Avalanche phase diagram for thermally activated yielding in amorphous solids

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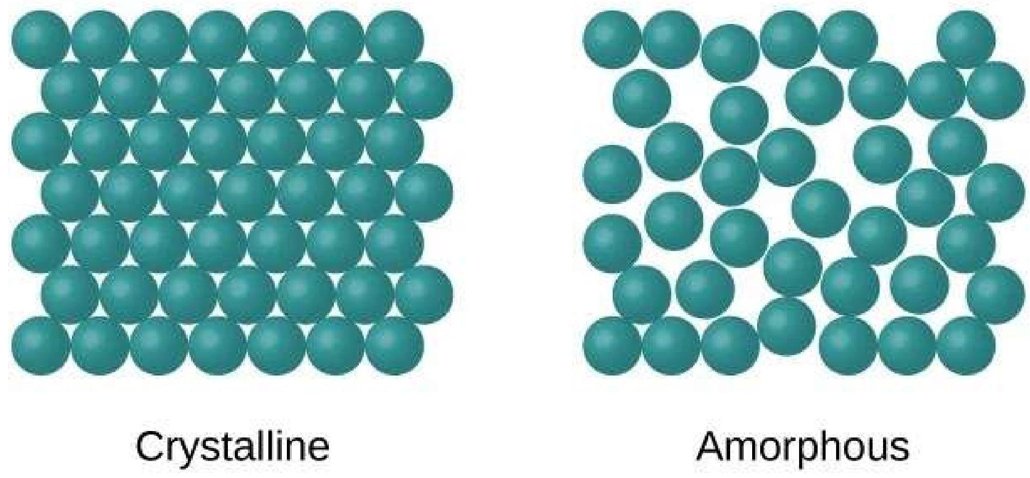
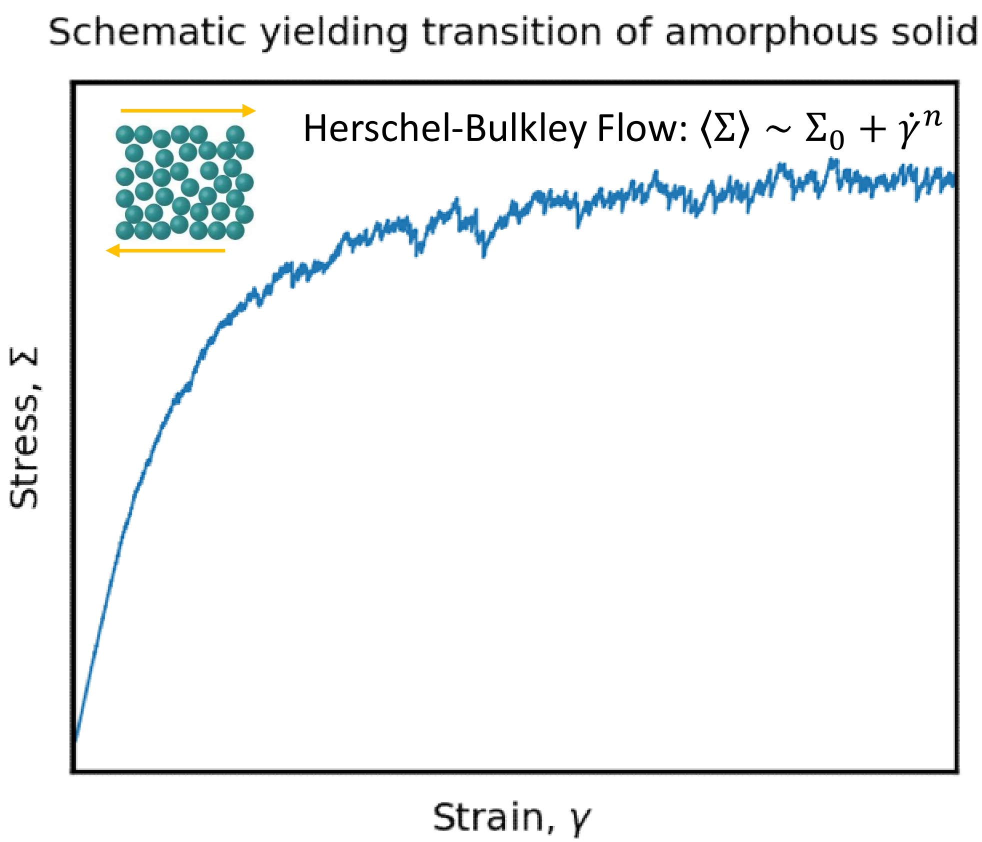
Amorphous yielding transition

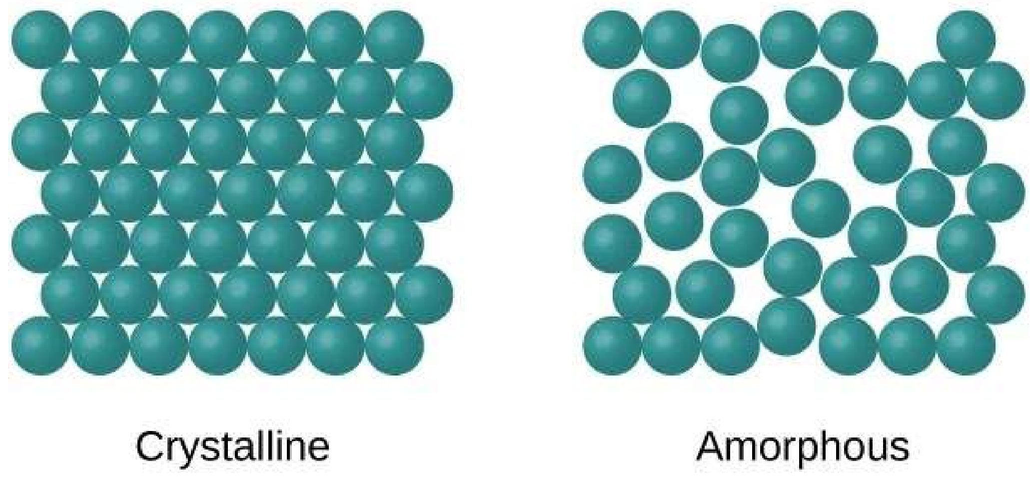


Amorphous solid graphic: opentextbc.ca

Alexandre Nicolas et al. 2018. *Review of Modern Physics*

Amorphous yielding transition





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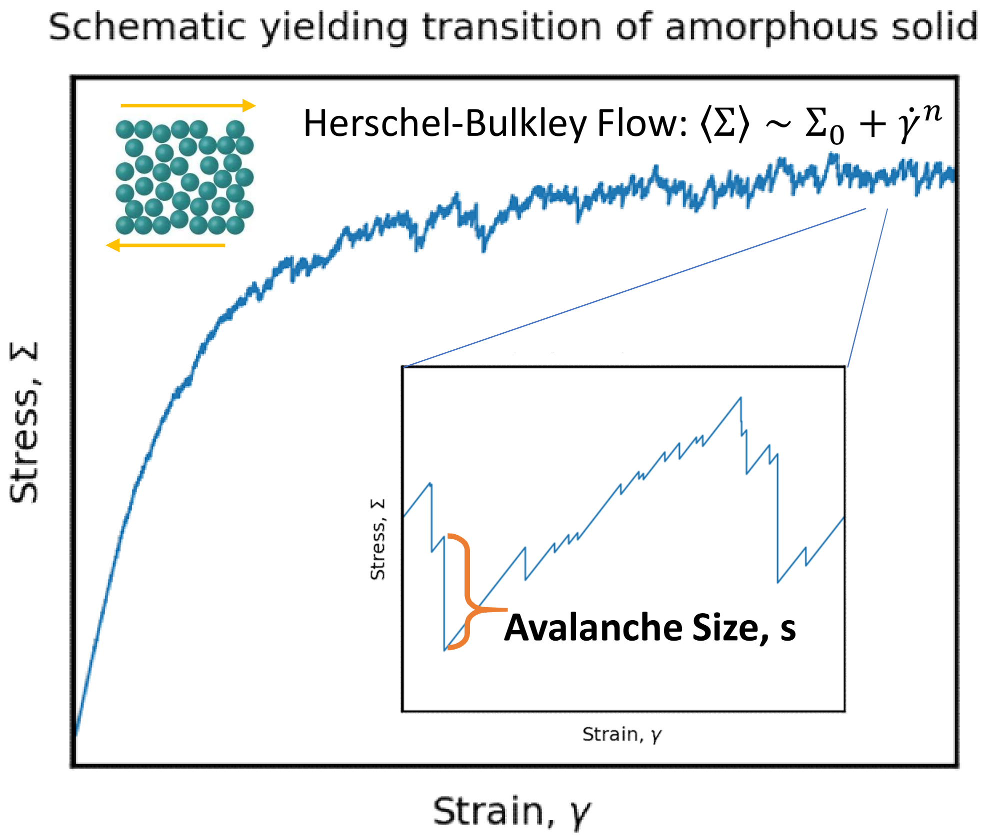

