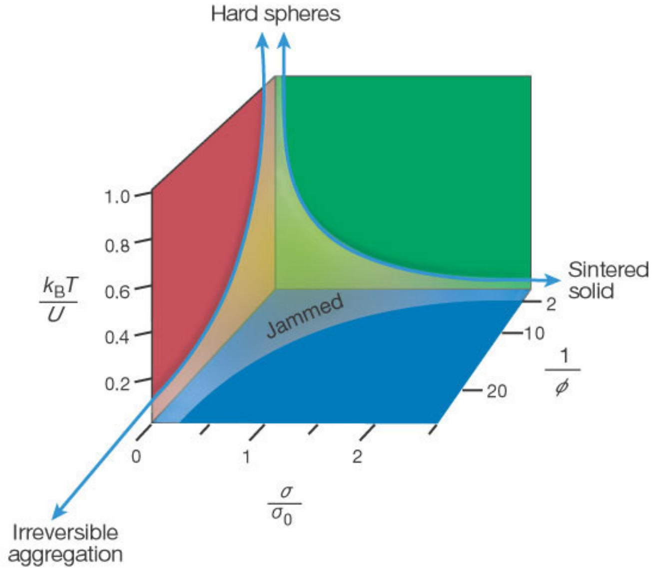
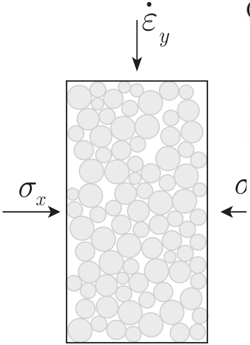
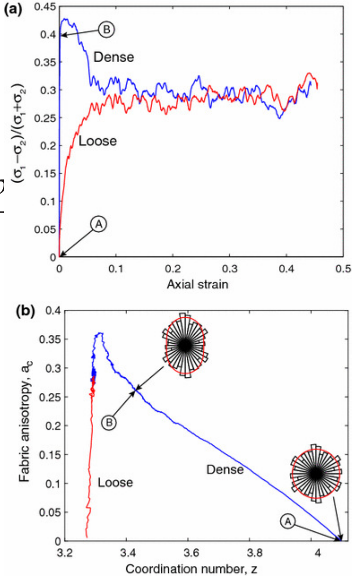
Jamming transition



[*V. Trappe et al., Nature (2001)*]

Stable Evolution Surface (**SES**)





- $Z \equiv$ average coord. number (*n. contacts*)
- $\Gamma \equiv$ rigidity ratio (*contact deformation*)
- $a_c \equiv$ fabric anisotropy (*cont. orientation*)

[*M. Pouragha & R Wan Granular Matter (2016)*]

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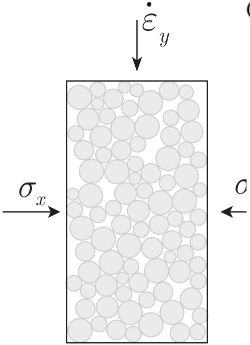

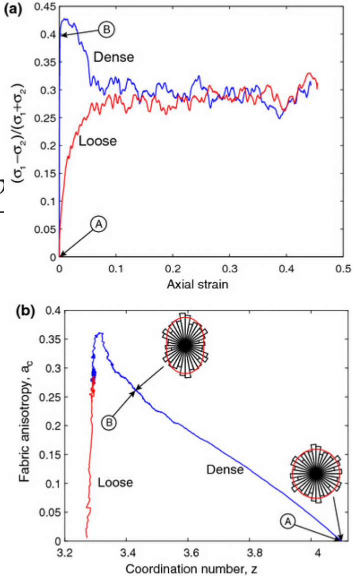

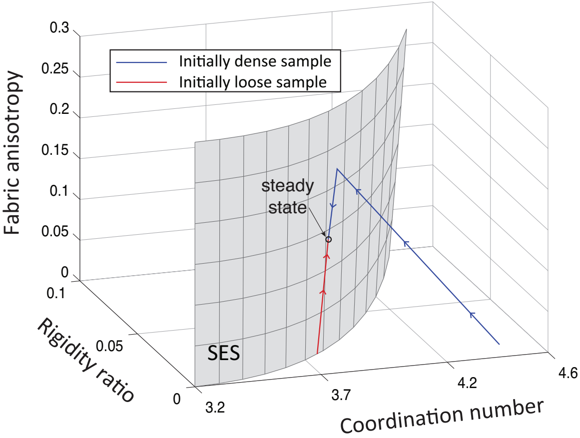

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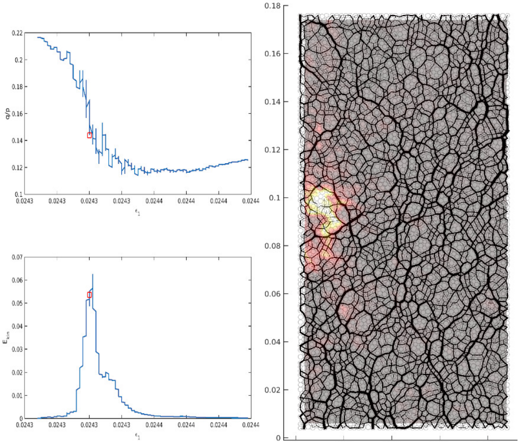
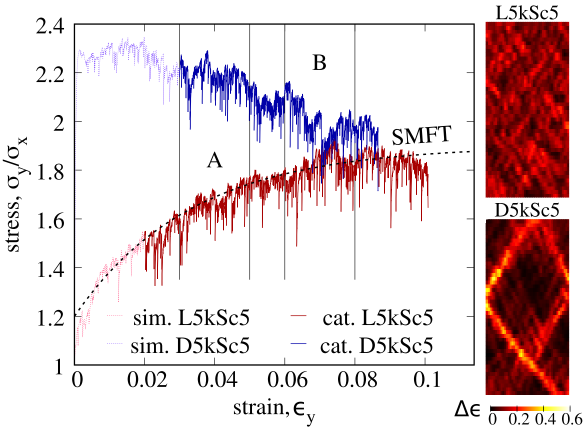
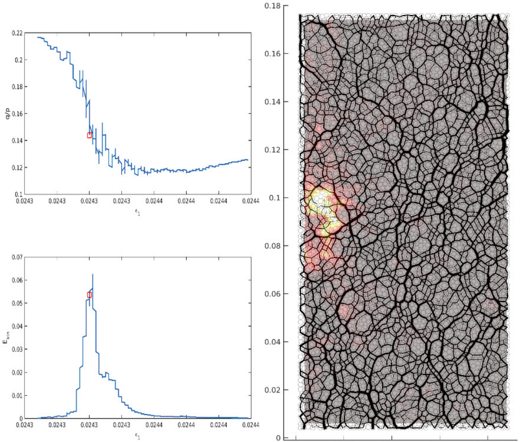
Non-linear dynamics at SES





Total kinetic E.: $E_K(t) = \frac{1}{2}\sum_{i=1}^{N}(m_i v_i^2 + I_i \omega_i^2)$

Non-linear dynamics at SES



- Are these avalanches?
- Can we define states from SES instead of $\{\sigma, T, \phi\}$ or $\sigma(\epsilon)$?
- SES $\rightarrow$ avalanche statistics?

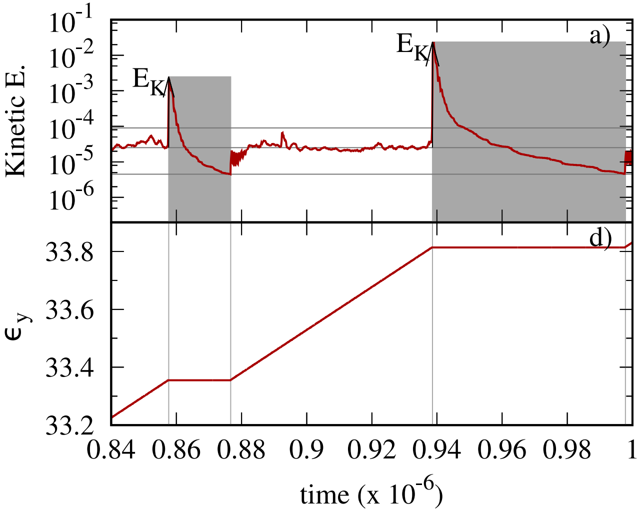
Total kinetic E.: $E_K(t) = \frac{1}{2}\sum_{i=1}^{N}(m_i v_i^2 + I_i \omega_i^2)$

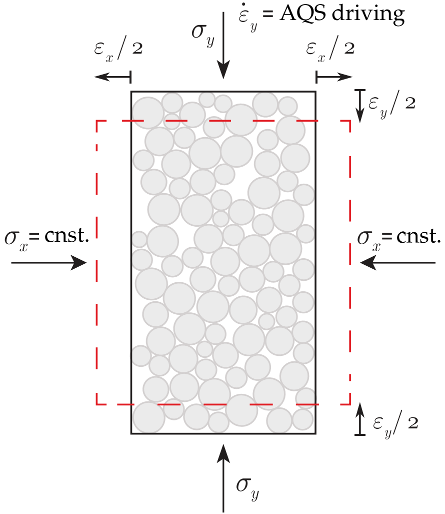
Summary:

1. Discrete Element (DEM) simulations.
2. Avalanche statistics.

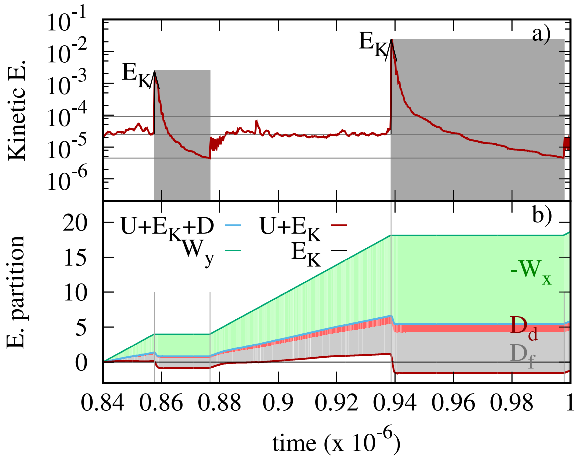
3. Origin of Mean Field (MF) exponents.
4. Comparison with acoustic emission (a.e.).

