Criticality in Neuronal Avalanches without Timescale Separation

Daniel Korchinski[1,2], Javier Orlandi[1,3], Seung-Woo Son[4], Jörn Davidsen[1,3]

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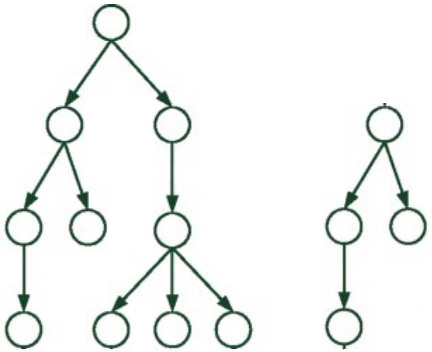

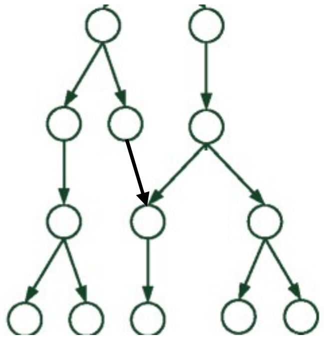

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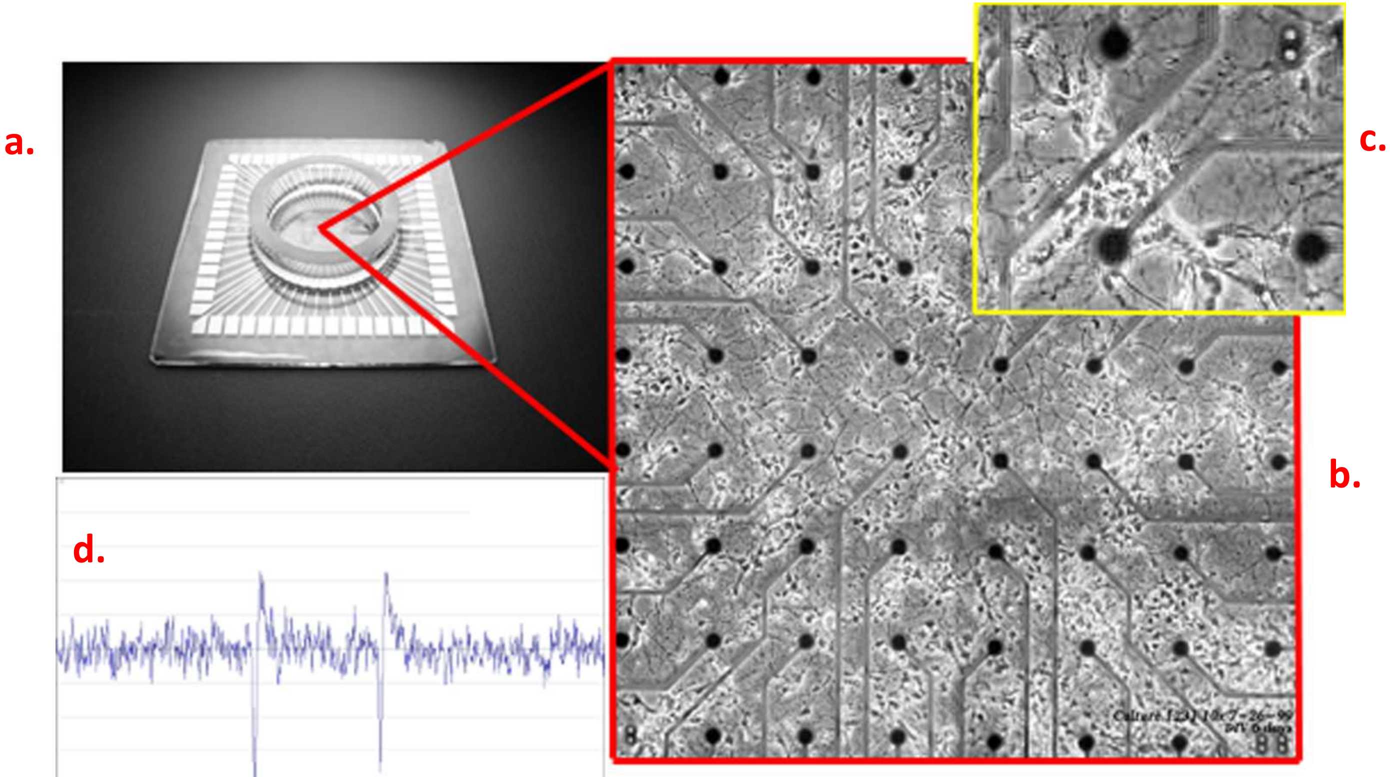
[3]University of Calgary, Hotchkiss Brain Institute
[4]Hanyang University, Department of Applied Physics