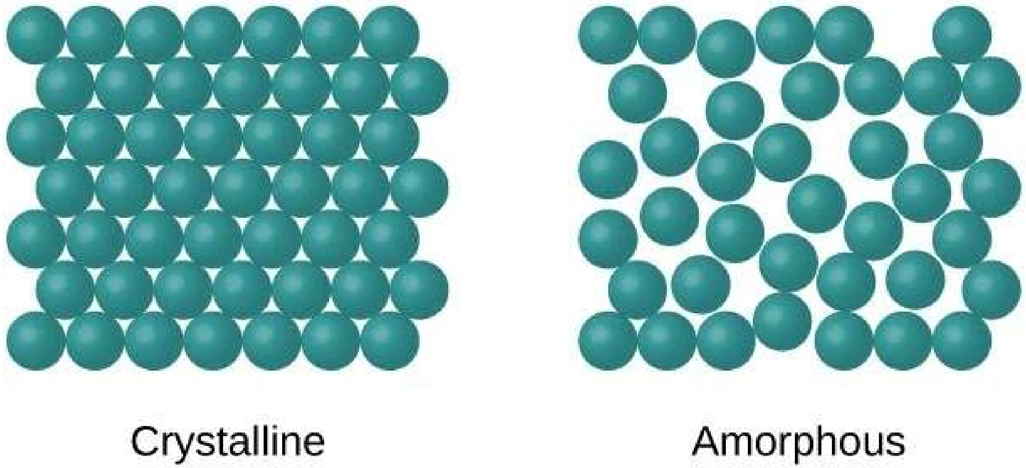

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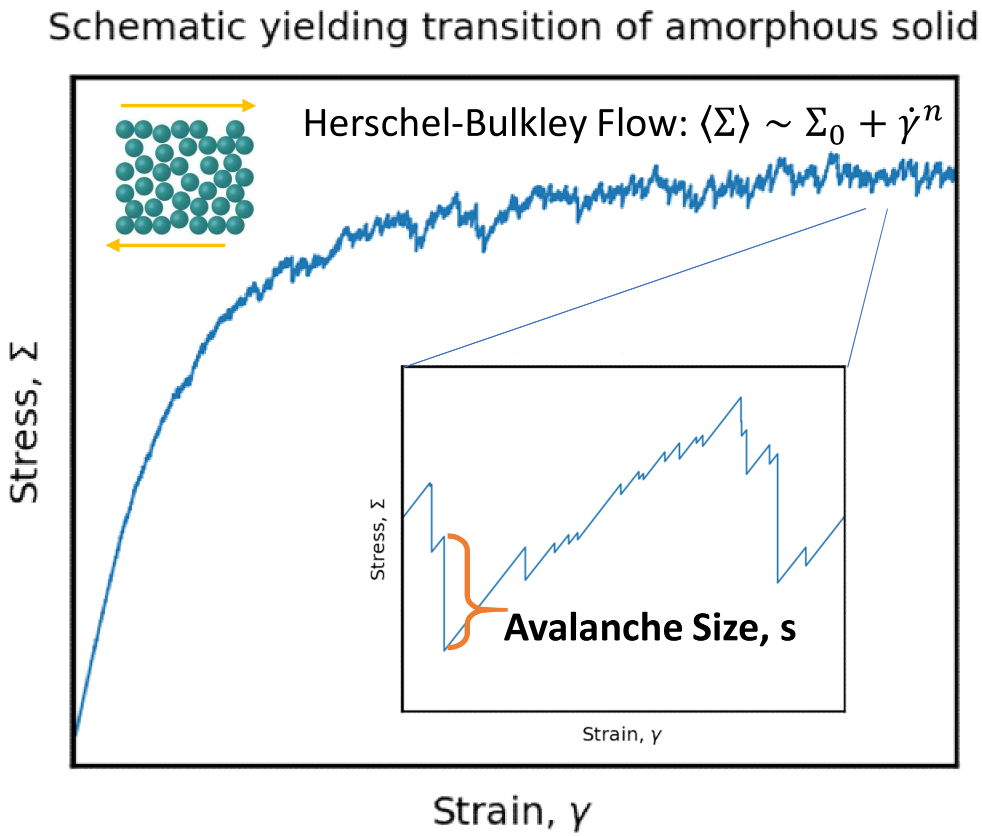
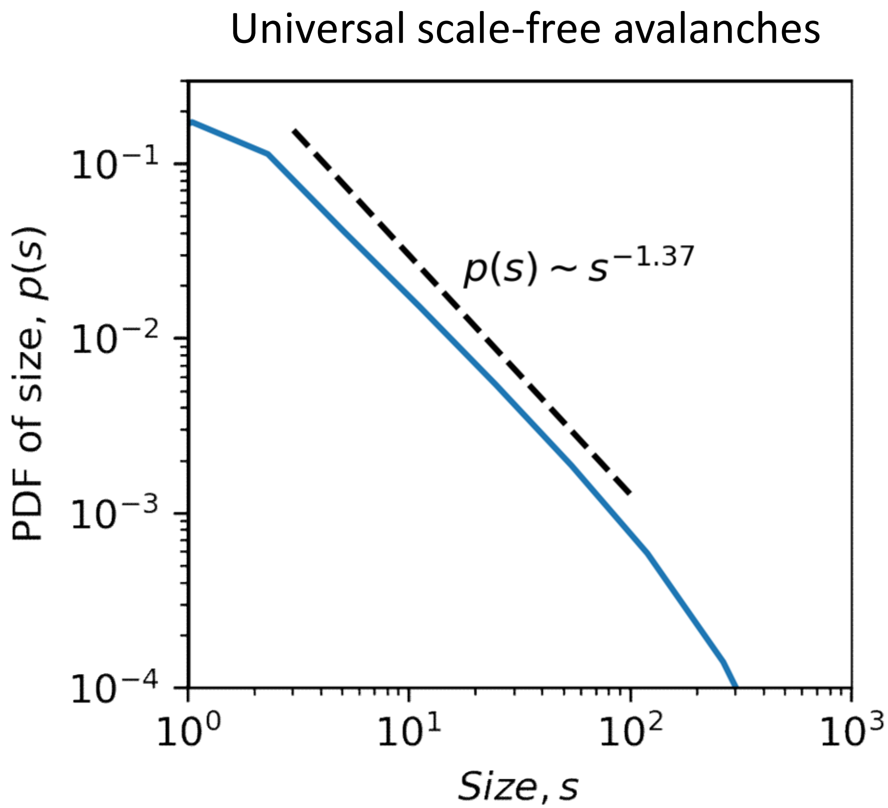
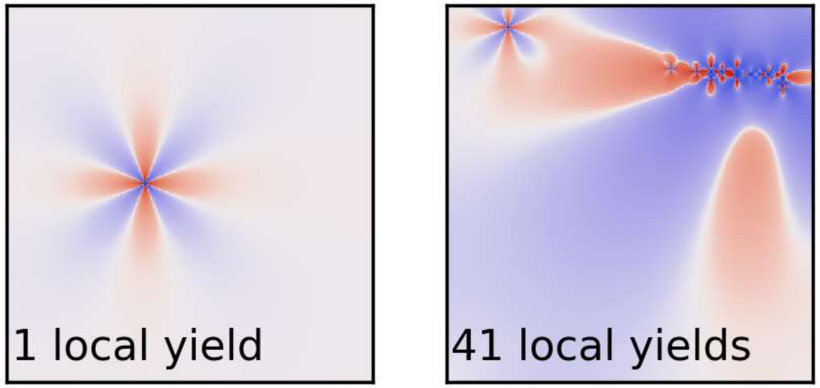




Amorphous solid graphic: opentextbc.ca

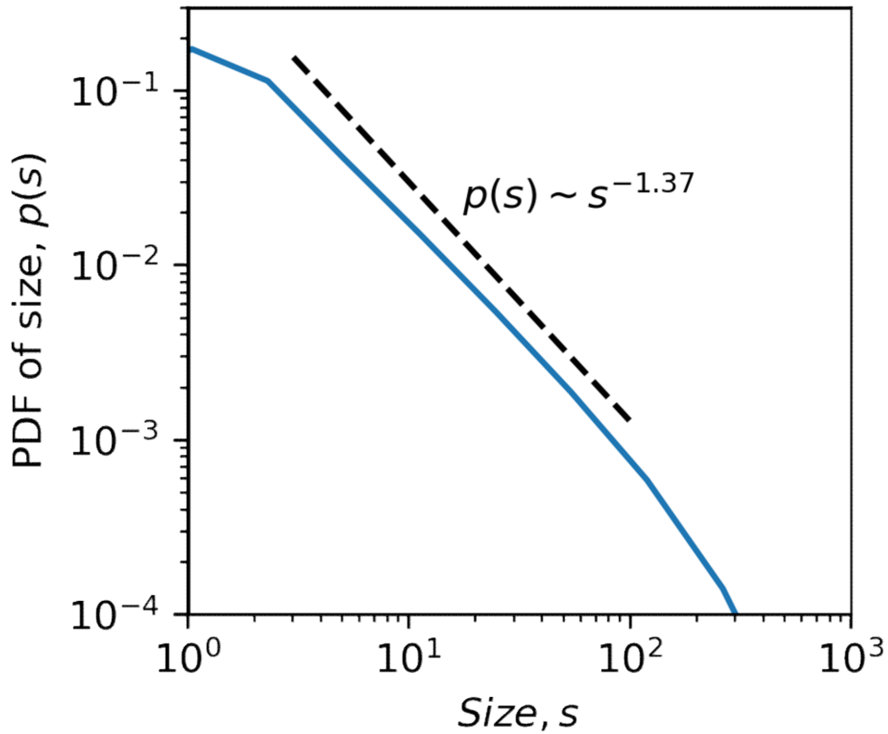
Amorphous yielding transition

- Beautiful scaling theory in athermal quasistatic (AQS) limit, distinct from depinning
- Avalanches proceed through shear-transformations with quadrupolar interactions



Stress Fields

Universal scale-free avalanches



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What happens to avalanches with temperature?

- Partly answered in molecular dynamics (See: Karmakar et al. *PRE.* 2010)
  - Expect driving rate / temperature dominated regimes
  - Crossovers depend on system size
  - Herschel-Bulkley stress-rise occurs as avalanches overlap

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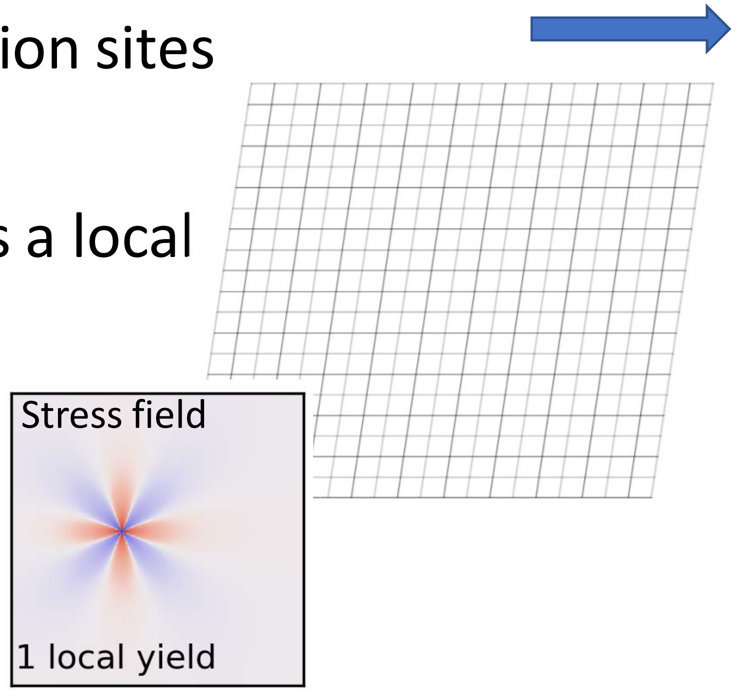
- Partly answered in molecular dynamics (See: Karmakar et al. *PRE.* 2010)
  - Expect driving rate / temperature dominated regimes
  - Crossovers depend on system size
  - Herschel-Bulkley stress-rise occurs as avalanches overlap
- Elastoplastic models expose several new aspects:
  - Residual stress distribution
  - Can probe very long timescales / low temperatures

Athermal Mesoscopic Model of Amorphous Yielding

Coarse grain to level of shear transformation sites

- Sites elastically coupled (finite element)
- Site i yields when local stress Σ_i exceeds a local threshold $\Sigma_{y,i}$, i.e. $x_i = \Sigma_{y,i} - |\Sigma_i| = 0$

Residual stress



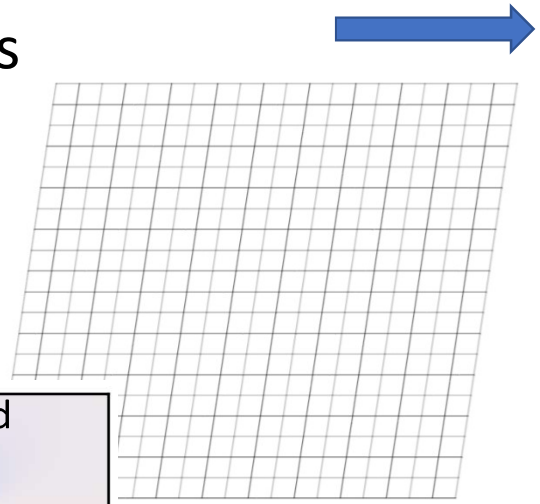
For review of mesoscopic models, see Nicolas et al. Rev. Mod. Phys. **90**, 045006

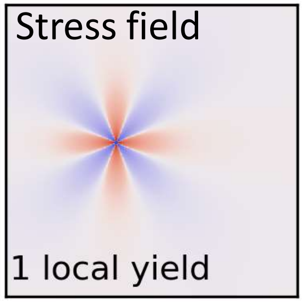
Thermal Mesoscopic Model of Amorphous Yielding

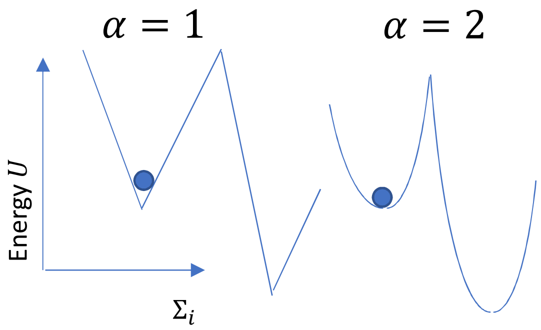
Coarse grain to level of shear transformation sites

- Sites elastically coupled (finite element)
- Site i yields when local stress Σ_i exceeds a local threshold $\Sigma_{y,i}$, i.e. $x_i = \Sigma_{y,i} - |\Sigma_i| = 0$
- Or stochastically, with Arrhenius rate

$$\lambda(x) = \frac{1}{\tau_{plastic}} \exp\left[-\frac{\mathrm{x}^{\alpha}}{\mathrm{T}}\right]$$





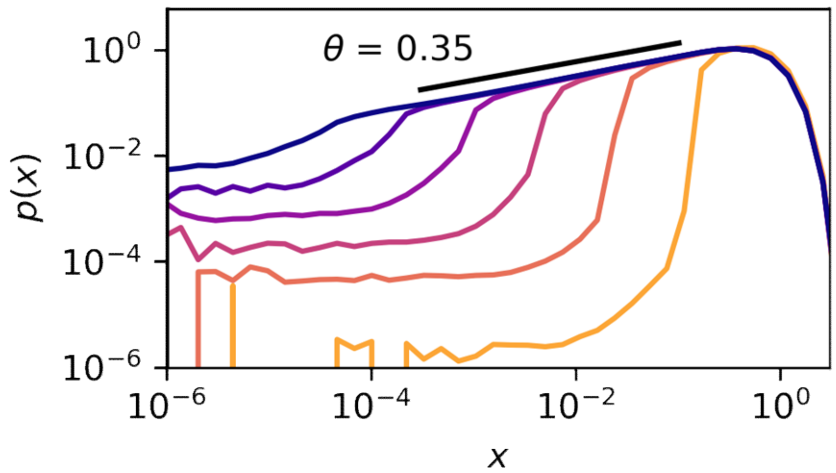


See:
Marko Popović et al. 2021
Ezequiel Ferrero et al. 2021
For studies of this model and Herschel-Bulkley temperature dependence

