Denouement of the energy-amplitude and size-amplitude enigma for acoustic emission avalanches

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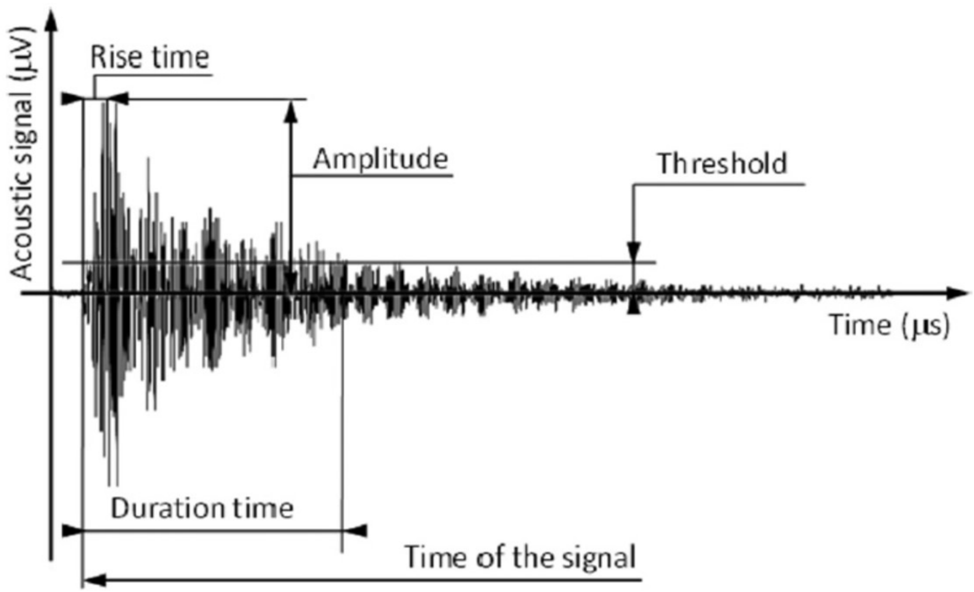
The enigma:

For acoustic emission (AE) avalanches the mean field theory (MFT) predicts power law relations between the energy, E, area, S, maximum amplitude, A_m, as well as duration time T (see e.g. [1]) :

$$S \propto T^{\gamma}, \qquad E \propto A_m^{\alpha} = A_m^{\frac{2\gamma-1}{\gamma-1}}, \quad S \propto A_m^{\beta} = A_m^{\frac{\gamma}{\gamma-1}} \text{, where } \gamma = 2. \qquad (1)$$

Thus, the power exponents α and β are 3 and 2, respectively .



$$E = \int_0^T U(t)^2 dt,$$

$$S = \int_0^T |U(t)| dt \qquad (2)$$

But, from experiments $\alpha \simeq 2$ and $\beta \cong 1$ ("enigma": [2])