The Critical Brain Hypothesis: Avalanches

- We can record and observe neurons in bulk using electrode arrays



Cadotte AJ, DeMarse TB, He P, Ding M (2008)

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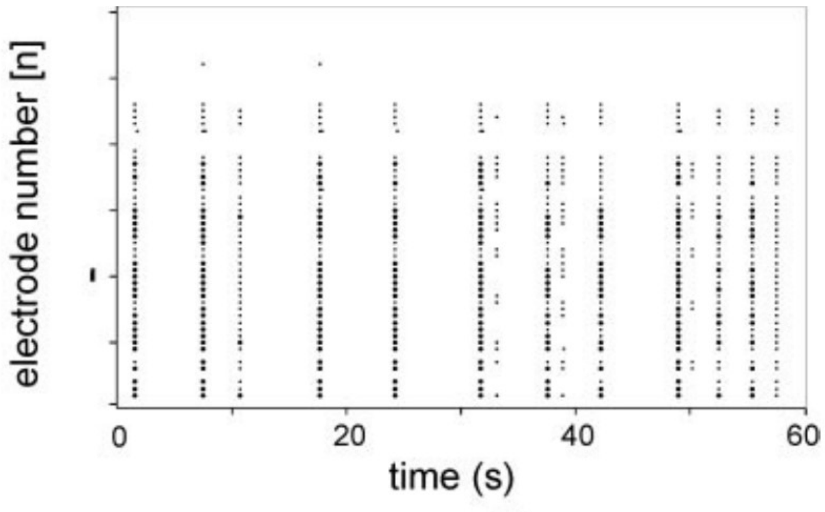
- You will see periods of activity that we can call “avalanches”.