For review of mesoscopic models, see Nicolas et al. Rev. Mod. Phys. **90**, 045006

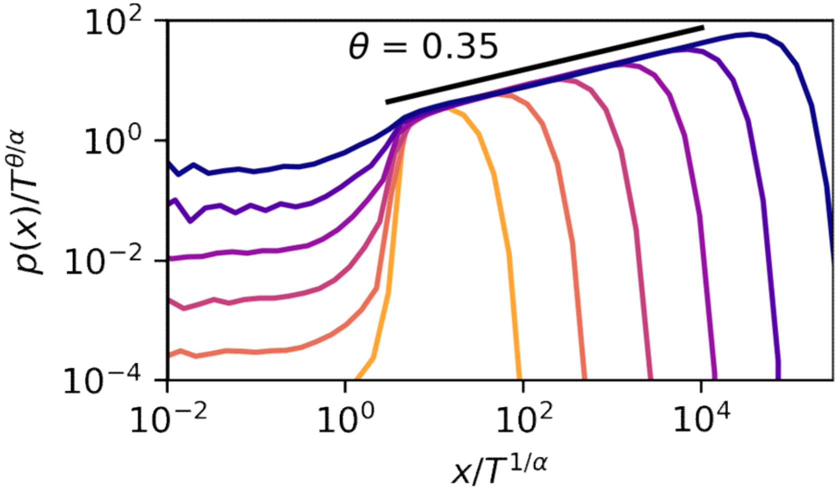
Results: Residual stress distribution

- $p(x) \sim x^{\theta}$ for $T = 0$ and large L



Results: Residual stress distribution

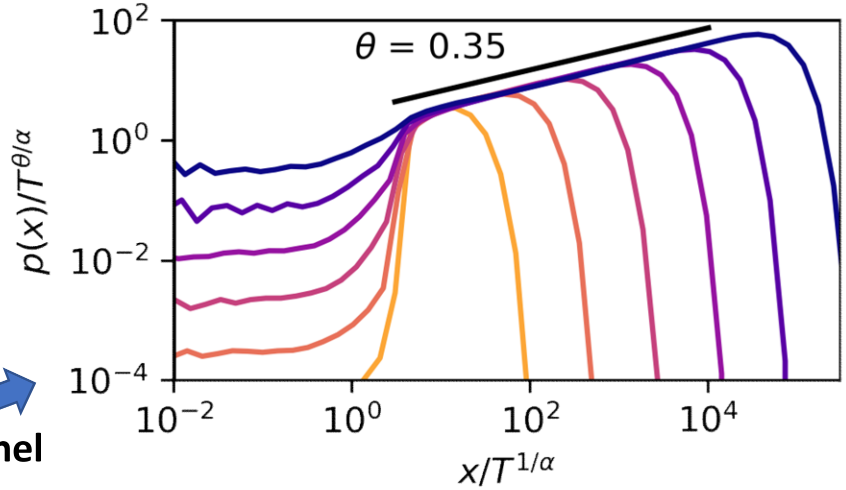
- $p(x) \sim x^{\theta}$ for $T = 0$ and large L
- Thermal activation scale: $x_c \sim T^{\frac{1}{\alpha}}$



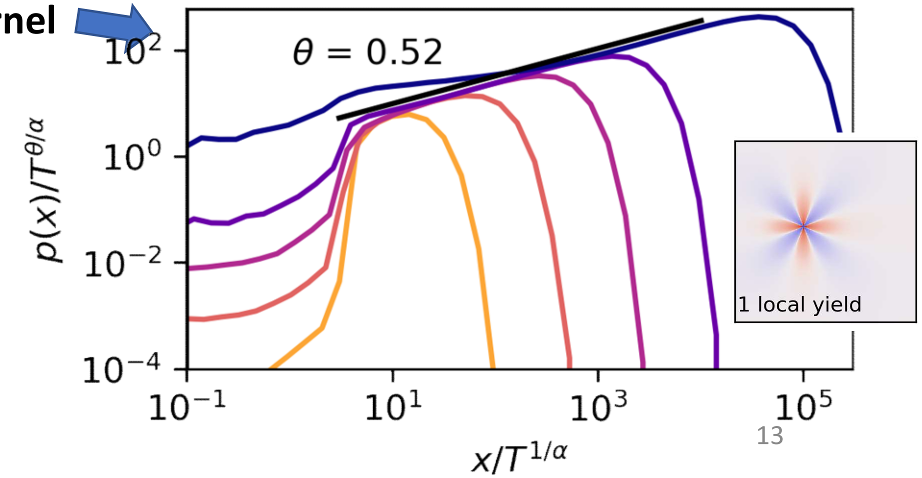
Results: Residual stress distribution

- $p(x) \sim x^{\theta}$ for $T = 0$ and large L
- Thermal activation scale: $x_c \sim T^{\frac{1}{\alpha}}$
- We vary θ to test scaling laws by shuffling the kernel

Mean-field (mf) shuffled kernel



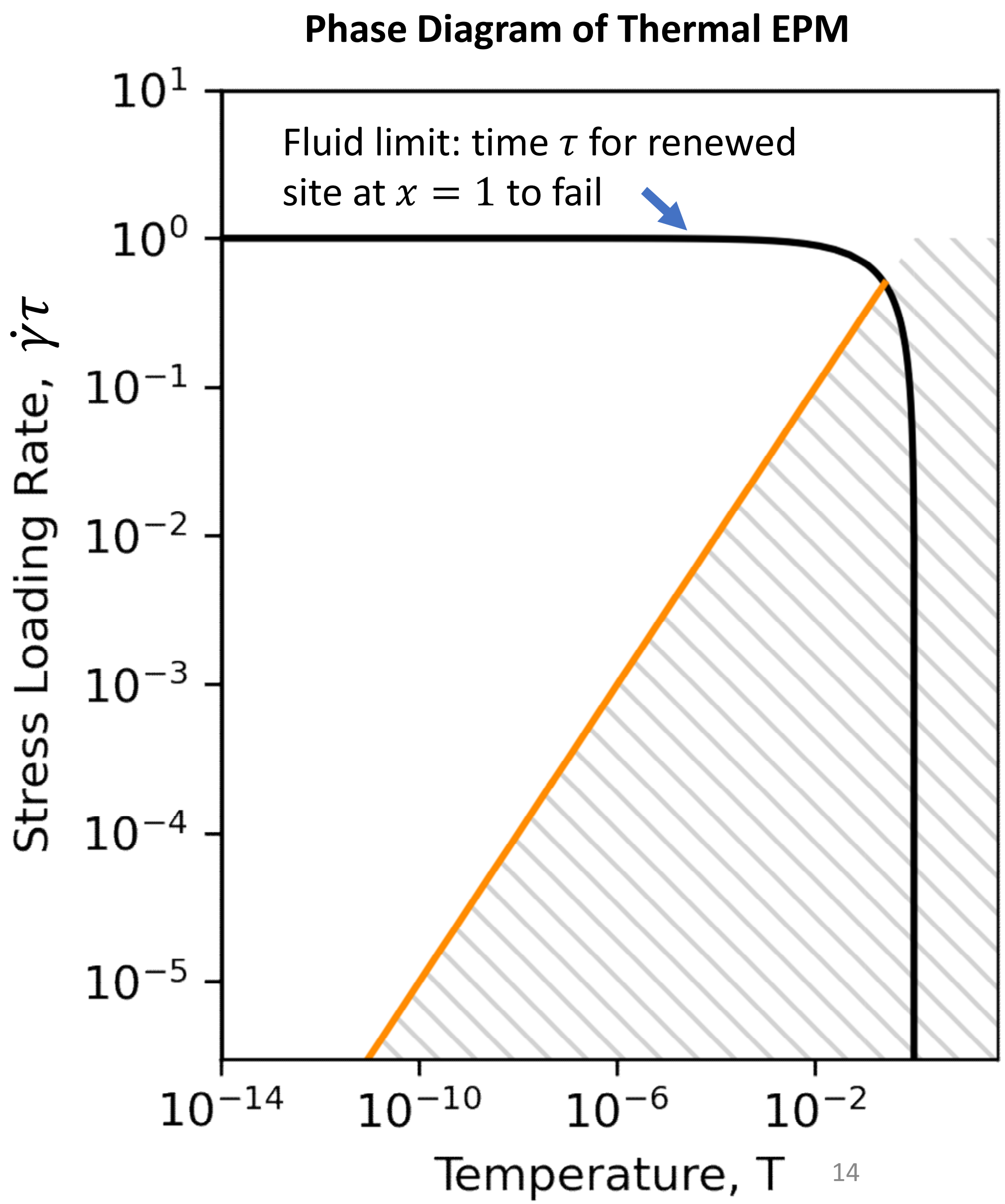
Two-dimensional (2d) kernel



Results: Phase diagram

- Most phase lines originate from competition of timescales
- Main timescales
  - t_{load} between avalanches
  - $\tau_{plastic}$ the ST plastic time

Temperature effects ≫ driving rate effects



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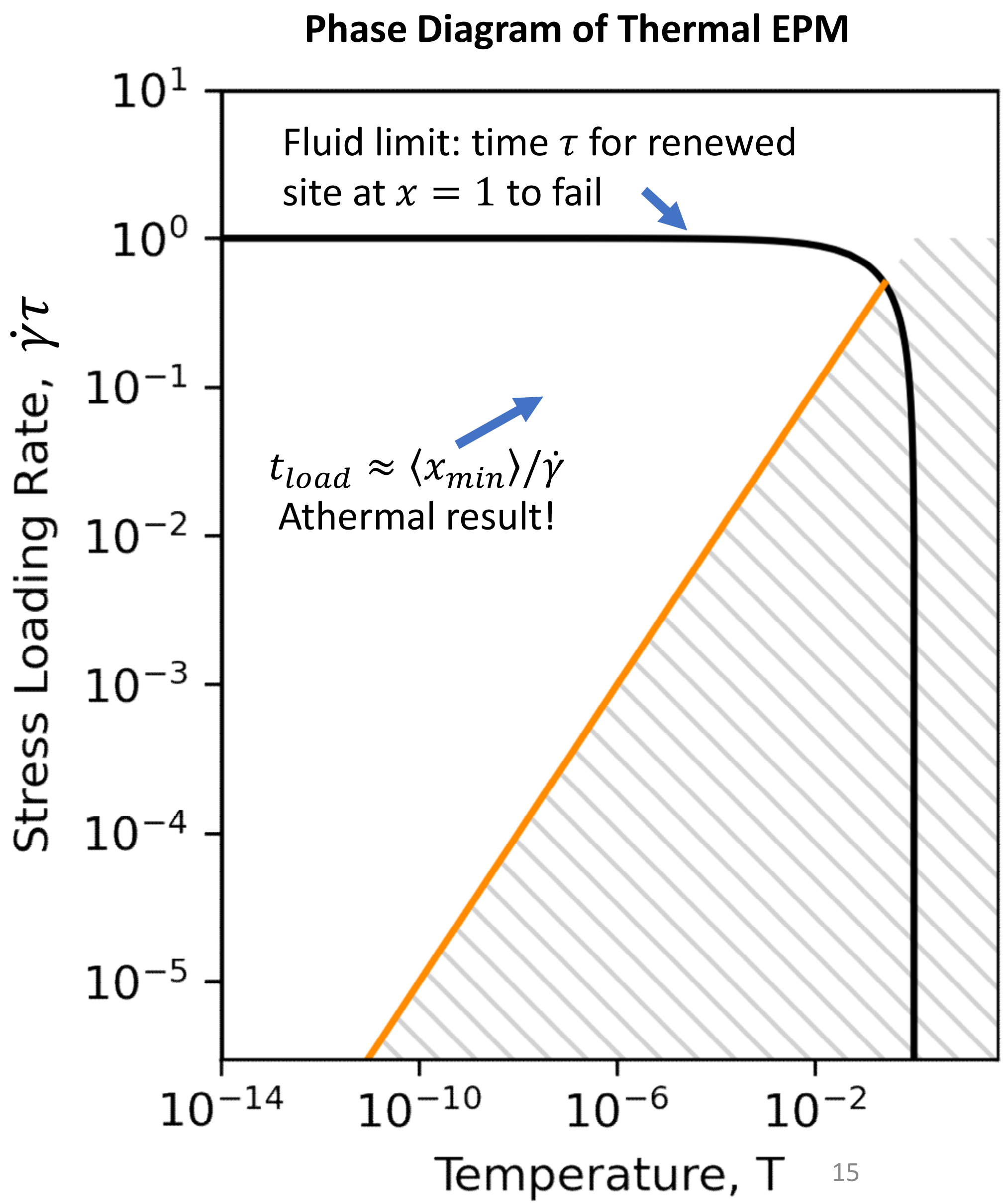