Quasistatic driving of elastic ($f_c = k_c\delta$) particles





Method by [*K. Salerno, M. Robbins PRE 2013*]



Energy Balance:

dW_y $\quad (= V\sigma_y d\epsilon_y)$ input work

$=$

dU $\quad \left(= \frac{1}{2}\sum \frac{f_c}{k_c} df_c\right)$

$+$

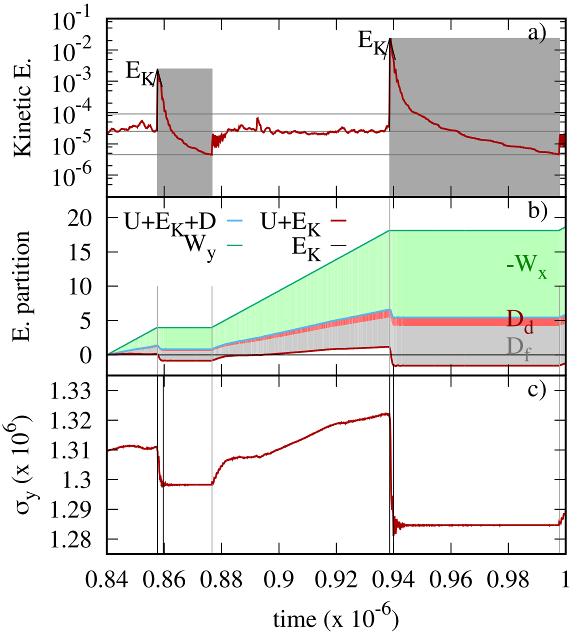
dD $\quad (= dD_f + dD_d)$ *(no heat)*

$+$

dE_k

$+$

dW_x $\quad (= V\sigma_x d\epsilon_x)$

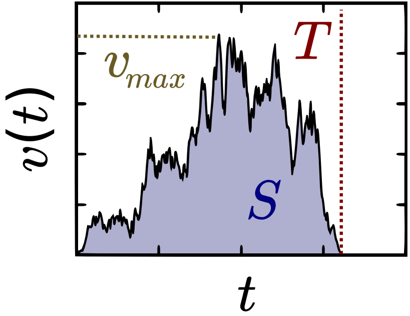


Avalanches as point process:

$$\mu(\epsilon_y, \sigma_y; T, K, \Delta\sigma_y, \Delta U, W_x)$$

- Duration: $T :=$ time of first rebound in $U(t)$
- Stress drop: $\Delta\sigma_y := \sigma_y(t_0) - \sigma_y(t_0 + T)$
- Potential E. drop: $\Delta U := U(t_0) - U(t_0 + T)$
- Kinetic energy: $K = E_K^{\max} - K_D$
- Lateral work: $W_x = \int_{t_0}^{t_0+T} V_0(1+\epsilon_v)\sigma_x \dot{\epsilon}_x dt$

Avalanche Sizes and Energies



Avalanche from vel. profile $v(t)$:

- Size: $S := \int_{t_0}^{t_0+T} v(t)dt$
- Duration T starting at time t_0
- Energy $E := \int_{t_0}^{t_0+T} v^2(t)dt$
- Energy peak $E_m := v^2_{\text{max}}(t)dt$

In terms of *internal* avalanche measures ...?

Avalanche Sizes

Expected $\Delta U \Leftrightarrow \Delta\sigma_y$

During avalanches ($d\sigma_x = 0$, $d\epsilon_y = 0$):

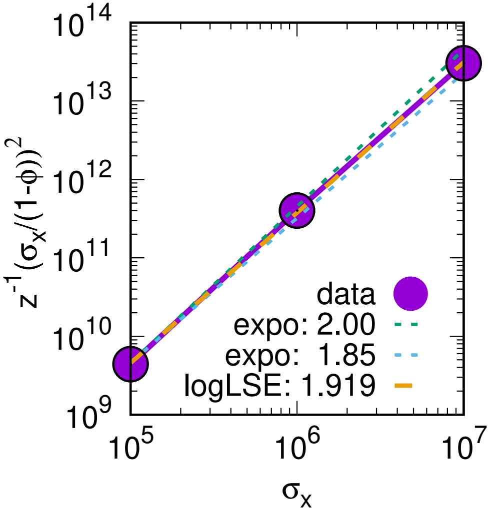
$$dU = V\left(\frac{\sigma_y}{\sigma_x}\frac{1}{E_y} - \frac{\nu_{xy}}{E_x}\right)\sigma_x^2 \frac{d\sigma_y}{\sigma_x}$$

$$V \propto N\pi\langle r^2\rangle/(1-\phi)$$

$$E \propto ZV^{-1} \propto Z(1-\phi)$$

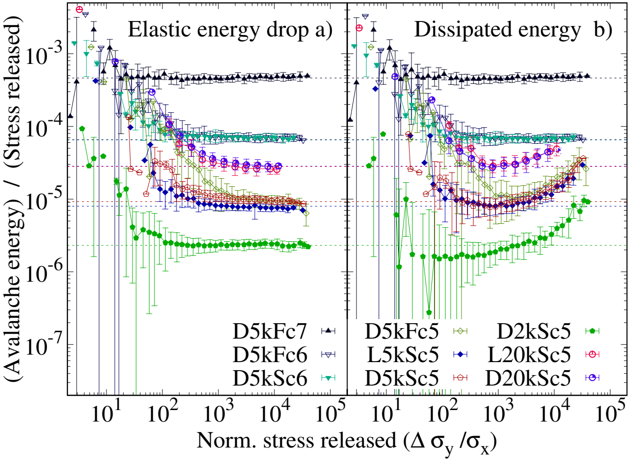
$$\Downarrow$$

$$\Delta U \propto N \boxed{Z^{-1}(1-\phi)^{-2}\sigma_x^2} \frac{\Delta\sigma_y}{\sigma_x}$$



Avalanche Sizes

Elastic E. vs Dissipation



$$\Delta U \propto NZ^{-1}(1-\phi)^{-2}\sigma_x^2 \frac{\Delta\sigma_y}{\sigma_x}$$

- Prop. $\Delta U \propto \Delta\sigma_y$:

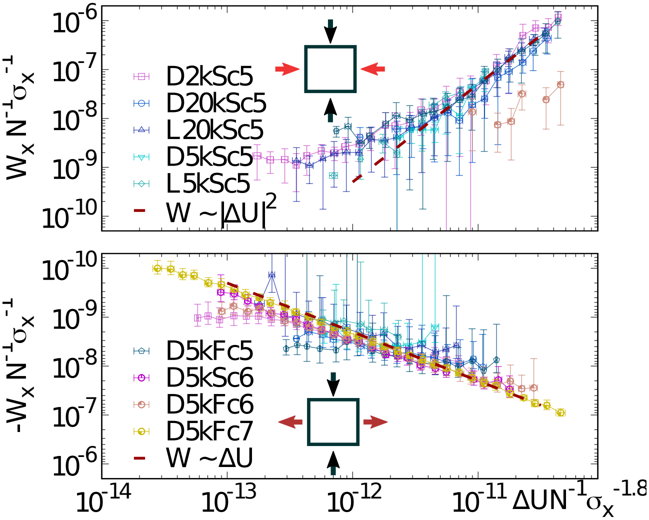
$$\boxed{\Delta U \propto N\sigma_x^{1.85(5)} \Delta\sigma_y/\sigma_x}$$