Beggs, J. M.; Plenz, D., 2003

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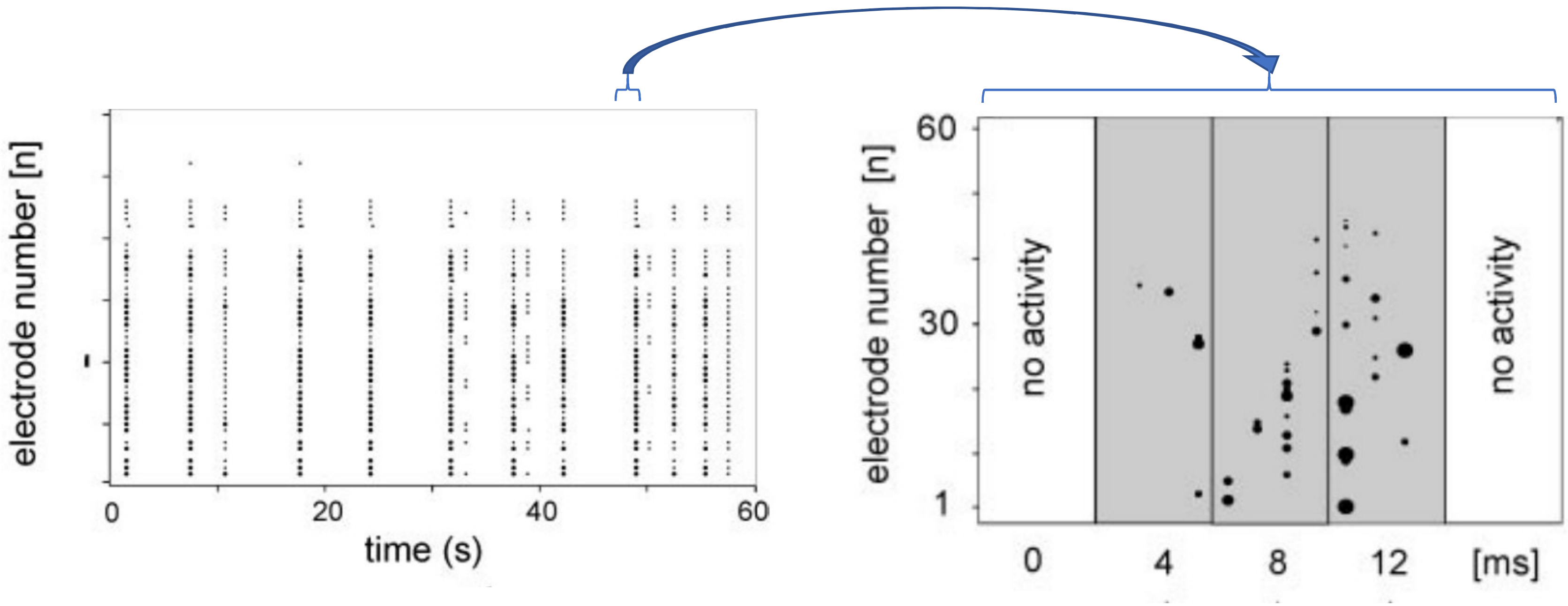

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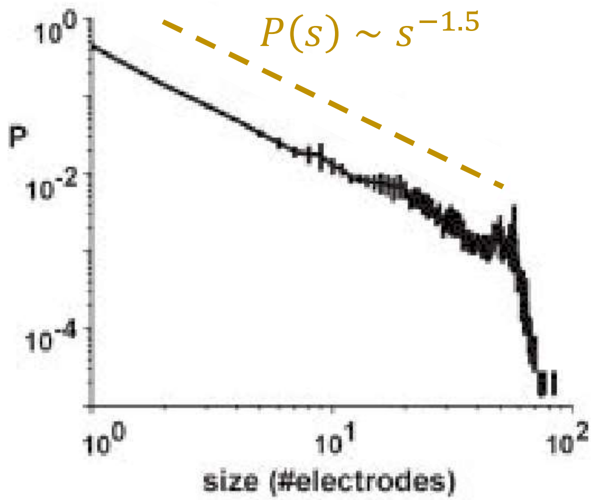
The Critical Brain Hypothesis: Avalanches

- The size distribution of avalanches appears to be scale free

Microelectrode array

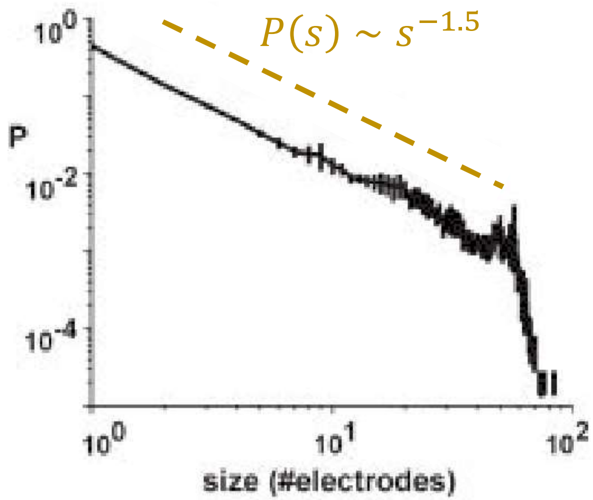


Electrode Array Data. Beggs, J. M.; Plenz, D., 2003

The Critical Brain Hypothesis: Avalanches

- The size distribution of avalanches appears to be scale free
- Avalanche exponents suggest a branching process i.e. directed percolation