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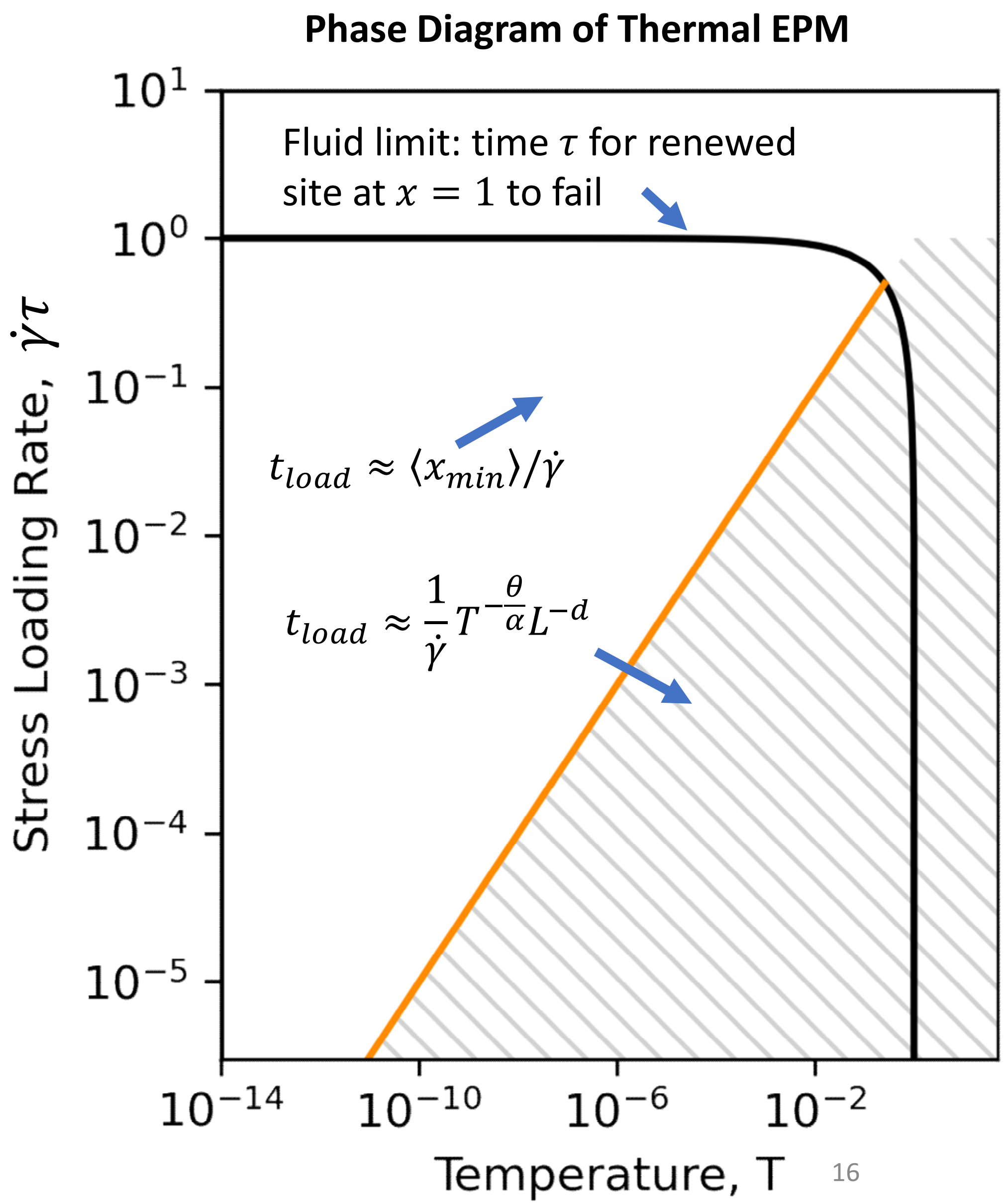

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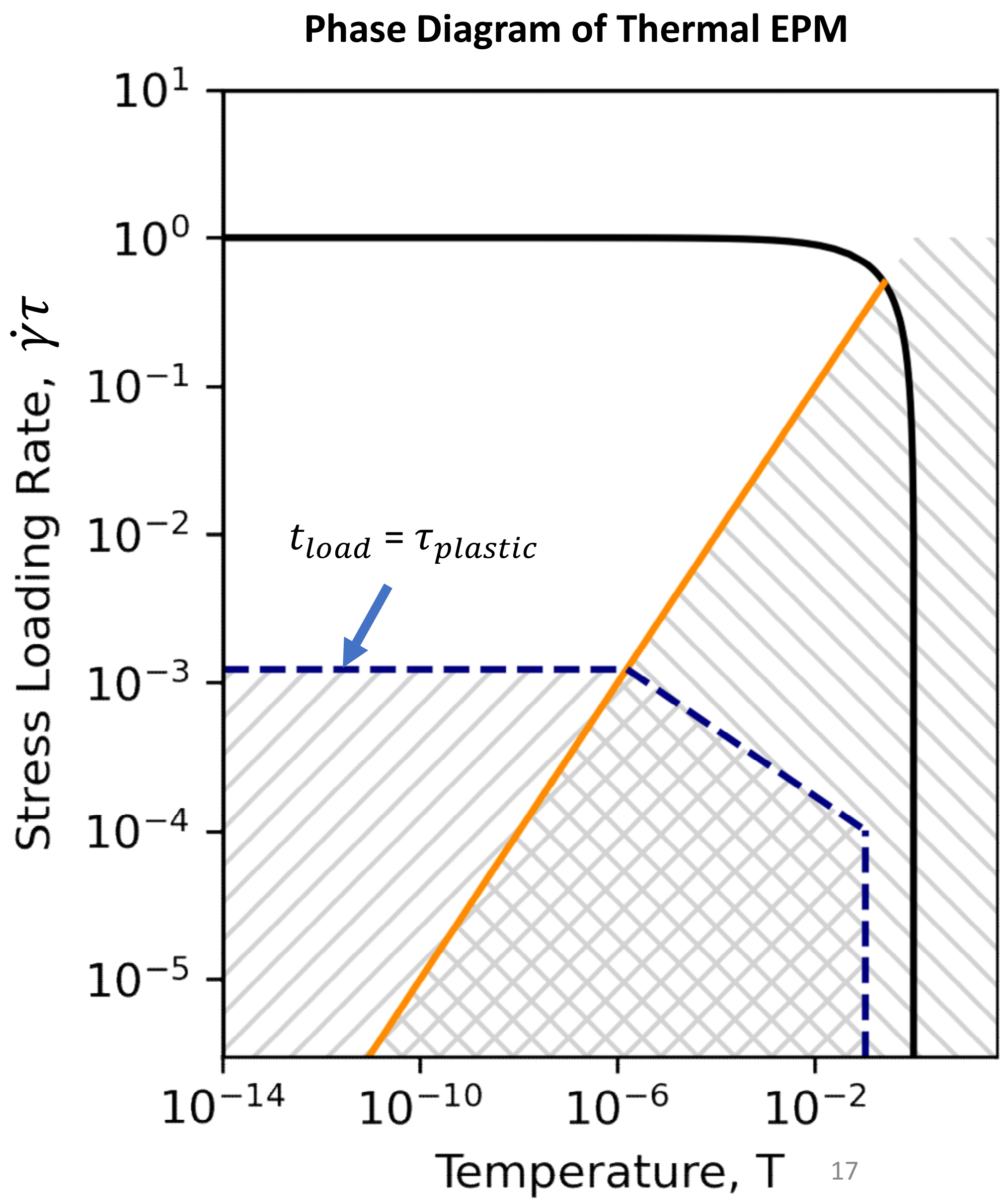
Temporally distinct avalanches (L dependent)



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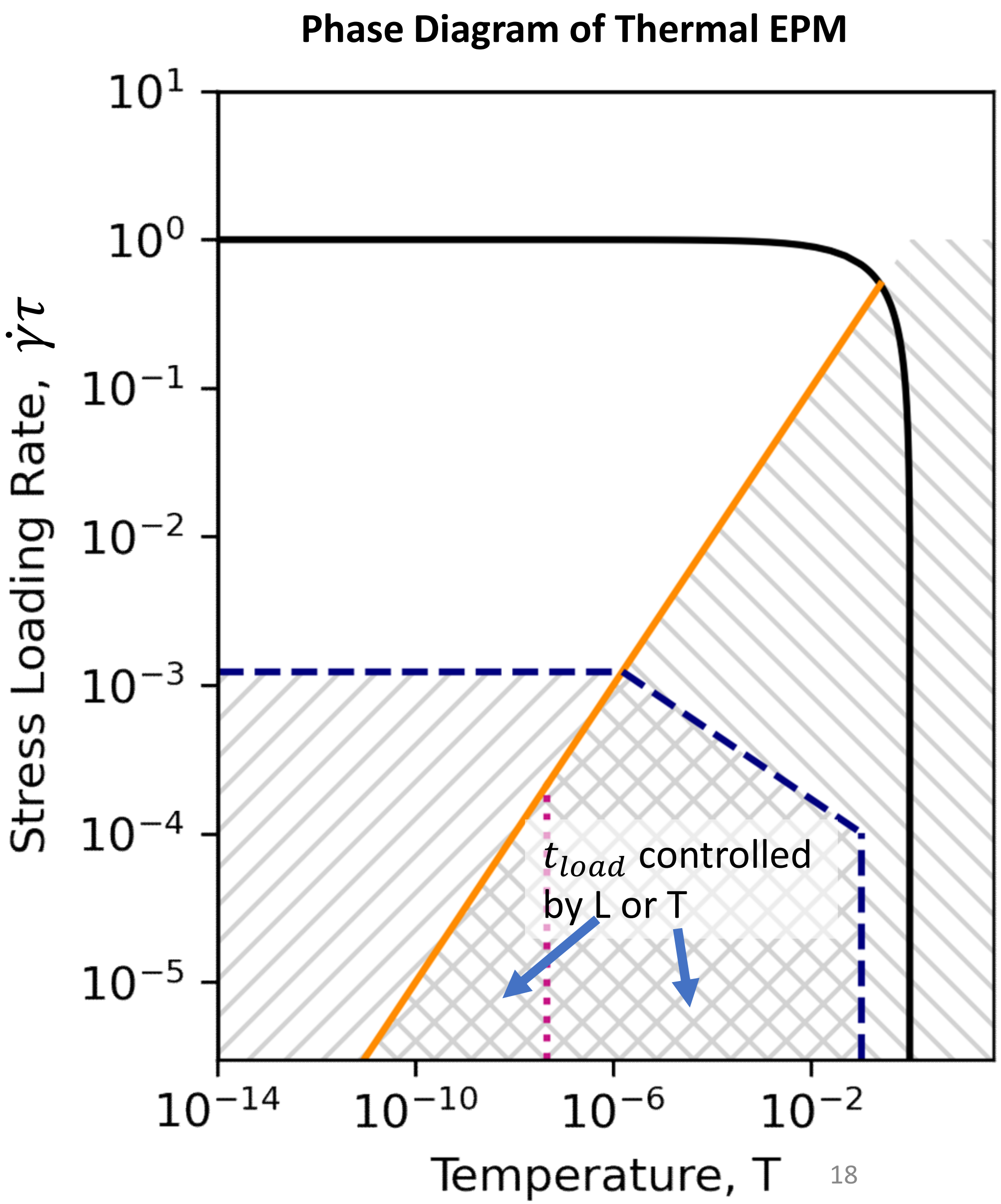