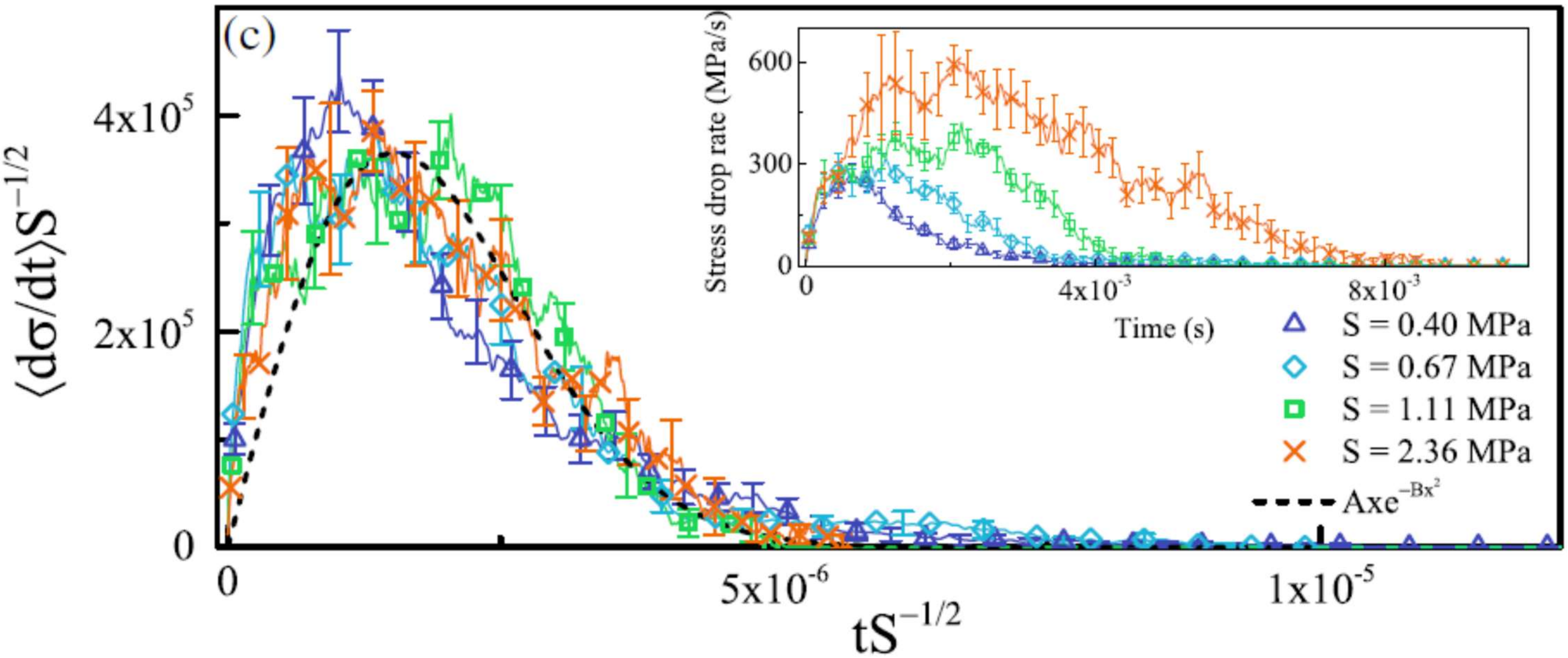
[1] M. Le Blanc et al. Phys. Rev. E 2013, 87, 022126; [2] B. Casals et al., Sci. Rep. 2021, 11, 5590

Similar problem:

Averaged, temporal shapes of avalanches, for fixed *S*, i.e. the *U(t)* functions (*U* is the detected voltage signal proportional to the interface velocity v(t), characteristic for the crackling noise emission, *t* is the time), has self-similar behaviour and thus, the properly normalized *U(t)* should be the same, independently of the type of materials. Normalization: e.g. by dividing both *U* and *t* by $\sqrt{S}$ [1] (based on the MF predictions (1), that $A_m \propto S^{\frac{1}{2}}$ and $T \propto S^{\frac{1}{2}}$)

But: there are experimental evidences and theoretical predictions that the average temporal shape of avalanches do not scale completely in a universal way [3,4].

[3] S. M. Kamel et al. Materials, 2022,15, 4556



Normalized stress drop rate (having similar meaning as U(t) for AE signals) belonging to slip avalanches in bulk metallic glass $Zr_{45}Hf_{12}Nb_5Cu_{15.4}Ni_{12.6}Al_{10}$: [4] J. Antonaglia et al., PRL. (2014) 112, 155501

In our recent paper (S. M. Kamel et al. Materials, 2022,15, 4556) we investigated the above two problems.

Let us start from the so called MF toy model for *U(t)* at fixed *S* (D.S. Fisher, Phys. Rep. (1998) 301, 4556, Dobrinevski et al. EPL 2014, 108, 66002, B. Casals et al., Sci. Rep. 2021, 11, 5590):

$\mathbf{U(t) = atexp(-\left(\frac{t}{\tau}\right)^2)}$, *a* and τ are material dependent constants (3)

Furthermore, for scaling the *U* and *t* axes, use the maximum amplitude, A_m and rising time, *R* (which are free from experimental distortions of AE signals: see also below): $U^*(t^*) = \frac{U}{A_m}\left(\frac{t}{R}\right)$. Since the maximum of (3) is at $R = \frac{\tau}{\sqrt{2}}$

$$A_m = ae^{\frac{1}{2}}R = BR,$$ i.e. the scaling parameters are interrelated.

We assumed that $B = \frac{A_m}{R} \propto A_m^{\varphi}$, where φ is material independent and is the same for the same mechanism.