- No prop. $D \propto \Delta\sigma_y$:

$$\boxed{D = \Delta U - W_x \not\propto \Delta U}$$

Avalanche Sizes

Contracting vs Expanding



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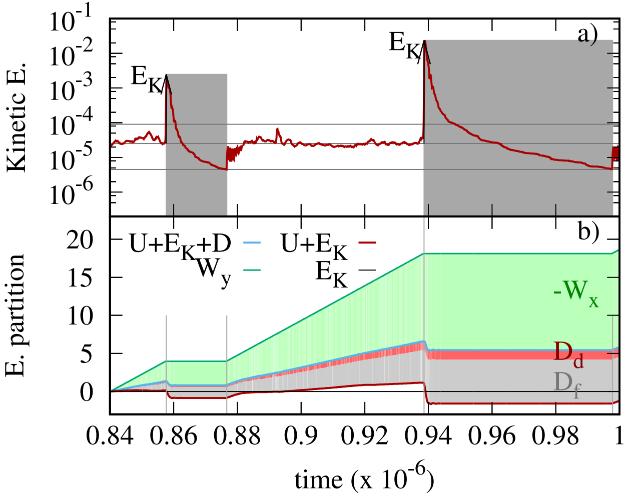
- No prop. $D \propto \Delta\sigma_y$:

$$\boxed{D = \Delta U - W_x \not\propto \Delta U}$$

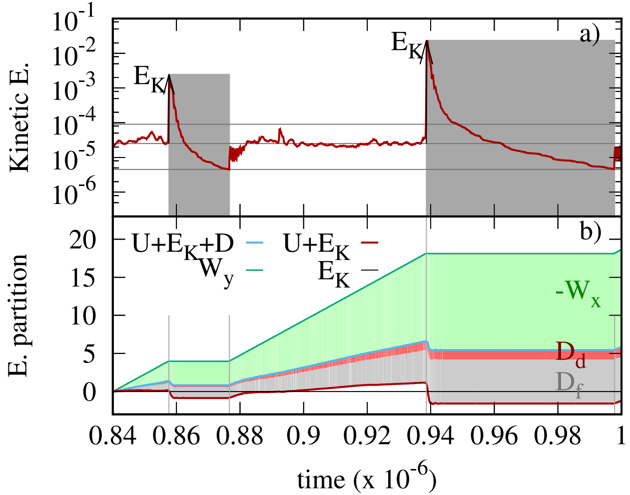
- Two pop. of avalanches in W_x
- $W_x \propto \Delta U$ if *expanding* (< 0)

Avalanche Energies



$K \propto E := \int v^2(t)dt$ or $E_m := v^2_{\max}(t)dt$?

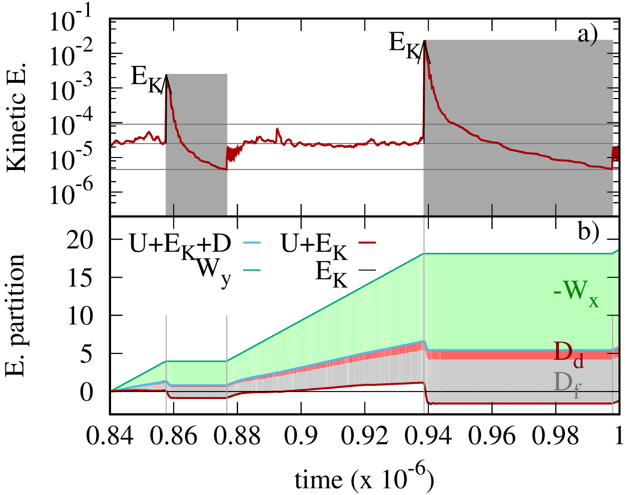
Avalanche Energies



$K \propto E := \int v^2(t)dt$ or $E_m := v^2_{\max}(t)dt$?

- Low dissipation between t_0 and T

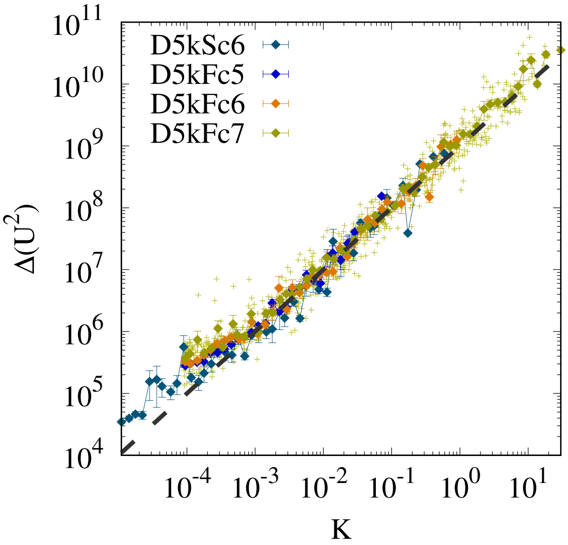
Avalanche Energies



$K \propto E := \int v^2(t)dt$ or $E_m := v^2_{\max}(t)dt$?

- Low dissipation between t_0 and T
- If $v(t) \propto \dot{U} \Rightarrow \dot{v}^2(t) \propto \dot{U}^2$

Avalanche Energies

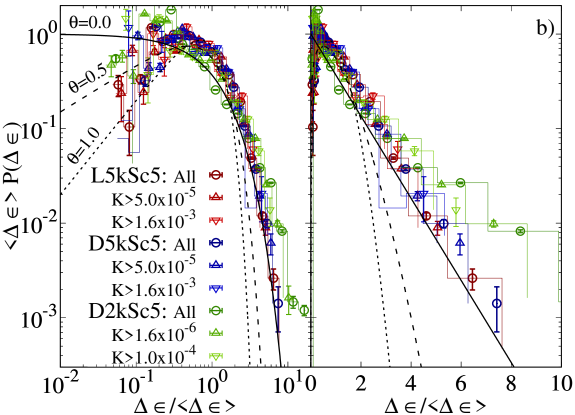


$$K \propto E := \int v^2(t)dt \text{ or } E_m := v^2_{\max}(t)dt \text{ ?}$$

- Low dissipation between t_0 and T
- If $v(t) \propto \dot{U} \quad \Rightarrow \quad \dot{v}^2(t) \propto \dot{U}^2$
- $K \propto \Delta(U^2) := \int_{t_0}^{t_0+T} \dot{U}^2 dt$

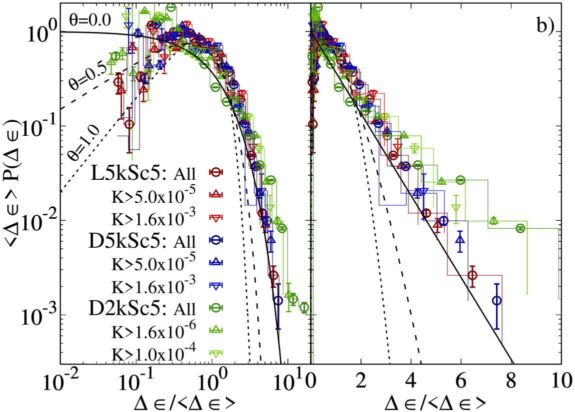
$$\Rightarrow \boxed{K \propto E}$$

Avalanche Interevent Times



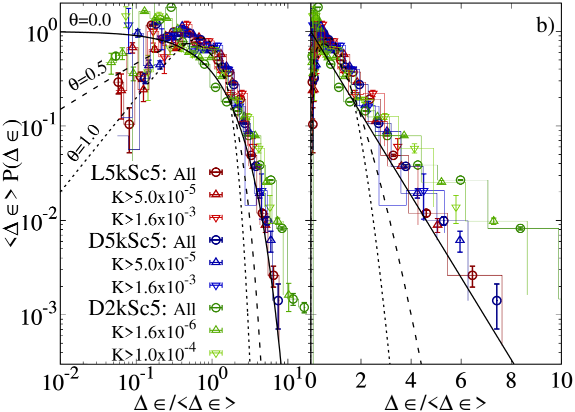
- Stationary (exp. decay at long $\Delta\epsilon$)

Avalanche Interevent Times



- Stationary (exp. decay at long $\Delta\epsilon$)
- Regularity (missing short $\Delta\epsilon$)

Avalanche Interevent Times



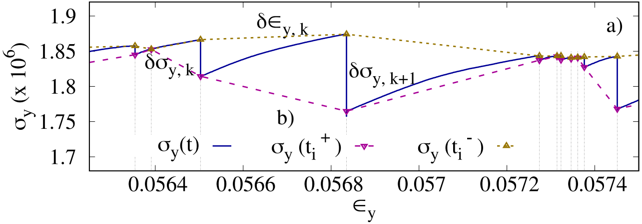
- Stationary (exp. decay at long $\Delta\epsilon$)
- Regularity (missing short $\Delta\epsilon$)
- Pseudo-gap from dynamic fields:

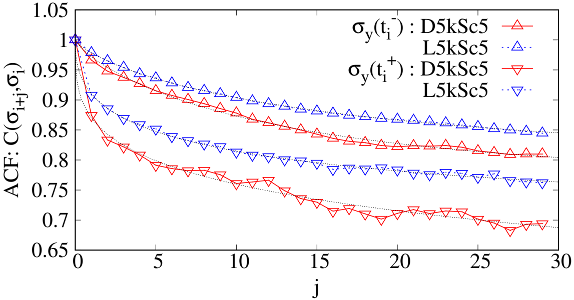
$$P(\Delta\epsilon) = \frac{1+\theta}{\langle\Delta\epsilon\rangle}\left(\frac{\Delta\epsilon}{\langle\Delta\epsilon\rangle}\right)^{\theta} e^{-\left(\frac{\Delta\epsilon}{\langle\Delta\epsilon\rangle}\right)^{\theta+1}}$$

Avalanche Interevent Times

Regularity → Time-predictability.