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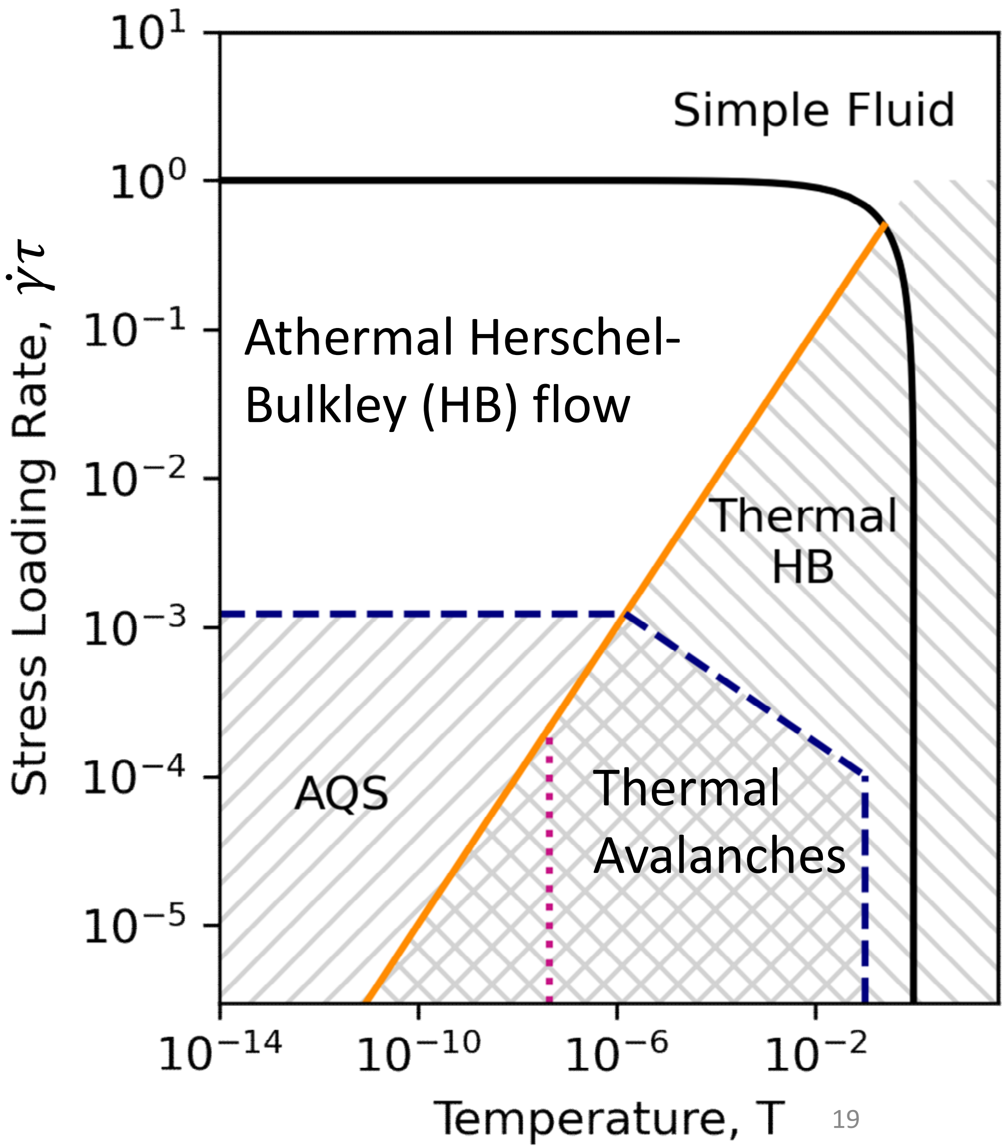
Phase Diagram of Thermal EPM



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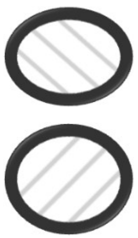
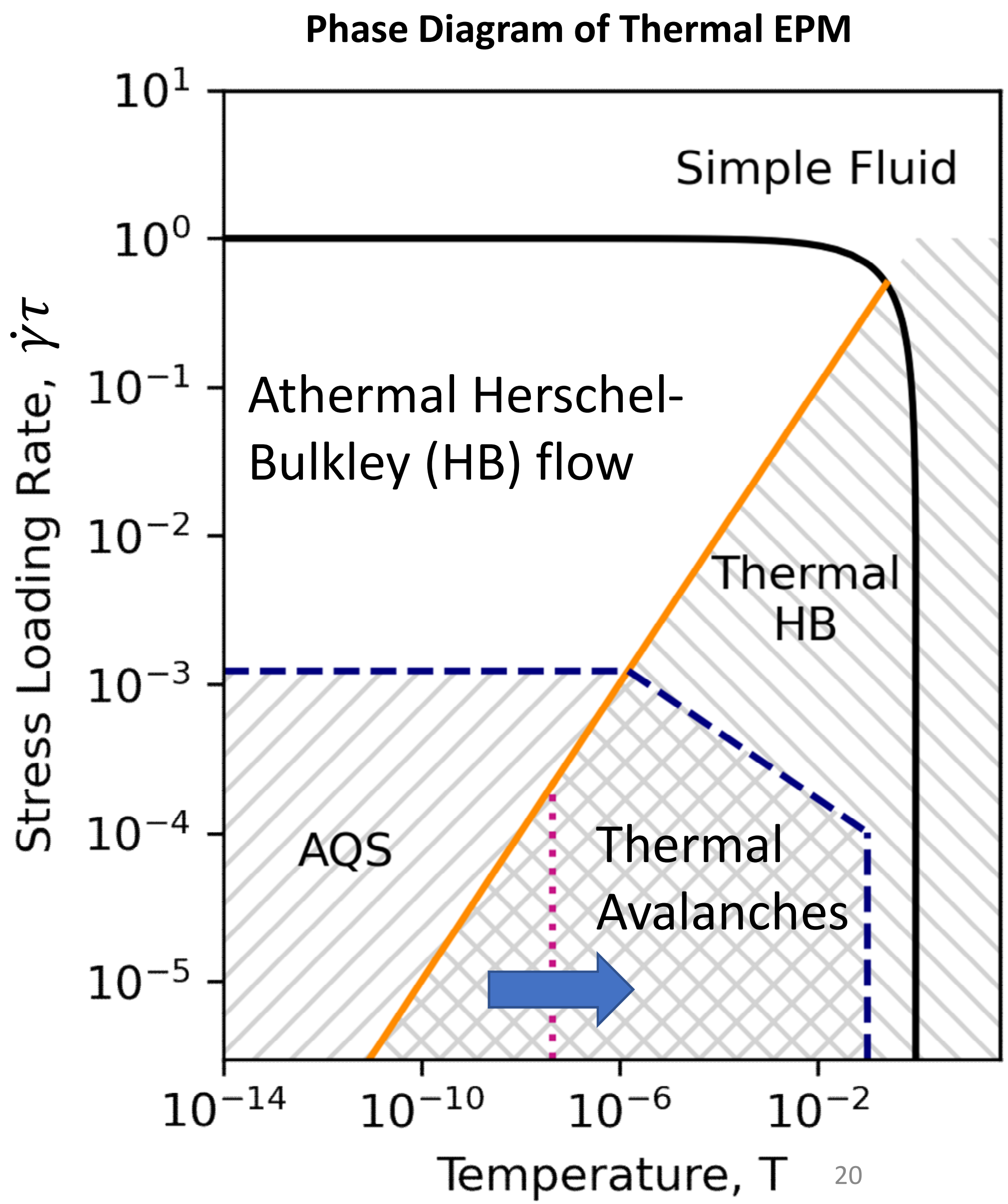


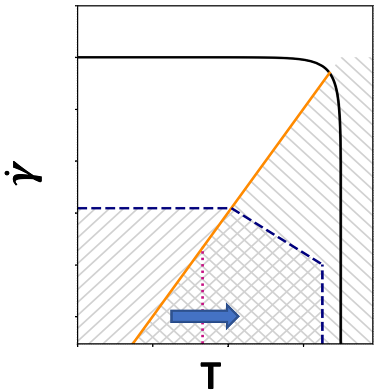


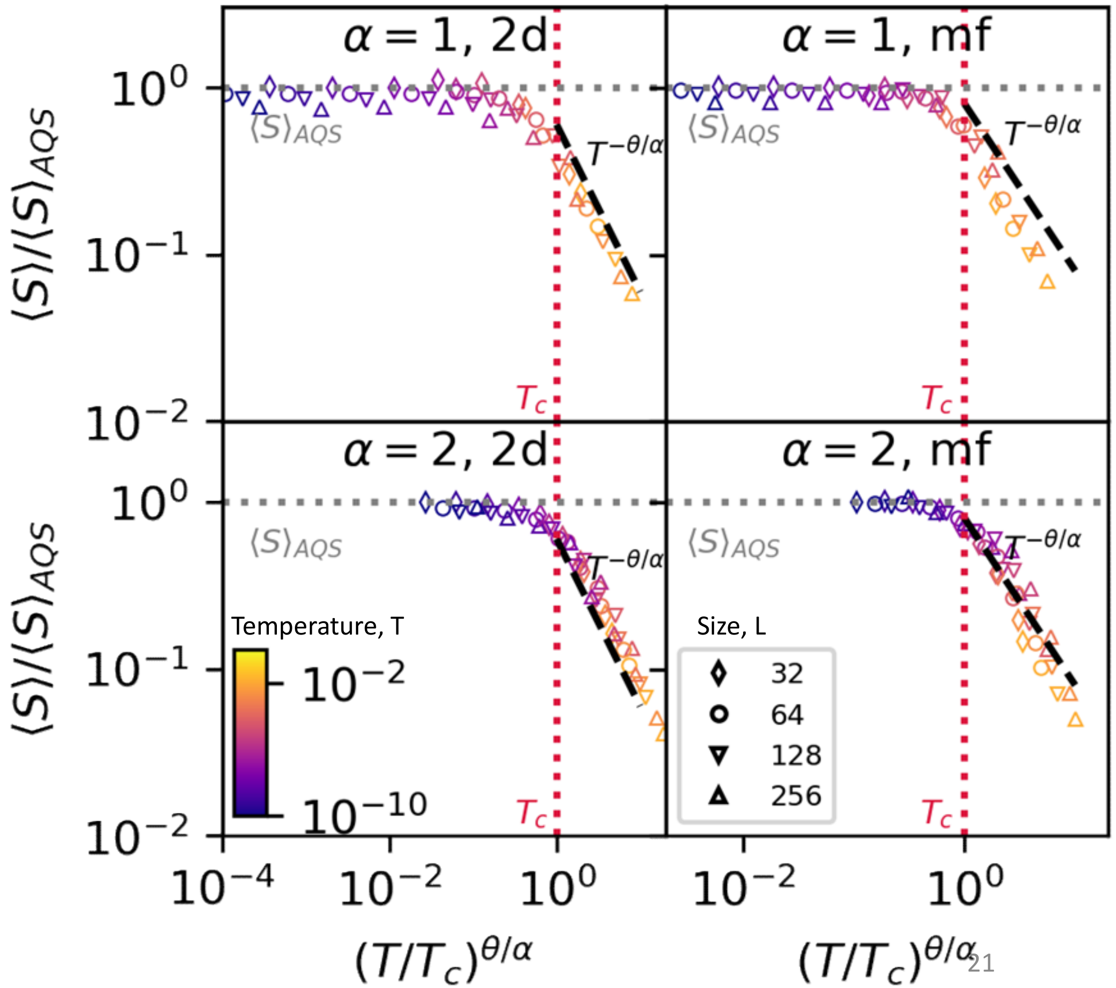
Results: Temperature truncated avalanches

- Temperature reduces avalanche size:

$$\langle S \rangle \sim T^{-\frac{\theta}{\alpha}}, \text{ for } T > T_c \sim L^{-\frac{d\alpha}{\theta+1}}$$

Crossing L, T phase line





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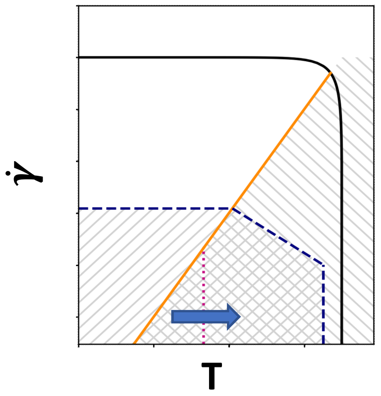