Using $\frac{A_m}{R} \propto A_m^{\varphi}$ and the dimensionless definitions of the energy and area

$$E^* = \frac{E}{A_m^2 R} = \int_0^{T^*} U(t)^{*2} dt^* \cong const. \propto \frac{E}{A_m^{3-\varphi}}$$

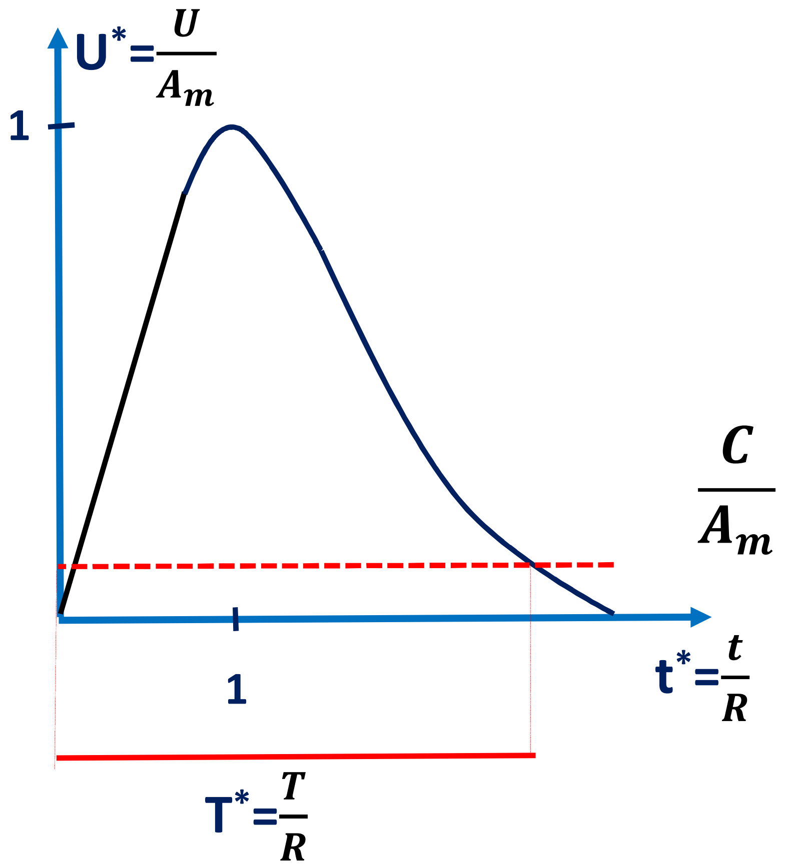
and

$$S^* = \frac{S}{A_m R} = \int_0^{T^*} |U^*(t^*)| dt^* \cong const. \propto \frac{S}{A_m^{2-\varphi}} \quad \text{(with } T^* = \frac{T}{R}\text{)}$$

DENOUEMENT OF ENIGMAS.

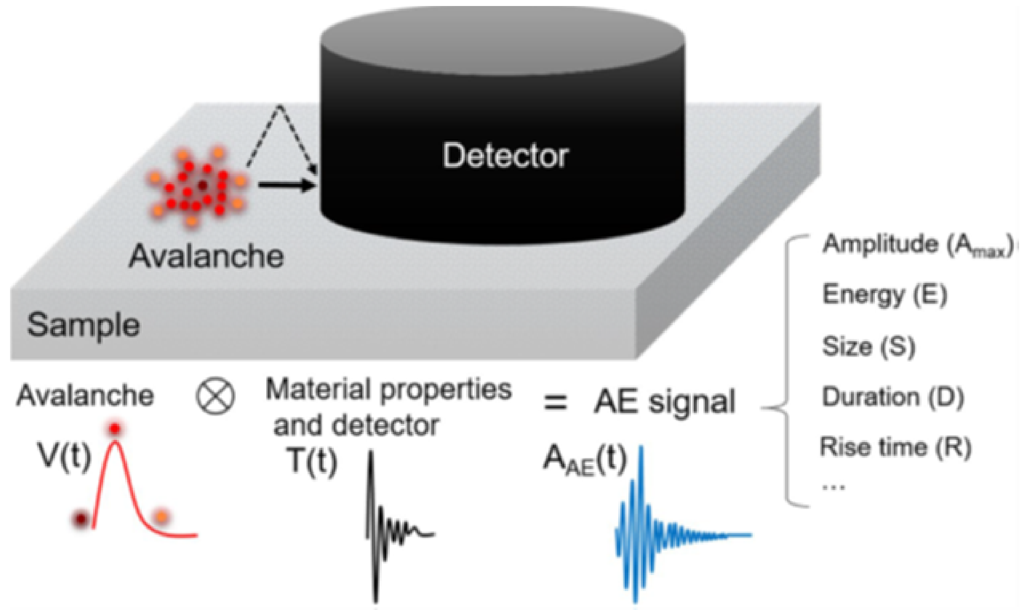
For experimental verification (e.g. for proving that the integrals above are indeed constants) we had also to deal with the problem of distortion of AE signals either by *finite threshold effects* and by *transfer problems*.