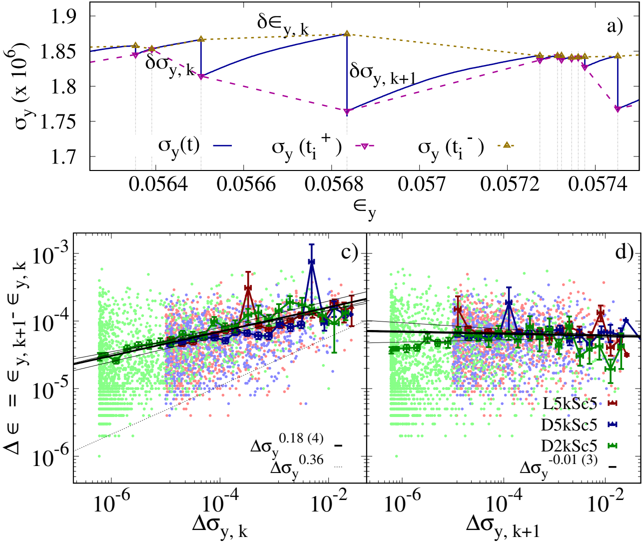
- More persistent σ_y (*rigidity* Γ) at avalanche **onset**.





Avalanche Interevent Times

Regularity $\rightarrow$ Time-predictability.

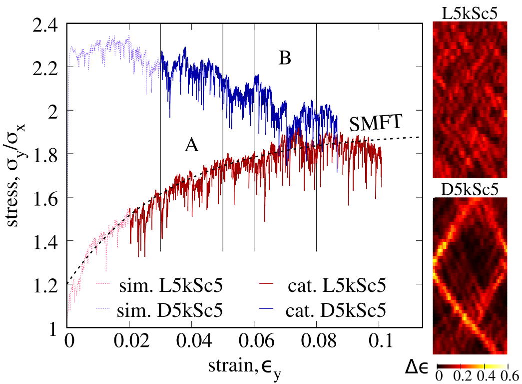


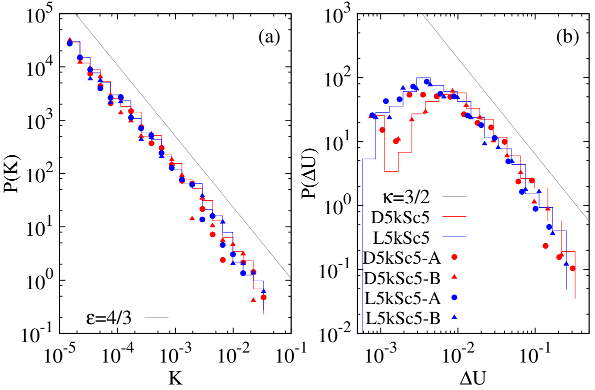
- More persistent σ_y (*rigidity* Γ) at avalanche **onset**.
- Minimum loading gap from last avalanche:

$$\Delta\epsilon_y \sim \Delta\sigma_y^{0.36}$$

- $\Rightarrow$ SES is a stability limit, triggering avalanches.

Size & Energy dist. stationary at SES





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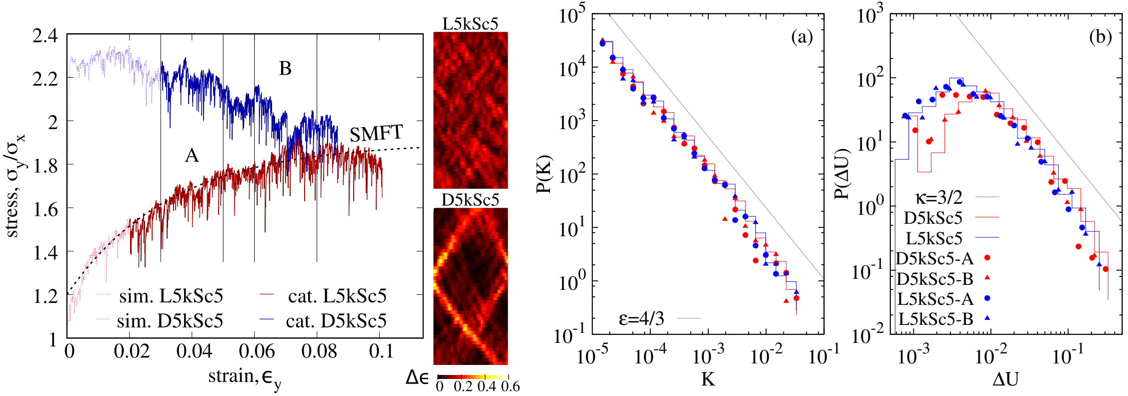


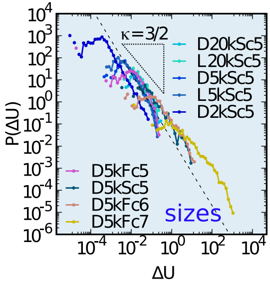
Effective modulus $\hat{E}_y := \Delta\sigma_y/\Delta\epsilon_y$ within the SES is non-stationary. *How?*

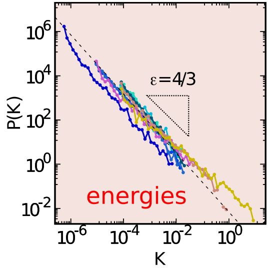
$$\rho_{\hat{E}_y,\text{ activity rate}} = 0.37 \quad \rho_{\hat{E}_y,\text{ inter-event reload in }\sigma_y} = 0.30 \quad \rho_{\hat{E}_y,\text{ avalanche size}} = 0.058$$

$$({}^{*}\quad \Delta\epsilon_y = 0.005)$$

Size & Energy dist. is scale-free

not scaled





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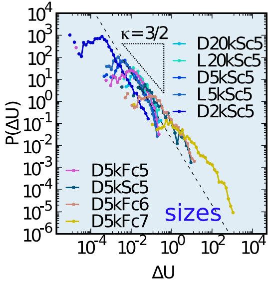
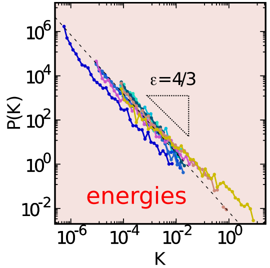


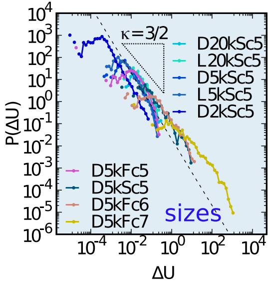


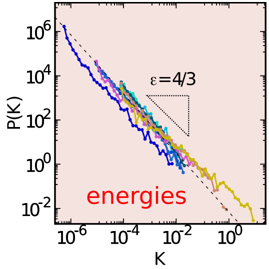
$$P(x)dx = x^{-\tau_x}\Phi_x(x/x^*)dx$$

$$x^*(N, \sigma_x, \dot{\epsilon}_y) = \tilde{x}^* N^{\gamma_N^x} \sigma_x^{\gamma_\sigma^x} \dot{\epsilon}_y^{\gamma_\epsilon^x}$$

Size & Energy dist. is scale-free

not scaled





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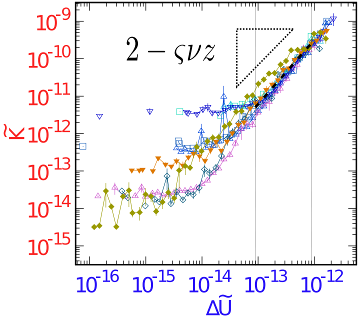
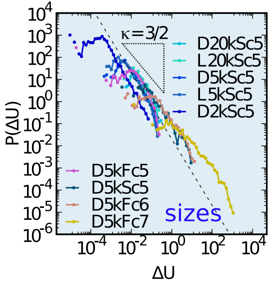
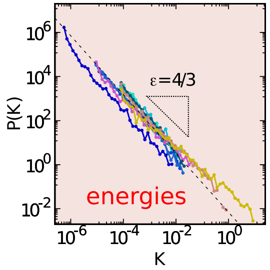


Size & Energy dist. is scale-free

not scaled





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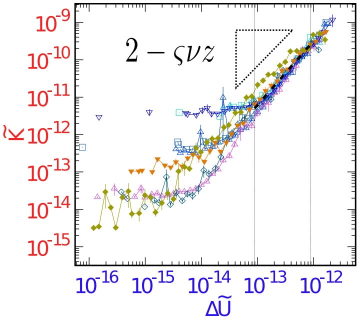

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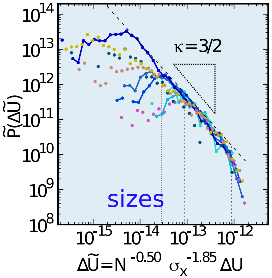
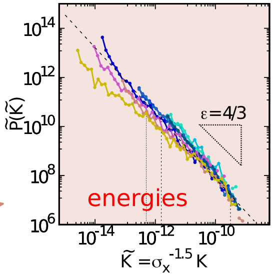
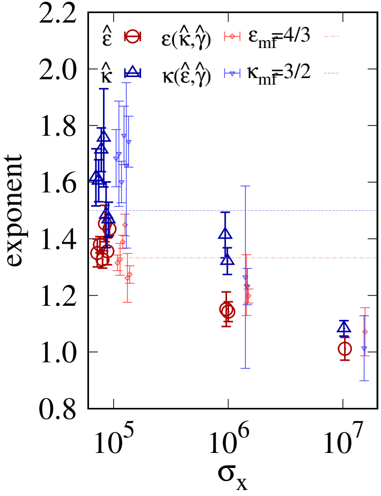




Estimated exponents by Max. Lik.