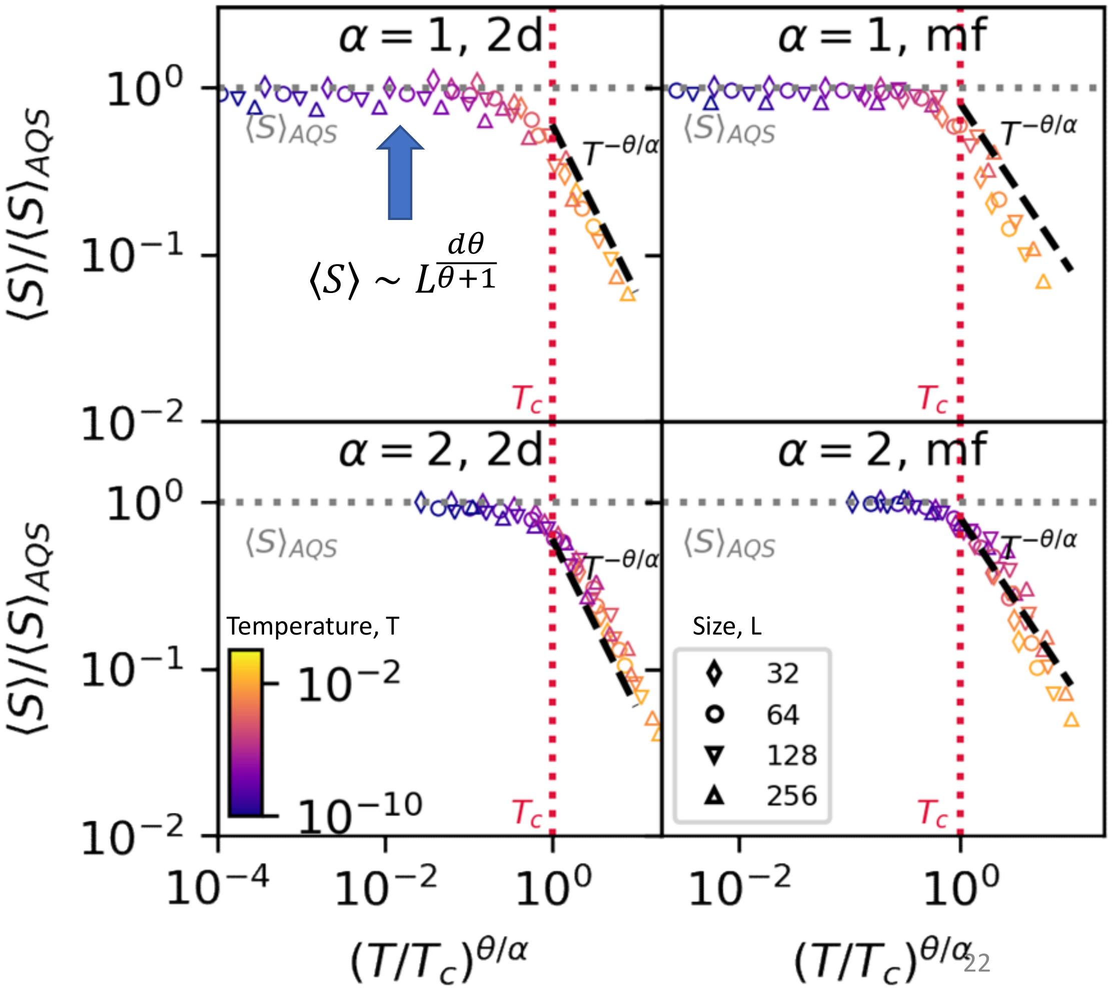

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- Interpretation: correlation length & avalanches truncated by either system size or temperature effects

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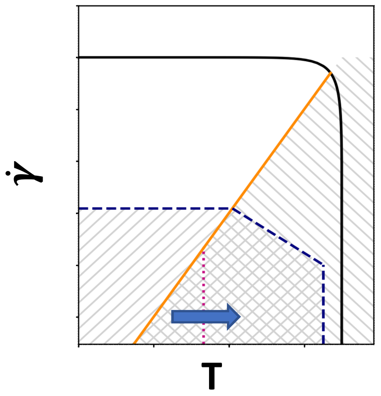
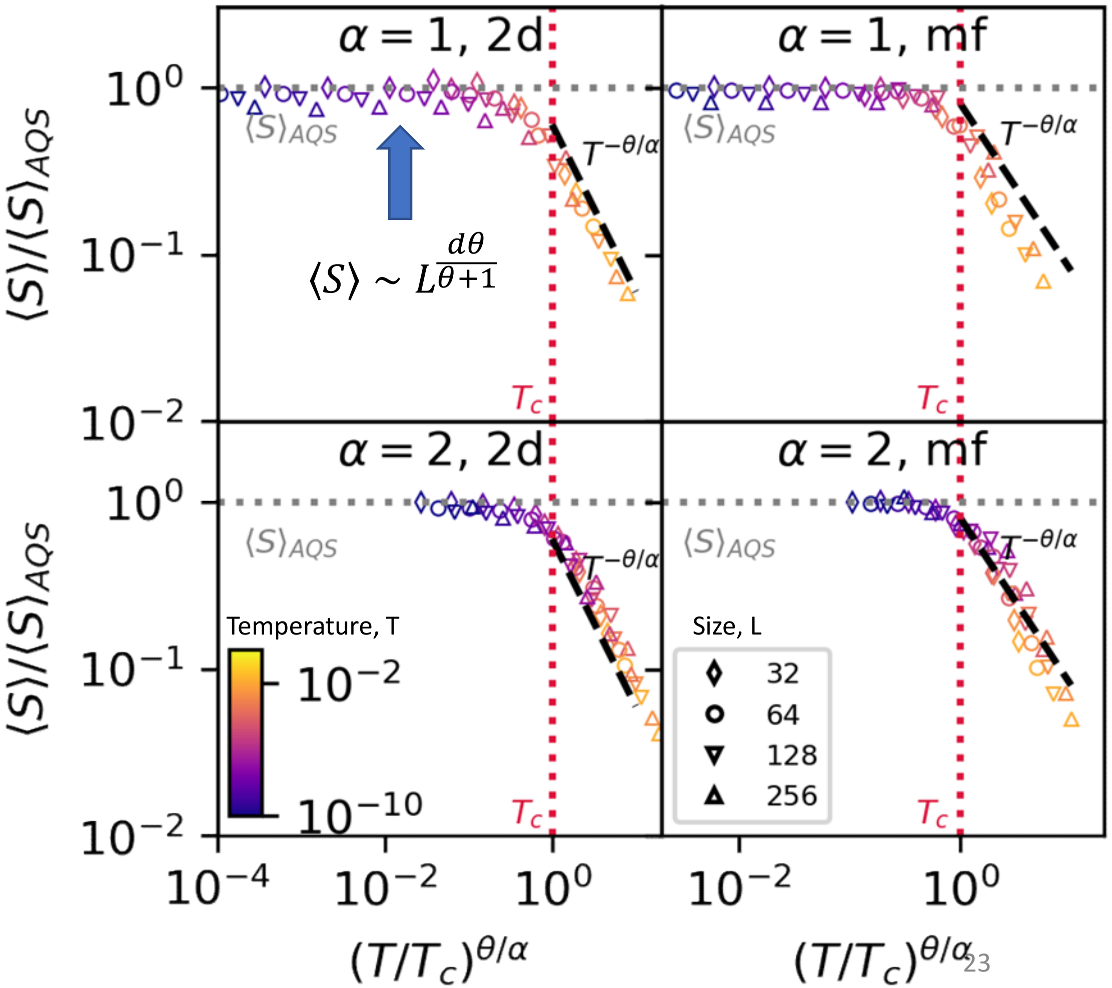
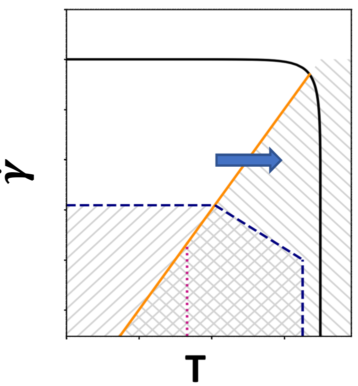