The *threshold effects* are measured by $\frac{C}{A_m}$



$T^* \sim \sqrt{\log \frac{A_m}{C}}$, i.e. T^* goes to an asymptotic limit as $\frac{A_m}{C}$ goes to infinity.

Transfer distortions (B. Casals et al. 2021) are measured by $\frac{\tau_a}{T}$

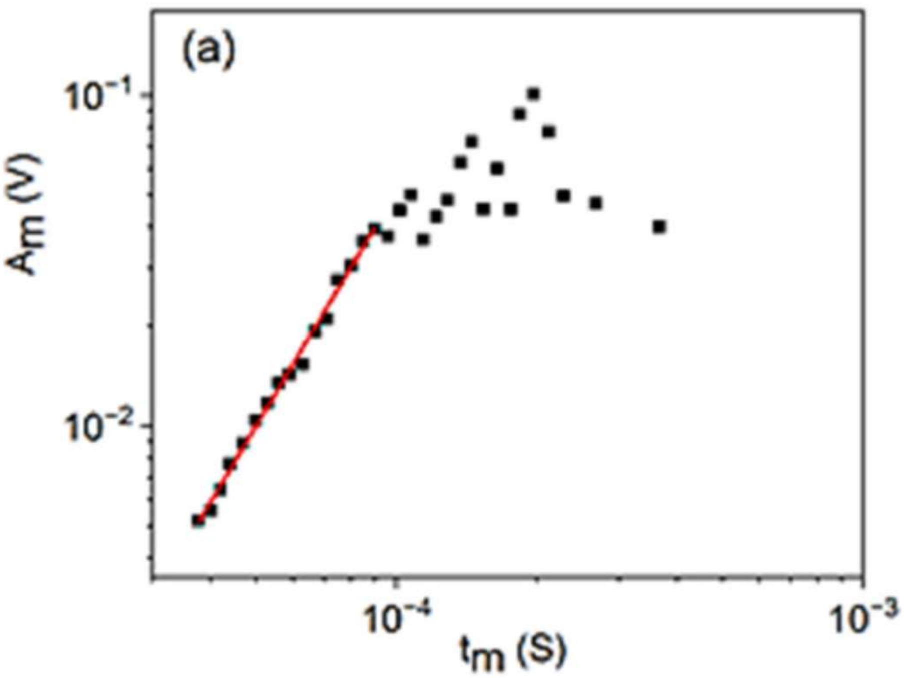


$A_{AE}(t) \equiv U(t)$, and $T(t) = \cos(\omega t)\exp(-t/\tau_a)$

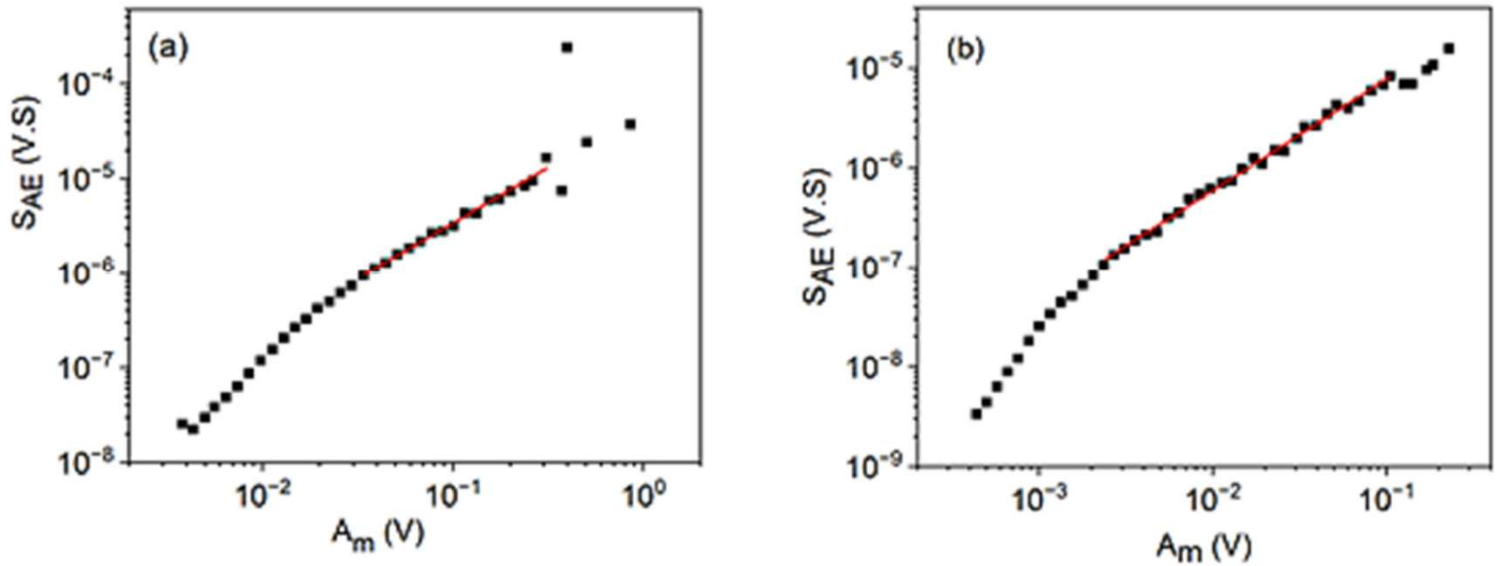
Barcelona's group (J. Baro, Thesis 2018; PRL, (2018) 120, 245501): these can be neglected if $\frac{\tau_a}{T} \ll 1$.