$\frac{\sigma_x}{k_n}(\sim \Gamma)$	#	κ	ε	$2-\varsigma\nu z$
10^{-4}	1684	1.62(10)	1.32(10)	1.95(5)
" "	979	1.60(7)	1.34(5)	1.85(10)
" "	788	1.71(8)	1.33(6)	1.83(4)
" "	130	1.77(17)	1.45(6)	1.85(15)
" "	236	1.49(11)	1.36(4)	1.69(6)
" "	1215	1.46(6)	1.36(4)	1.71(5)
10^{-3}	396	1.41(8)	1.14(11)	1.65(8)
" "	851	1.32(5)	1.14(6)	1.71(4)
10^{-2}	633	1.08(3)	1.02(8)	1.48(7)
SMFT$^{(1)}$		1.5	$1+\frac{\kappa-1}{2-\varsigma\nu z}=1.33$	1.5
2D EPM		1.25–1.28	~1.2 [*]	~1.45 [*]

[*] [*Budrikis et al. (2017)*]

Results:

Avalanches at SES are scale-free.

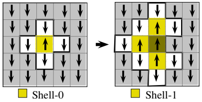
Within SES critical exponents depend (*at least*) on rigidity Γ:

Stiff particles $\to$ MF *Soft* particles $\to$ EPM

Discussion:

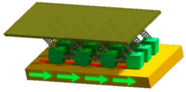
Why mean field in granular and a.e.?

Avalanches in **mean-field models:** E.g. RFIM:



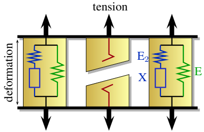
Random Field Ising (RFIM)

[J. Sethna PRL (1993)]



Slip Mean Field Theory (SMFT)

[K. Dahmen PRL (2009)]



Democratic Fiber Bundle Model (DFBM)

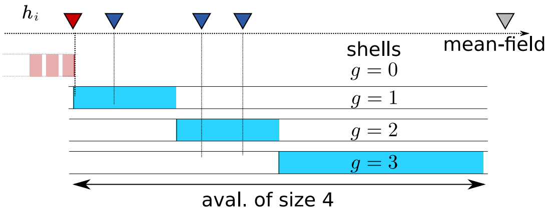
[JB, J. Davidsen, PRE (2018)]

$$\mathcal{H}(\{S\}) = \sum_i S_i \left(J \sum_{\langle ij \rangle} S_j + H_{\text{ext.}} + \mathbf{h_i} \right) \qquad \sum_{<j,i>} J_{j,i} S_j \to JM$$

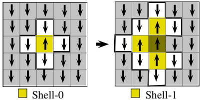
$\equiv$ **Random Thresolds (***shell model [Sethna PRL, 1993]*).

when **one** element $h_1$is activated:

$H_{\text{ext}}(t) + M \to H_{\text{ext}}(t) + M \mathbf{+2J/N}$

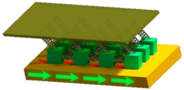


Avalanches in **mean-field models:** E.g. RFIM:



Random Field Ising (RFIM)

[*J. Sethna PRL (1993)*]



Slip Mean Field Theory (SMFT)

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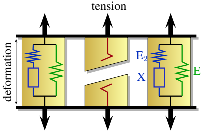

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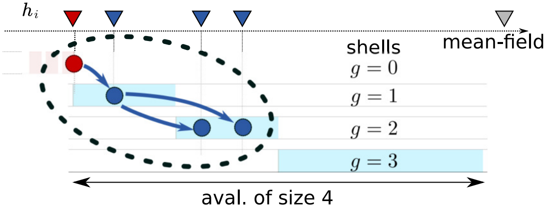
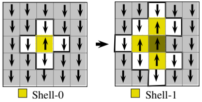


- Avalanches grow as a **branching** process.
- For $N \to \infty$: **MF-avalanche size** $\equiv$ **tree-size** in **Poisson G.W.**:

$$D(\Delta; n) = \frac{(n\Delta)^{\Delta-1} \exp(-n\Delta)}{\Delta!} \sim \boxed{\Delta^{-3/2}\, \mathcal{D}(n\Delta)}$$

Avalanches in **loopless trees:** E.g. RFIM:



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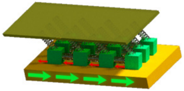

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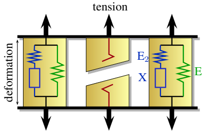

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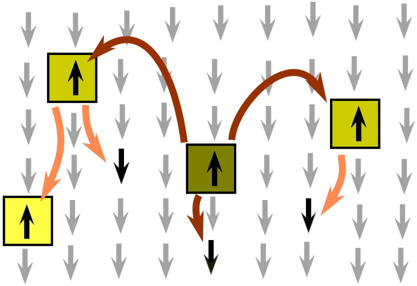


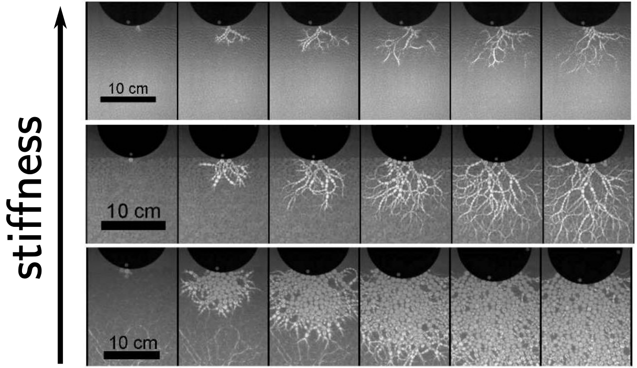
- Avalanches grow as a **percolation** process.
- For $N \to \infty$: **cluster size** $\approx$ **tree-size** in **Binomial G.W.**:

$$D(\Delta; n) \sim \boxed{\Delta^{-3/2}\, \mathcal{D}(n\Delta)}$$

- Similar in a BTW version: [*HM Brker, P Grassberger, EPL (1995)*] [*P. Grassberger, EPL (2022)*]

Structure of force chains in **granular** materials

short range vs. long-range

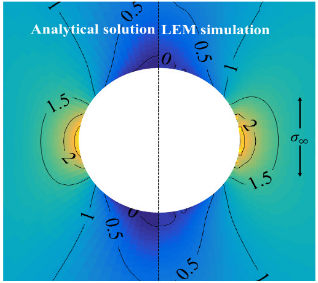


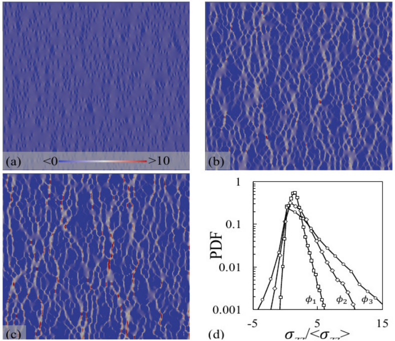
PRL **114,** 144502 (2015) PHYSICAL REVIEW LETTERS week ending 10 APRIL 2015

Nonlinear Force Propagation During Granular Impact

Abram H. Clark,[1,*] Alec J. Petersen,[1] Lou Kondic,[2] and Robert P. Behringer[1]

Structure of force chains in **porous** materials





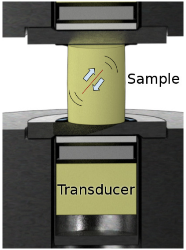
Selected for a Viewpoint in *Physics*

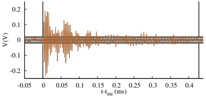
PRL **119,** 075501 (2017) PHYSICAL REVIEW LETTERS week ending 18 AUGUST 2017

Stress Transmission and Failure in Disordered Porous Media

Hadrien Laubie,[1,*] Farhang Radjai,[2,3,†] Roland Pellenq,[1,2,4,‡] and Franz-Josef Ulm[1,2,§]

Results in acoustic emission (a.e.) $\rightarrow$ SiO_2





$\Downarrow$

$$E \sim \int |\text{Signal}(t)|^2 dt$$

$N \sim 10^4$ pairs: $\{t_i, E_i\}$

[*JB, et al., PRL (2013)*]

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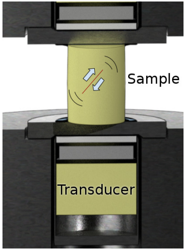