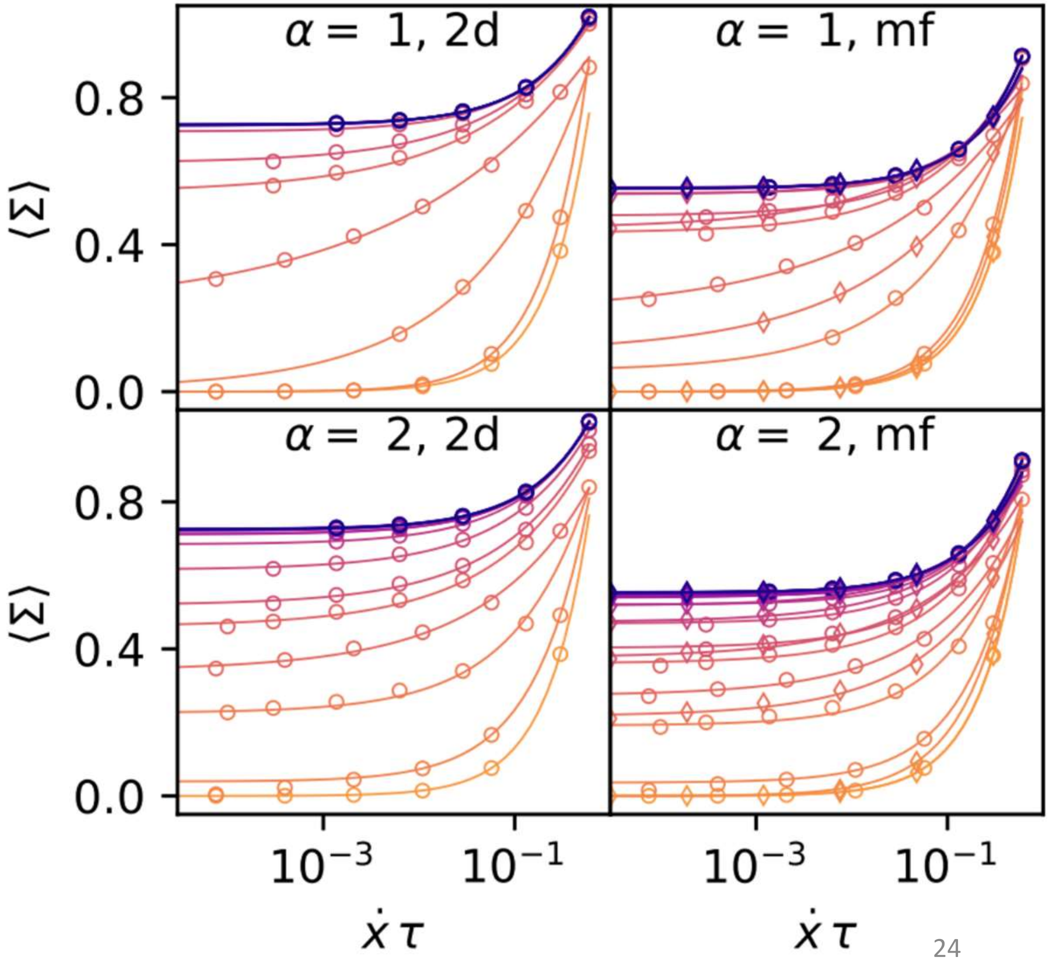


Results: Thermal HB exponent

- Temperature reduces flow-stress

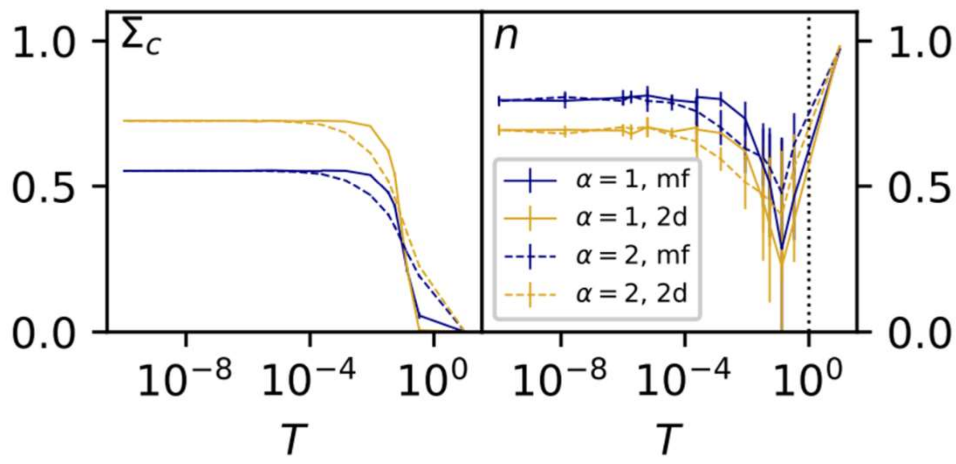
Crossing T,$\dot{\gamma}$ phase line

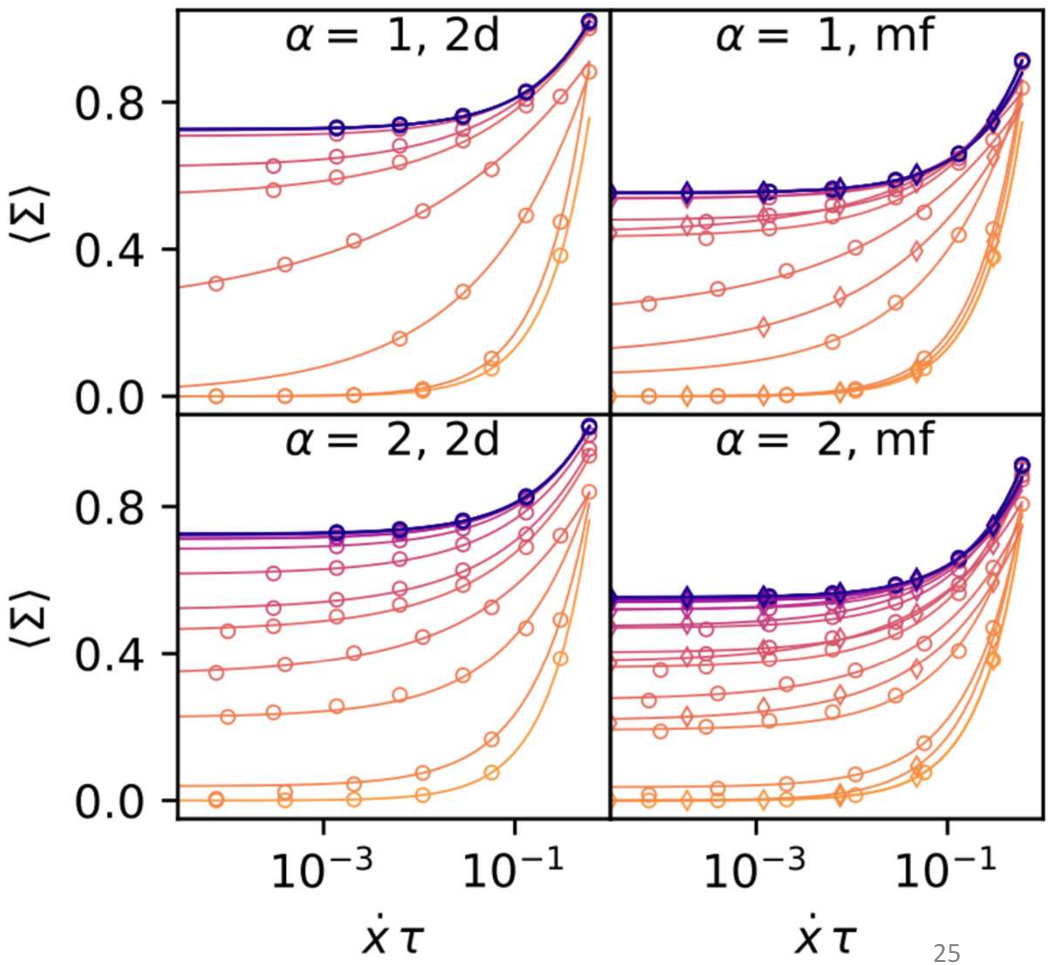




Results: Thermal HB exponent

- Temperature reduces flow-stress
- Naïve Herschel-Bulkley fits
  $$\langle\Sigma\rangle(T) = \Sigma_c(T) + C\,\dot{\gamma}^n$$





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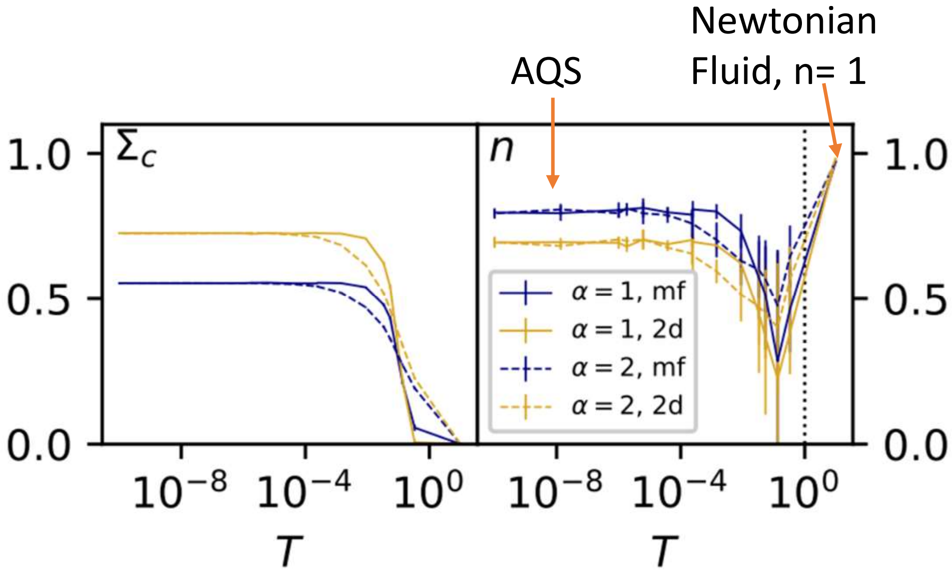

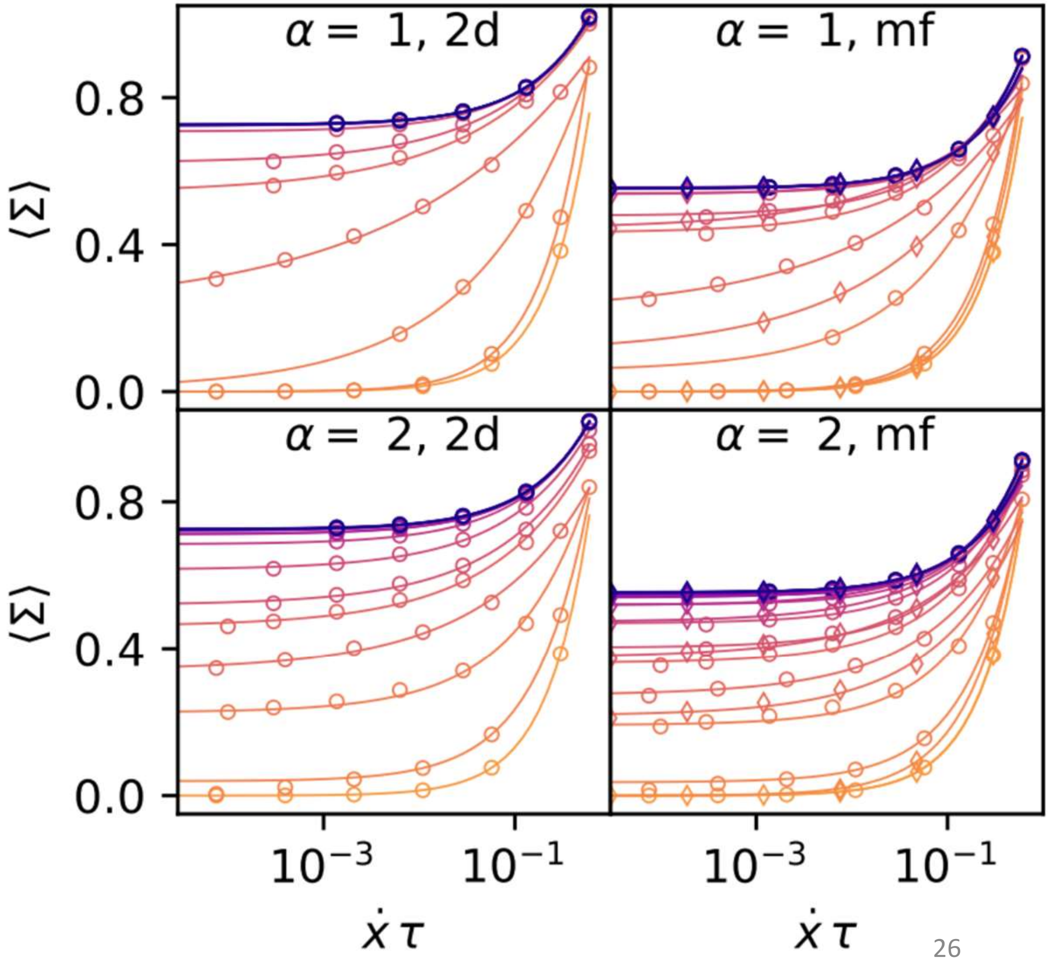

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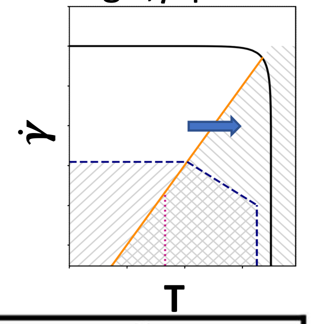
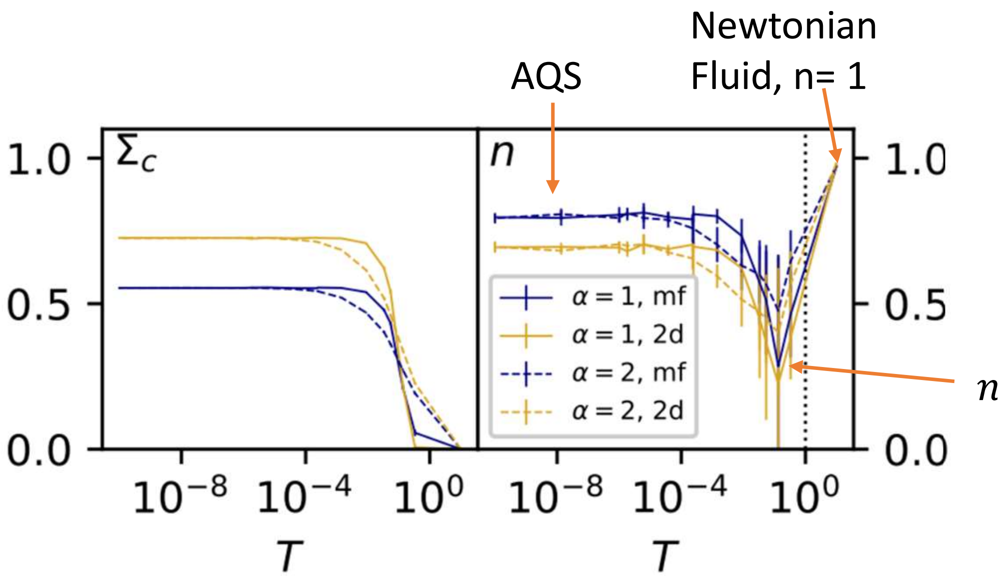
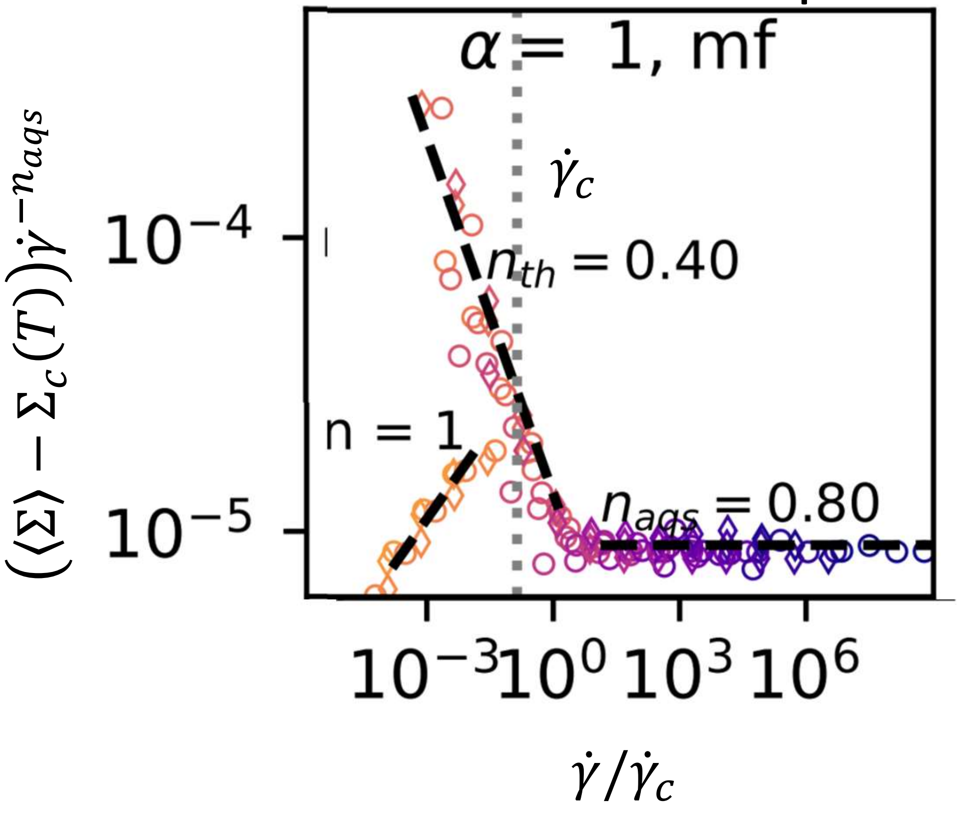