Microelectrode array



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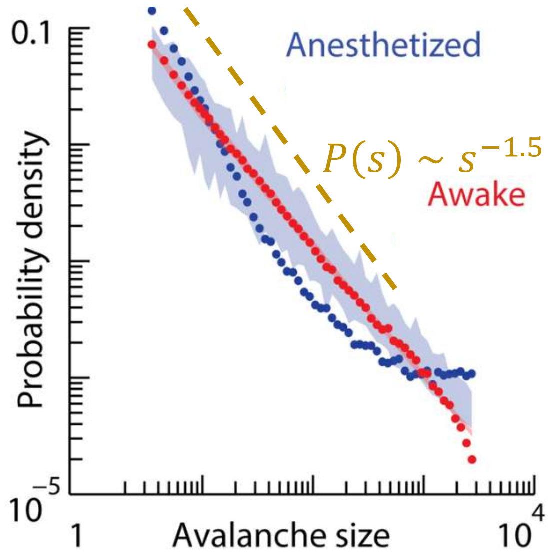
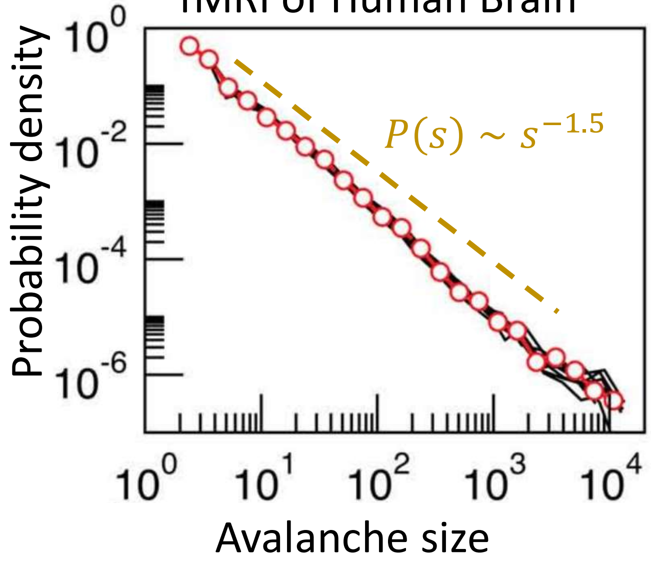
Voltage Imaging of Mouse Cortex



Gregory Scott et al. 2014

fMRI of Human Brain



Tagliazucchi et al. 2012

Increasing Scale

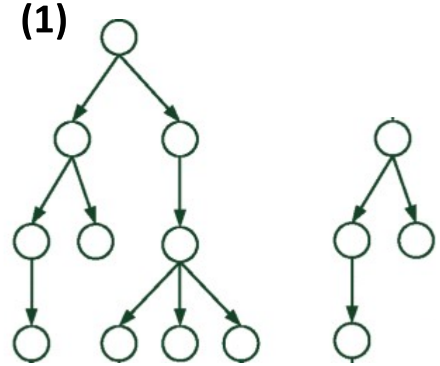
Can input-driven avalanches be critical?