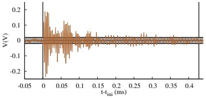

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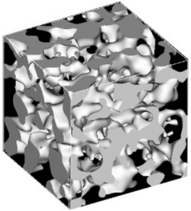
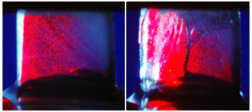
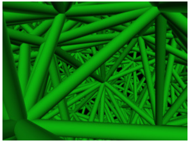
[JB, et al., PRL (2013)]

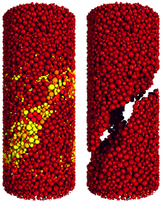
Vycor (SiO2):
JB at. al PRL (2013)





Computational Model:
F. Kun, I. Varga, S. Lennartz-Sassinek, I.G. Main
Phys. Rev. Lett. 112, 065501 (2014)





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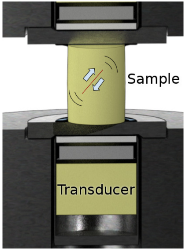

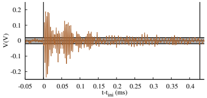

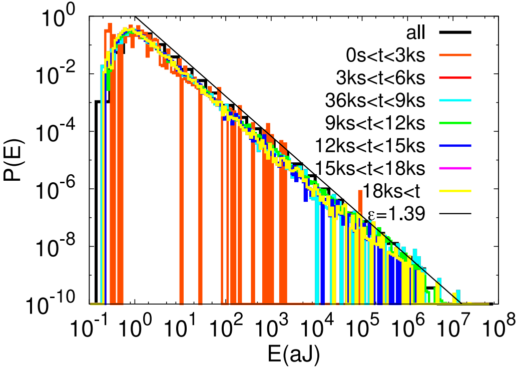

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[*JB, et al., PRL (2013)*]

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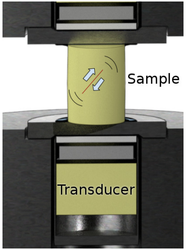

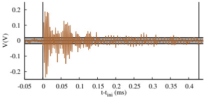

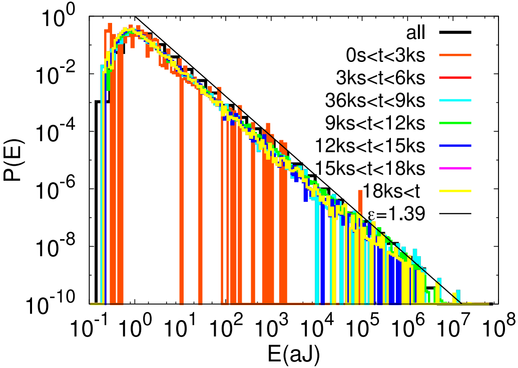

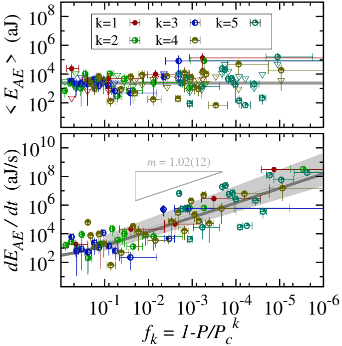

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Foreshocks preceding failure $\mu(t) \approx (t - t_f)^m$

[*JB & Davidsen, PRE (2018)*]

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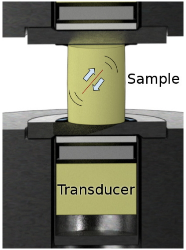

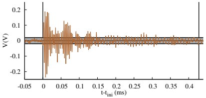

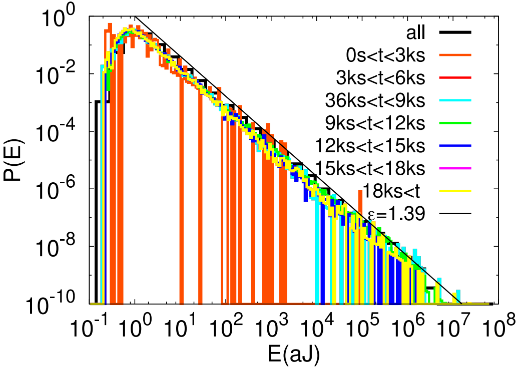

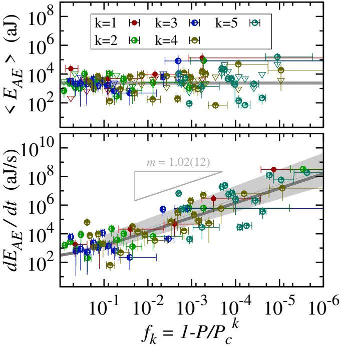

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[JB, et al., PRL (2013)]
[JB, et al., PRL (2018)]

	V32	G26	SR2	slip MF	fracture MF
$\gamma*$	3.0 (4)	3.4 (4)	3.2 (4)	3	3
ε	1.40 (5)	1.40 (5)	1.50 (5)	4/3	4/3
m	1.02 (13)	1.11 (20)	0.99 (8)	1	1/2
$\varsigma\nu z$	0.50 (6)	0.45 (6)	0.48 (5)	1/2	1/2
κ	1.60 (8)	1.62 (8)	1.76 (8)	3/2	3/2

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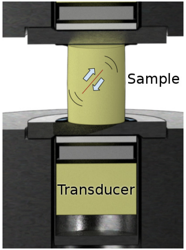

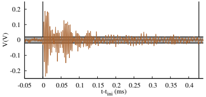

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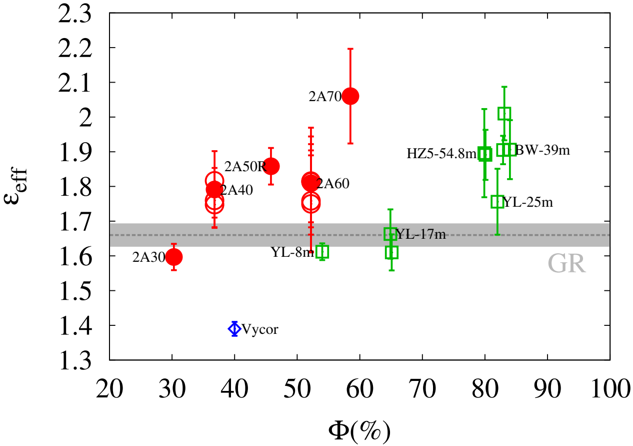
[*JB, et al., PRL (2013)*]
[*JB, et al., PRL (2018)*]

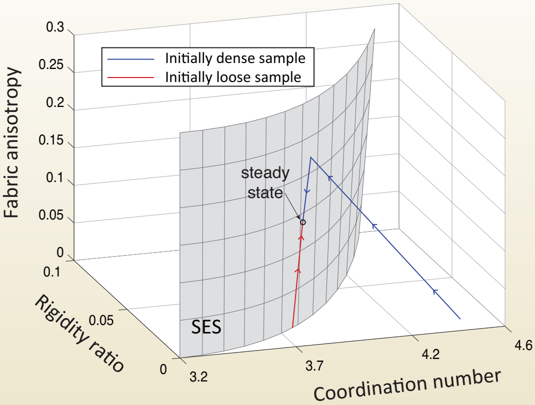
Typically $\varepsilon > \varepsilon_{MF}$. Porous pure $SiO_2 \rightarrow$ MF.

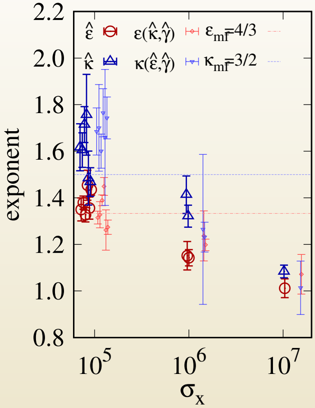


[*P. Castillo-Villa, JB, et al., J. Phys: Cond. Matt. (2013)*]

Conclusions

- Internal measures and theory: $\Delta U \propto S$, $K \propto E$.
- SES behaves as an state-attractor with SOC properties.
- Non-universal exponents depend on rigidity ($\Gamma \sim \sigma_x$)





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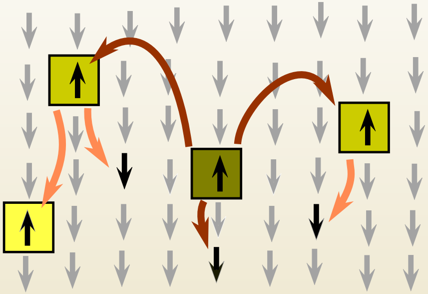