Conclusions

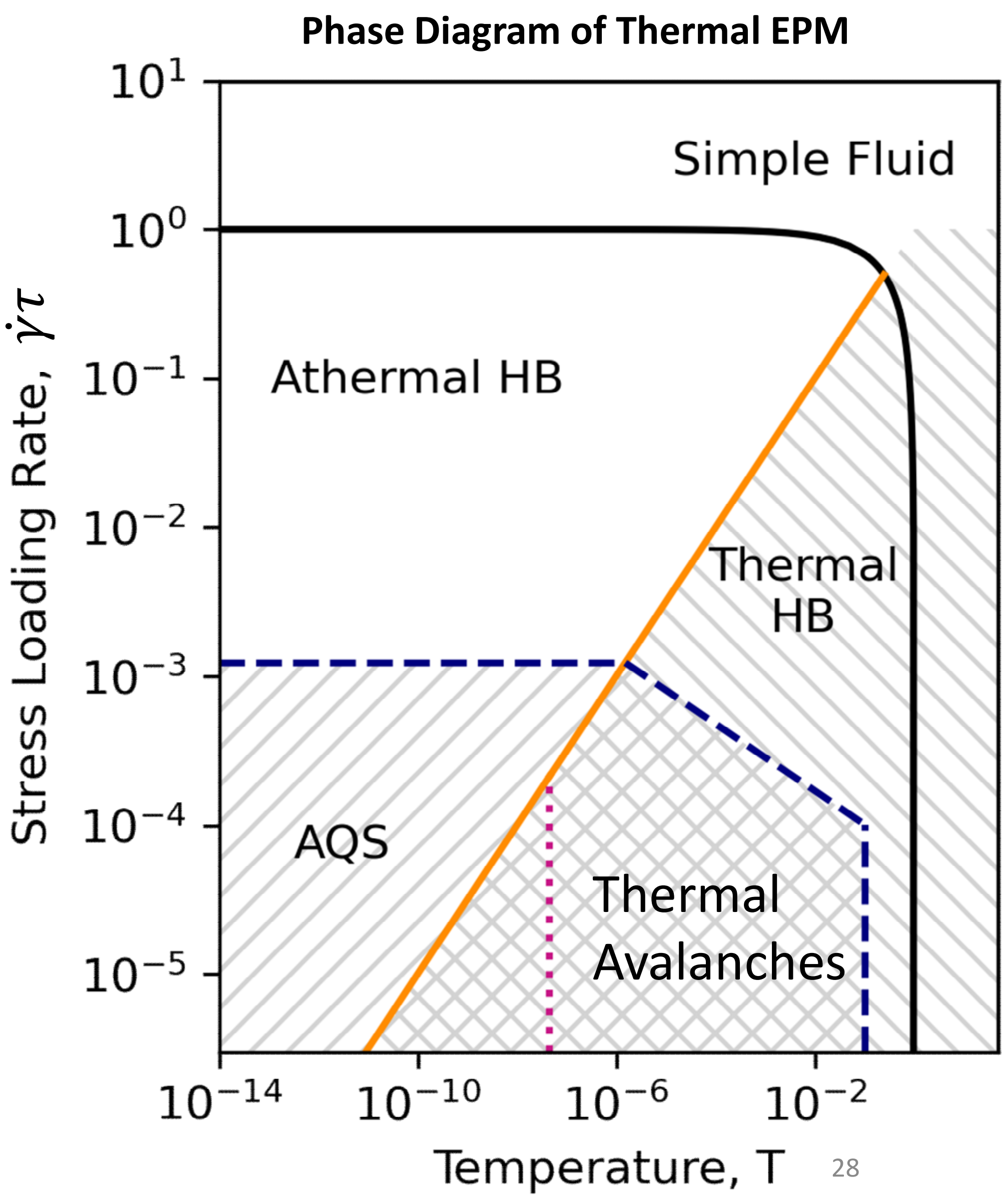
- When do thermal effects appear?

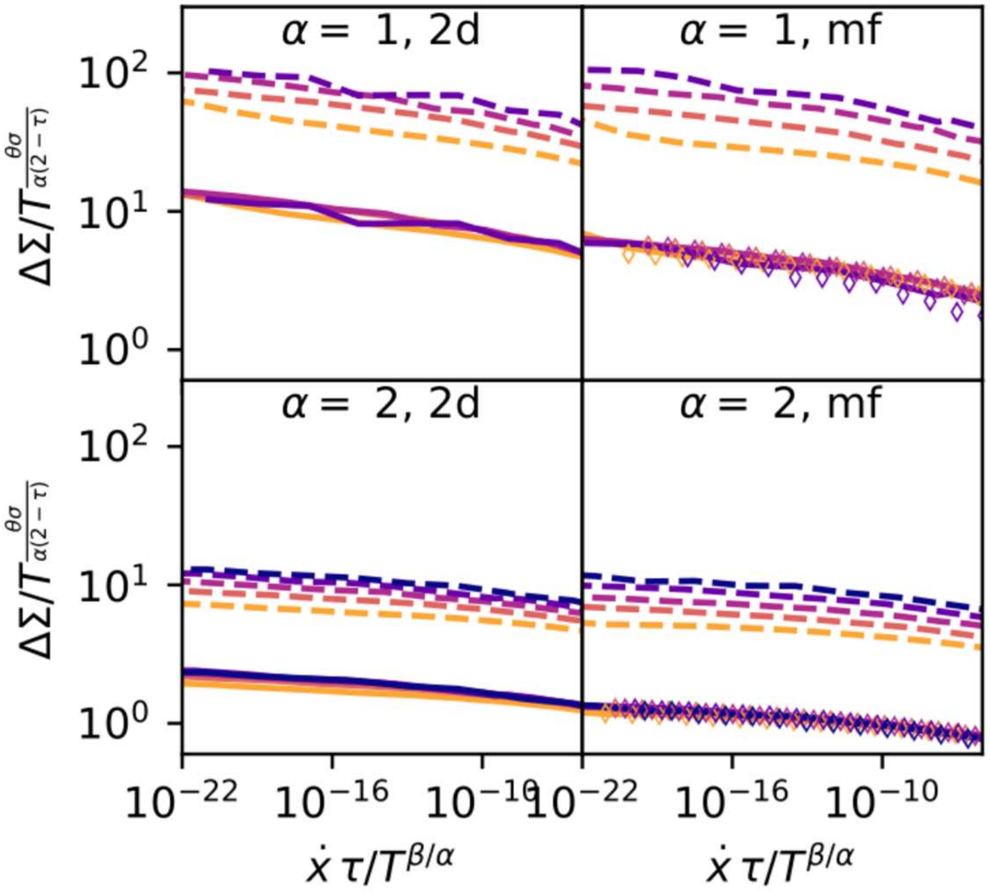
$$\dot{\gamma} < \dot{\gamma}_c = \frac{1}{\tau} T^{\frac{1}{\alpha}}$$

- When do avalanches overlap? (L, T, $\dot{\gamma}$)
- Correlation length truncated by L or T
- Temperature dependent Herschel-Bulkley n exponent
  - Can this be tied to avalanche exponents? (scaling theory: $\frac{1}{n} = 1 + z/(d - d_f)$)

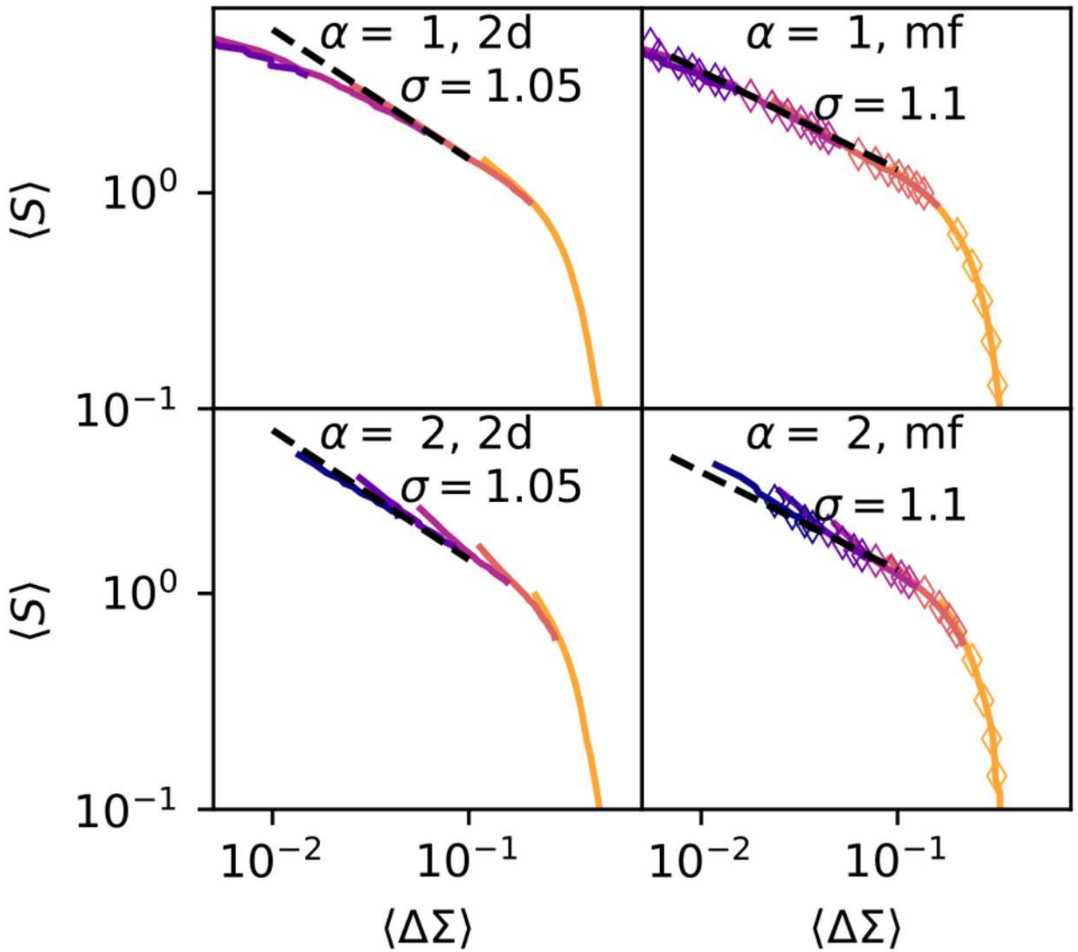
Paper to appear in PRE

Preprint at: arxiv.org/abs/2204.07545





$\alpha =$ 1, 2d
$\alpha =$ 1, mf
$\alpha =$ 2, 2d
$\alpha =$ 2, mf
10^2
10^1
10^0
$\Delta\Sigma/T^{\frac{\theta\sigma}{\alpha(2-\tau)}}$
10^{-22}
10^{-16}
10^{-10}
$\dot{x}\,\tau/T^{\beta/\alpha}$



$\alpha =$ 1, 2d
$\sigma = 1.05$
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$\sigma = 1.05$
$\alpha =$ 2, mf
$\sigma = 1.1$
$\langle S \rangle$
10^0
10^{-1}
10^{-2}
$\langle \Delta\Sigma \rangle$