Input, Avalanches, and “Coalescence”

Input introduces two problems:

Input, Avalanches, and "Coalescence"

(1)



Input introduces two problems:

1) Mixed-timescales means independent cascades

Input, Avalanches, and "Coalescence"

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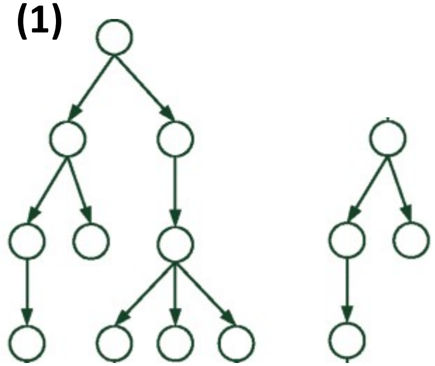

Input introduces two problems:

1) Mixed-timescales means independent cascades
2) Initially independent cascades can then overlap (coalesce)

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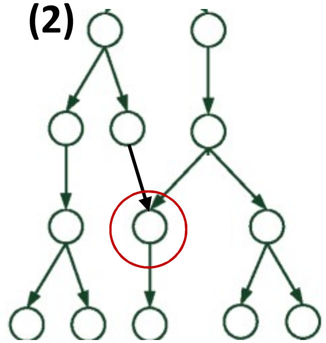

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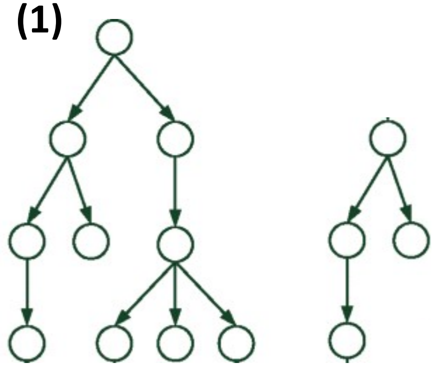

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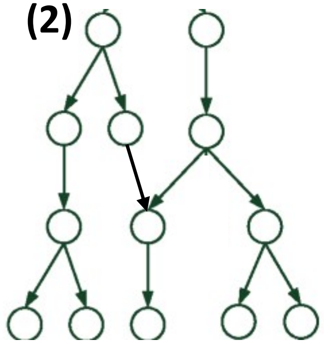


We use “**causal webs**” to generalize avalanches

- Use network structure to identify independent cascades
- Overlapping cascades form a single “causal web”
- Details, see: R. V. Williams-Garcia 2017

Model: The Branching Process

The branching process:

Start a network, activate single neuron.
Each time step, each active neuron independently activates its daughters with probability q

Basic properties of neurons

- Are directed computational elements
- All or nothing electrical response
- Sum inputs from multiple parents

In the absence of noise, the probability neuron j activates at time $t + 1$, given that it has $m_{j,t}$ parents active at time t is:

$$P_{j,t+1} = 1 - (1 - q)^{m_j,t}$$

Model: The Branching Process with Input

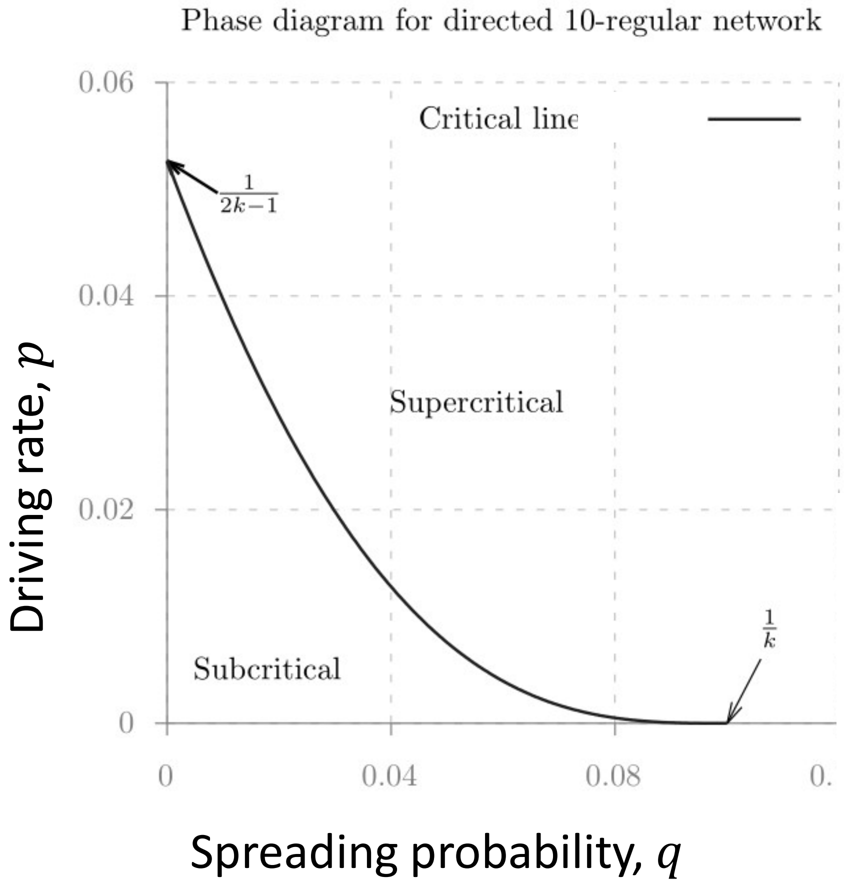
The branching process with input:

- Each time step, each active neuron independently activates its daughters with probability q.
- Each time step, each neuron independently activates with probability p.

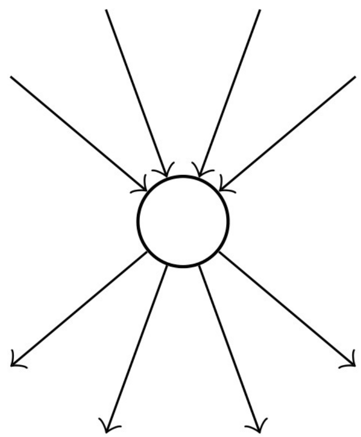
The probability neuron j activates at time $t + 1$, given that it has $m_{j,t}$ parents active at time t is:

$$P_{j,t+1} = 1 - (1-p)(1-q)^{m_{j,t}}$$

Avalanches without timescale separation

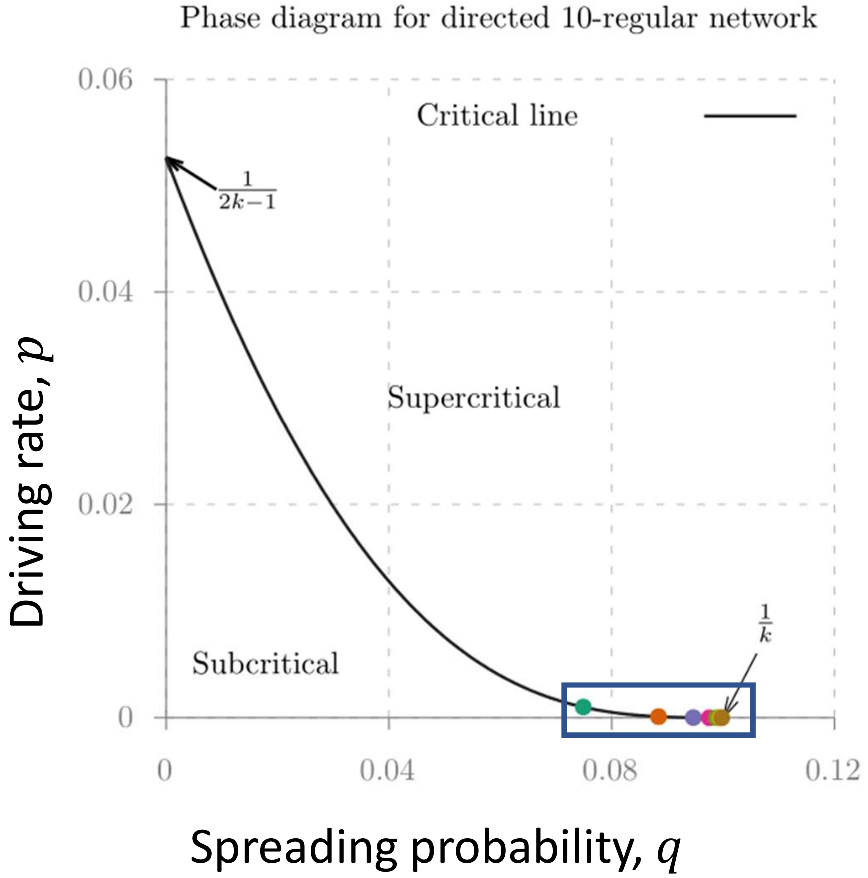