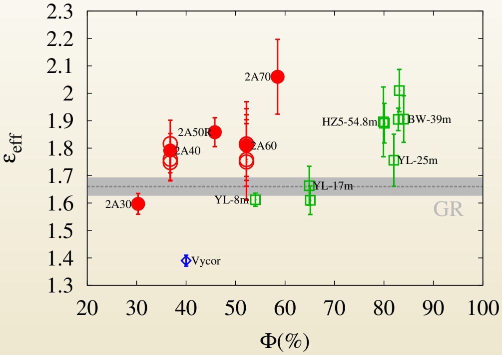

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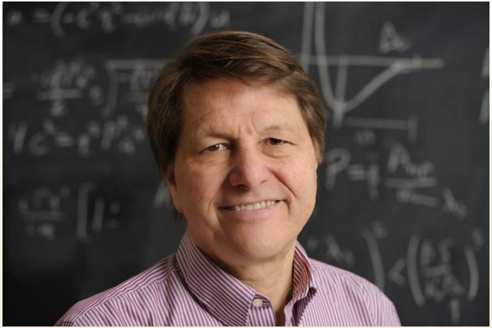
Ongoing research:

- Understand behavior expanding, contracting avalanches.
- MF $\leftrightarrow$ EPM: *Smooth transition? sharp transition? finite size effect (only exact at $\Gamma = 0$)? Hidden universal function? New finite size scaling techniques?*
- Determine avalanche properties in terms of SES (*different from classic avalanche statistics*). *Relation between avalanches at SES and potential energy landscape and kinematics.*
- Archaeology: Can we translate legacy results to SES? *Additional effects of friction, kinematics, rate, temperature, etc..*



Mark O. Robbins (1956-2020)

- **J. Baró, M. Pouragha, R. Wan, J. Davidsen** *Quasistatic kinetic avalanches and self-organized criticality in deviatorically loaded granular media* PRE 104 (2), 024901 (2021)

Stable Evolution Surface:

- M. Pouragha and R. Wan, Granular Matter 18, 38 (2016).

Experiments a.e.:

- J. Baró, et al., Phys. Rev. Lett. 110, 088702 (2013).
- J. Baró, et al., Phys. Rev. Lett. 120, 245501 (2018).
- P.O. Castillo-Villa, et al., J. Phys.: Cond. Matt. 25 292202 (2013)

solutions Mean Field stats.:

- J.P. Sethna, et al., PRL 70, 3347 (1993)
- K.A. Dahmen, Y. Ben-Zion, and J.T. Uhl, Phys. Rev. Lett. 102, 175501 (2009).
- J. Baró and J. Davidsen, Phys. Rev. E 97, 033002 (2018).

Mean Field from loopless tree:

- H.M. Brker, P. Grassberger, EPL 30 319 (1995)
- P. Grassberger, EPL, 136 26002 (2022)

Force chains in granular and porous mat.:

- A.H. Clark A.J. Petersen, L. Kondic, R.P. Behringer, PRL 114 144502 (2015)
- H. Laubie, F. Radjai, R. Pellenq, F.J.Ulm, PRL 119 075501 (2017)

Amorphous & LJ:

- K.M. Salerno, M.O. Robbins PRE, 88, 062206 (2013).
- Z. Budrikis, et al., Nat. Commun. 8, 15928 (2017).

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$\leftarrow$ More on SES: talk by M. Pouragha:
https://youtu.be/JclTxuJspQk?t=10650
(2:57:30 s)

$$\kappa = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$$

$$\begin{aligned} \langle T|S\rangle &\sim S^{\varsigma\nu z} && \text{where} \quad \varsigma\nu z = 1/2 \\ \langle E|S\rangle &\sim S^{2-\varsigma\nu z} && \text{where} \quad 2-\varsigma\nu z = 3/2 \\ \langle E_m|S\rangle &\sim S^{2\varsigma\rho} && \text{where} \quad 2\varsigma\rho = 1\ . \end{aligned} \tag{1}$$

$$\begin{aligned} P(S) &\sim S^{-\kappa} && \text{where} \quad \kappa = 3/2 \\ P(E) &\sim E^{-1-\frac{\kappa-1}{2-\varsigma\nu z}} && \text{where} \quad 1+\frac{\kappa-1}{2-\varsigma\nu z} = 4/3 \\ P(E_m) &\sim E_m^{-\frac{1+\mu}{2}} && \text{where} \quad \frac{1+\mu}{2} = 3/2\ . \end{aligned} \tag{2}$$

$$\begin{gathered} P(\Delta U)\, d\Delta U = \Delta U^{-\kappa}\, \Phi_{\Delta U}(\Delta U/\Delta U^*)\, d\Delta U, \\ P(K)\, dK = K^{-\varepsilon}\, \Phi_K(K/K^*)\, dK, \end{gathered} \tag{3}$$

	stiffness	#	κ	ε	γ
D2kSc5	(stiff)	1684	1.62(10)	1.32(10)	1.95(5)
L5kSc5	" "	979	1.60(7)	1.34(5)	1.85(10)
D5kSc5	" "	788	1.71(8)	1.33(6)	1.83(4)
L20kSc5	" "	130	1.77(17)	1.45(6)	1.85(15)
D20kSc5	" "	236	1.49(11)	1.36(4)	1.69(6)
D5kFc5	" "	1215	1.46(6)	1.36(4)	1.71(5)
D5kSc6		396	1.41(8)	1.14(11)	1.65(8)
D5kFc6		851	1.32(5)	1.14(6)	1.71(4)
D5kFc7	(soft)	633	1.08(3)	1.02(8)	1.48(7)
SMFT$^{(1)}$			1.5	$1 + \frac{\kappa - 1}{2 - \varsigma\nu z} = 1.33$	$2 - \varsigma\nu z = 1.5$
2D EPM			1.25–1.28	$\sim$1.2 [**?**]	$\sim$1.45[**?**]

name	num. of particles N	confining pressure $\sigma_x(N/m)$	driving rate $\dot{\epsilon}_y(\times 10^{-9}s^{-1})$	initial porosity ϕ_0
D20kSc5	19520	10^5	2.3	0.156
L20kSc5	19353	10^5	2.3	0.190
D5kSc5	6374	10^5	2.4	0.159
L5kSc5	5504	10^5	2.4	0.192
D2kSc5	1593	10^5	1.3	0.165
D5kSc6	6374	10^6	2.4	0.154
D5kFc5	6374	10^5	7.0	0.159
D5kFc7	6374	10^6	7.0	0.154
D5kFc7	6374	10^7	7.0	0.120

σ_x	σ_x/k_n	$\approx$ porosity
1e5	1e-4	0.1685
1e6	1e-3	0.1644
1e7	1e-2	0.1233

Magnitude Relations:

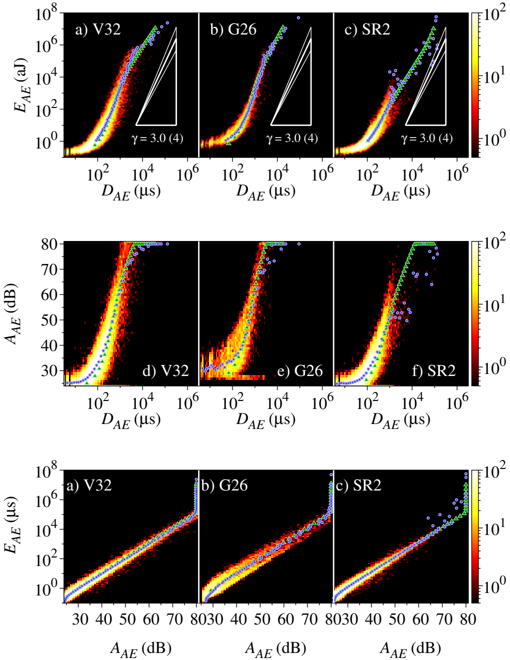
- AE magn. $\begin{cases} D_{AE} = t - t_i \| V < V_{th} \\ A_{AE} = \max(V(t)) \\ E_{AE} = \int_{t_i}^{t_i + D_{AE}} \|V(t)\|^2 dt \end{cases}$

- Signal Hypothesis:

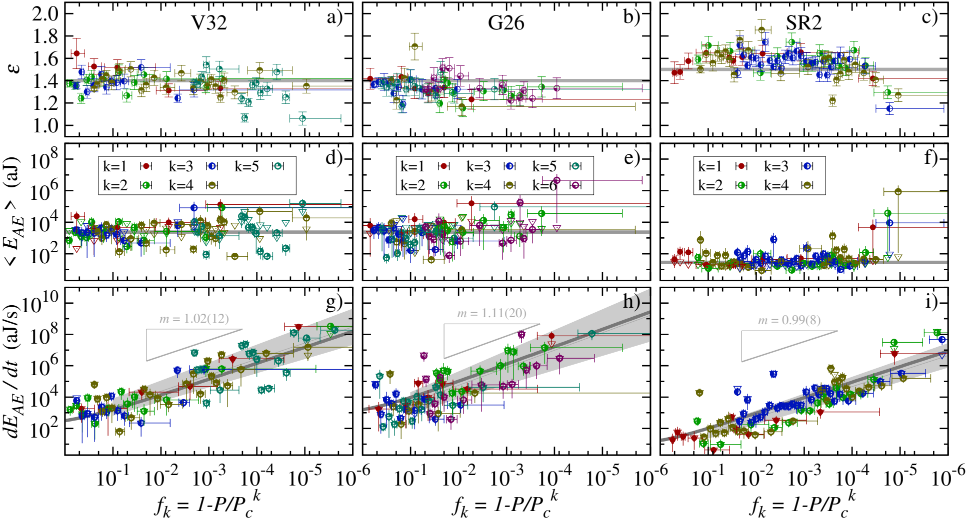
$$V(t) = G \int_{-\infty}^{t} v(t) e^{i\omega_0 t - \frac{t-t'}{\tau}} dt'$$