We have demonstrated that its is possible to choose proper windows of fit on the A_m axis to get reliable exponents (lower bounds are given by $\frac{C}{A_m} \ll 1$ and $\frac{\tau_a}{T} \ll 1$, while the upper bounds are determined by the overlaps of avalanches and/or by small numbers of hits).

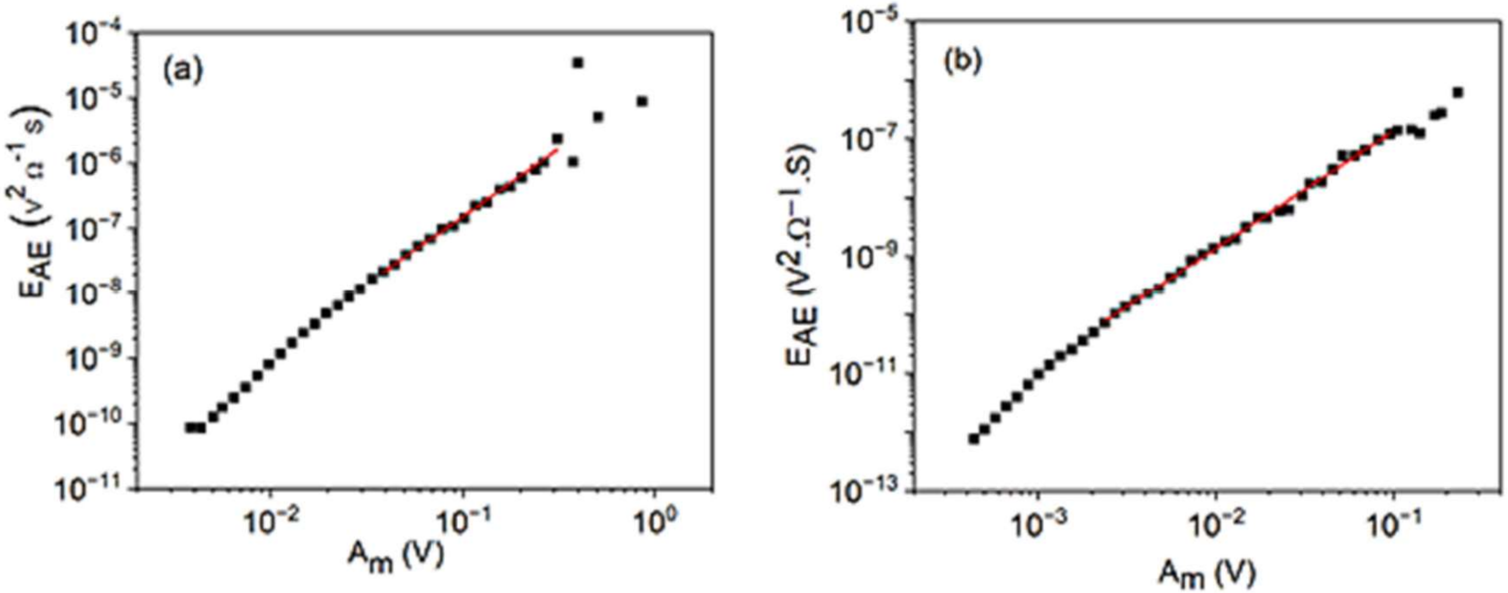
Experimental results for martensitic transformation in two shape memory single crystals ($Ni_{45}Co_5Mn_{36.6}In_{13.4}$; alloy *A,* and $Ni_{49}Fe_{18}Ga_{27}$ Co_6; alloy *B*)



Log A_m **versus** ***log R (where*** $R \equiv t_m$) **for alloy *A* in cooling at small constant magnetic field *B=250 mT*). The slope is $2.4 = \frac{1}{1-\varphi}$, which gives φ *=0.6±0.1.***



Log *S* versus log A_m for cooling of alloy *A* at (B=250 mT) (a) as well as for alloy *B* for heating. The slopes are 1.2±0.1 and 1.11±0.05, i.e. φ=*0.8±0.1* and φ=*0.9±0.08*, respectively.

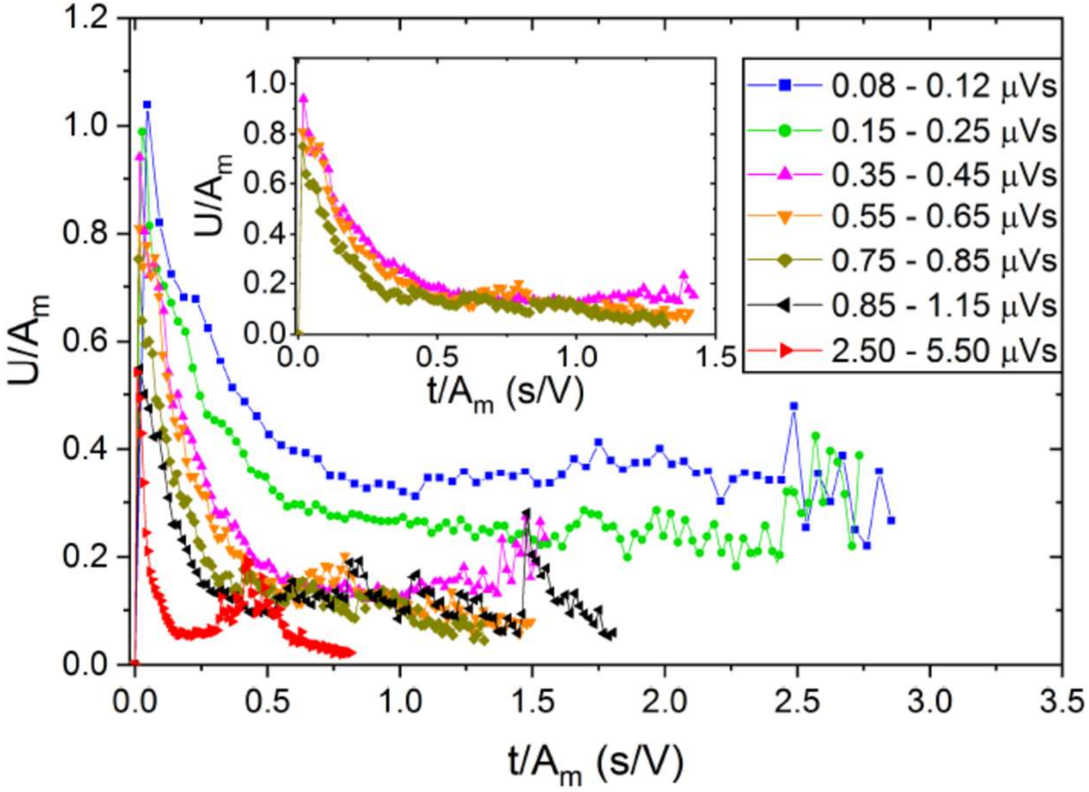


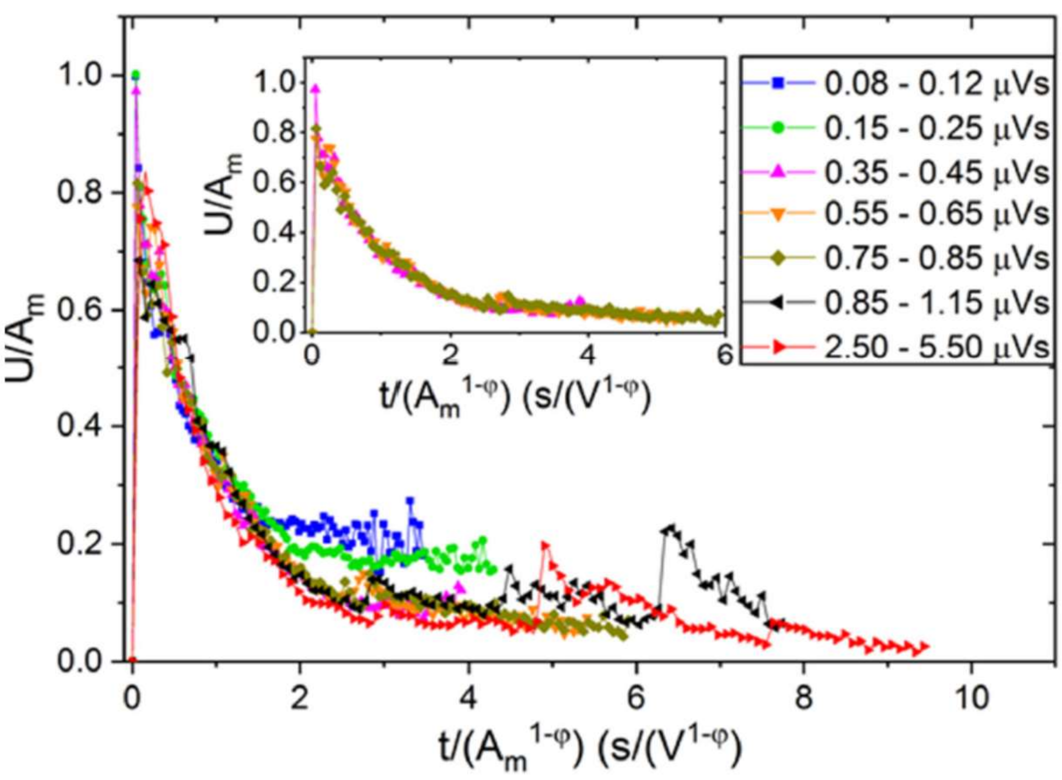
Log *E* versus log A_m for cooling of alloy *A* at (B=250 mT) (a) as well as for alloy *B* for heating. The slopes are 2.00±0.2 and 2.08±0.08, i.e. φ=*0.9±0.1* and φ=*1.0±0.1*, respectively.

1 Equation	Value of φ	
	alloy A	**alloy B**
$\mathbf{B}= \frac{A_m}{R} \propto A_m^{\varphi}$	0.6±0.1	0.6±0.1
$S \propto A_m^{2-\varphi}$	0.8±0.1	0.90±0.08
$E \propto A_m^{3-\varphi}$	0.9±0.1	1.0±0.1
average	0.77±0.11	0.83±0.13

φ is the same for both alloys: 0.8±0.1

Construction of the $U^*(t^*)$ function: since $R \propto A_m^{1-\varphi}$ the U and the t axes are normalized by A_m as well as by $A_m^{1-\varphi}$, respectively. For alloy B