- Parabolic shape:

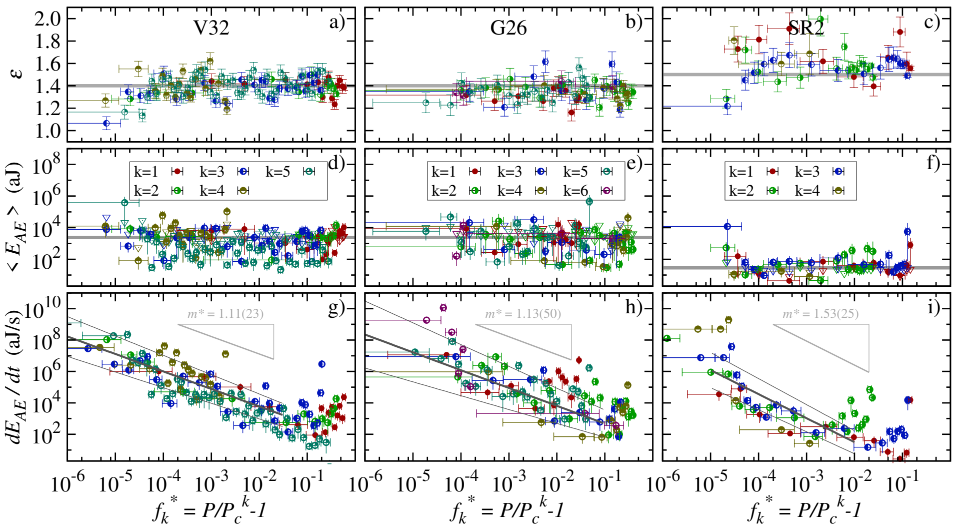
$$\tilde{v}(t/T) = 4\left(t/T - (t/T)^2\right)$$



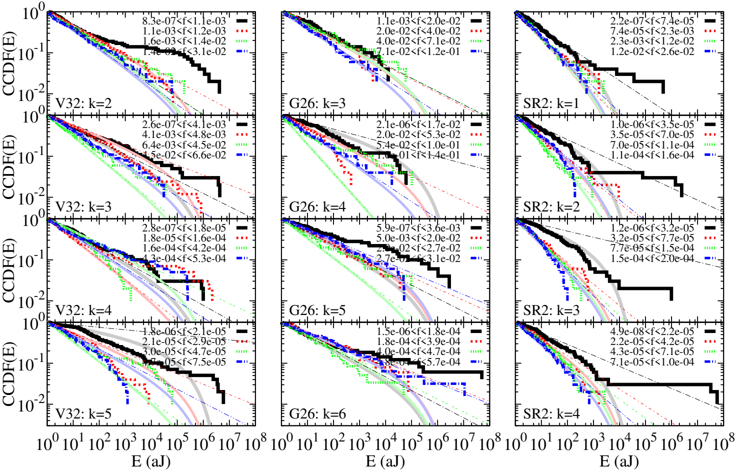
- Acceleration and energy exponent before failure:

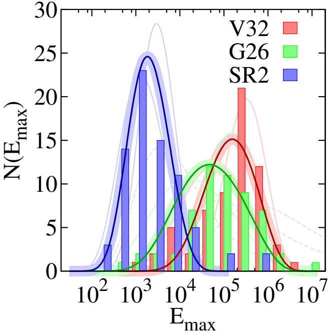


- Deceleration and energy exponent after failure:



Complementary Cumulative Distribution of Energies:





	V32	G26	SR2	slip MF	fracture MF
γ	3.0 (4)	3.4 (4)	3.2 (4)	3	3
ε	1.40 (5)	1.40 (5)	1.50 (5)	4/3	4/3
m	1.02 (13)	1.11 (20)	0.99 (8)	1^a 2^b	$1/2^a$ 1^b
$\sigma\nu z$	0.50 (6)	0.45 (6)	0.48 (5)	1/2	1/2
κ	1.60 (8)	1.62 (8)	1.76 (8)	3/2	3/2
σ^a	0.40 (9)	0.34 (9)	0.24 (8)	1/2	1
σ^b	0.88 (12)	0.80 (16)	0.76 (7)	1/2	1
β^a	3.7 ± 0.8	4.6 ± 1.2	6.3 ± 2.1	3	3/2
β^b	1.67 (24)	1.83 (37)	2.00 (25)	3	3/2

Table: First three top rows: fitted exponents in experimental data, compared to the MF exponents for slip and fracture MF models. Bottom rows: fundamental exponents estimated from MF theory. Superscripts a and b denote two different interpretations of ASR in terms of MF theory.

	area A (mm^2)	height h (mm)	driving rate dP/dt (kPa/s)	Th (dB)	N
Vycor (V32)	17.0	5.65	5.7	23	34138
Gelsil (G26)	46.7	6.2	0.7	26	5412
Sands. (SR2)	17.0	4.3	2.4	23	27271

Table: Sample details: crossectional area A; height h; compression rate dP/dt; number N of recorded signals above threshold Th.

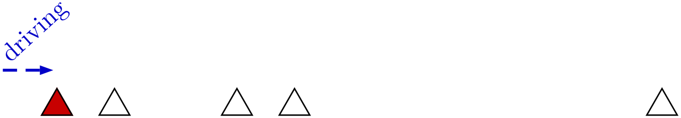
... mean-field ...
... exponent 3/2
?

Mean-field avalanche from positive-feedback.

when one element is activated:

$$\sigma_l \to \sigma_l + \Delta\sigma_l$$

(increment $\Delta\sigma_l \approx$ constant)



... mean-field ...

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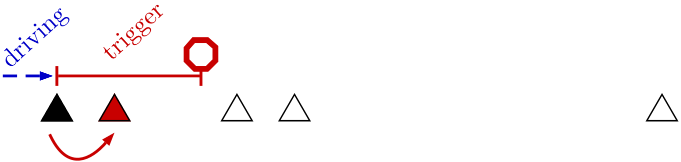

... mean-field ...

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Mean-field avalanche from positive-feedback.

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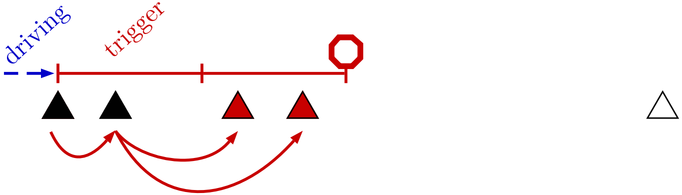

- The MF avalanche grows as a **branching** process.

Mean-field avalanche from positive-feedback.

... mean-field ...
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?

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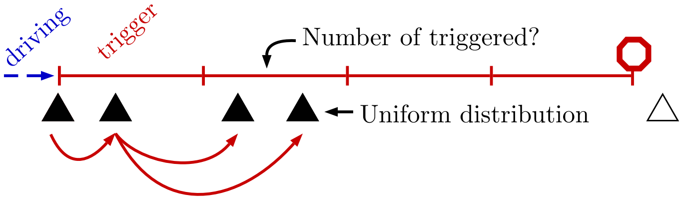

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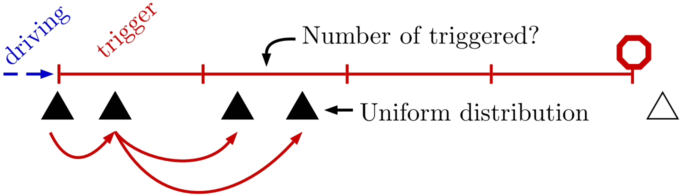


... mean-field ...
... exponent 3/2
?

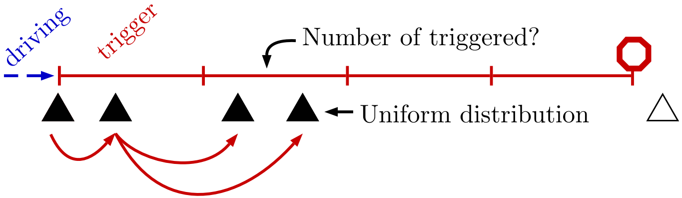
- The MF avalanche grows as a **branching** process.
- All elements can trigger a number of elements with the same **Poisson** distribution:

Mean-field avalanche from positive-feedback.

when one element is activated:

$$\sigma_l \to \sigma_l + \Delta\sigma_l$$

(increment $\Delta\sigma_l \approx$ constant)



... mean-field ...

... exponent 3/2

?

- The MF avalanche grows as a **branching** process.
- All elements can trigger a number of elements with the same **Poisson** distribution:
- **MF-avalanche size** $\equiv$ **tree-size** in **Poisson Galton-Watson**:

$$P(\Delta; n) = \frac{(n\Delta)^{\Delta-1}\exp(-n\Delta)}{\Delta!} \sim \boxed{\Delta^{-3/2}\,\mathcal{D}(n\Delta)}$$