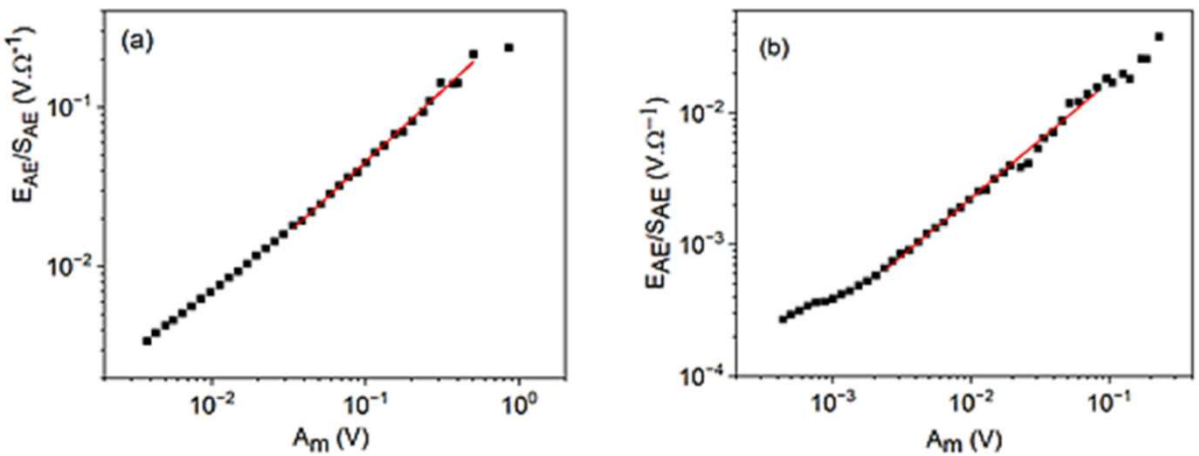
Conclusions:

i) If the voltage scale and the time scale are normalized by A_m and R, then the toy model predicts a relation between the scaling parameters as

$$B = \frac{A_m}{R} \propto A_m^{\varphi}. \qquad \text{and}$$

$$S \propto A_m^{2-\varphi} \qquad \text{as well as} \qquad E \propto A_m^{3-\varphi}.$$

The mean field results are obtained only if $\varphi=0$, which also would mean that $A_m \propto R \propto \tau$. Interestingly the E/A_m ratio does not contain φ and γ thus the linear dependence of this ratio can be a good tool to check the reliability of a given measurement.



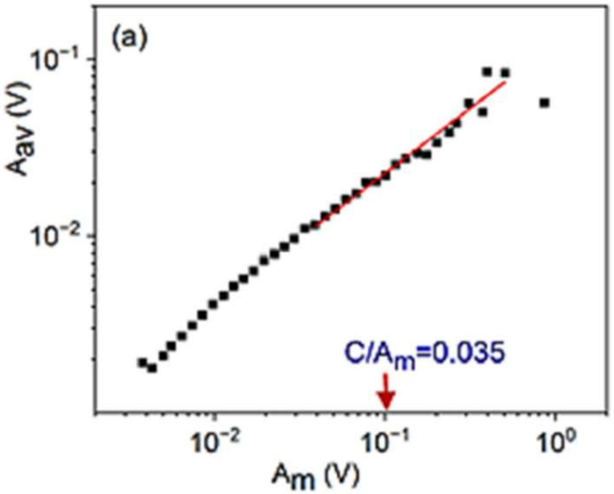
The slopes are 0.9±0.1 for both alloys.

Figure 4. $log\frac{E_{AE}}{S_{AE}}$ versus $log A_m$ plots for cooling in alloy A (**a**) at small, constant, external magnetic field (B = 250 mT) and for heating in alloy B (**b**) (at B = 0).

ii) Using that $T^{*} \sim \sqrt{\log \frac{A_m}{C}}$, i.e. T^{*} goes to an asymptotic limit as C goes to zero, it can be shown (in contrast to M. Le Blanc et al. Phys. Rev. E 2013, 87, 022126) that the average amplitude $A_{av} = \frac{S}{T}$ is not linearly related to A_m but

$$\frac{\partial lnA_{av}}{\partial lnA_m} = z = 2 - \varphi + \frac{1}{2ln\frac{Am}{C}} - \theta,$$

where θ characterizes the transfer effects ($\theta = \frac{logT}{log\ A_m}$, for θ=1 these are neglected)



Relation between A_{av} and A_m for cooling transformation in alloy A , the slope is z= 0.74; the arrow shows the centre of fit ($\theta = 0.8$).

iii) Using A_m and $A_m^{1-\varphi}$ parameters for reducing the voltage and time scales, respectively nice common temporal avalanche shapes were obtained for different bins of area

Thank you very much for your attention!

Comparison of scaling both scales with A_m, or with $S^{\frac{1}{2}} \sim (A_m t_m)^{\frac{1}{2}} \sim A_m^{1-\frac{\psi}{2}}$, or scaling U by A_m and t by $A_m^{1-\varphi}$