- Subcritical region has finite avalanches
- Supercritical region has a percolating cluster
- Correlation length and average cluster size diverge on critical line



Node in directed 4-regular graph

Avalanches without timescale separation



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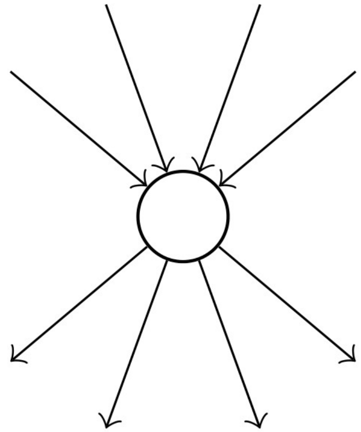

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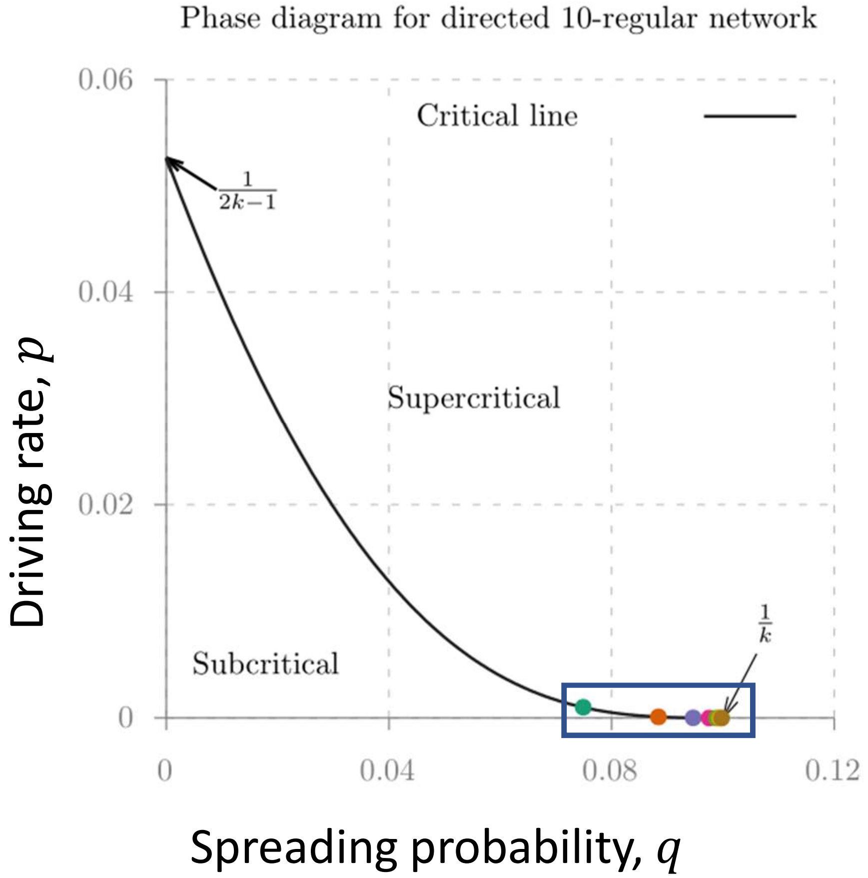
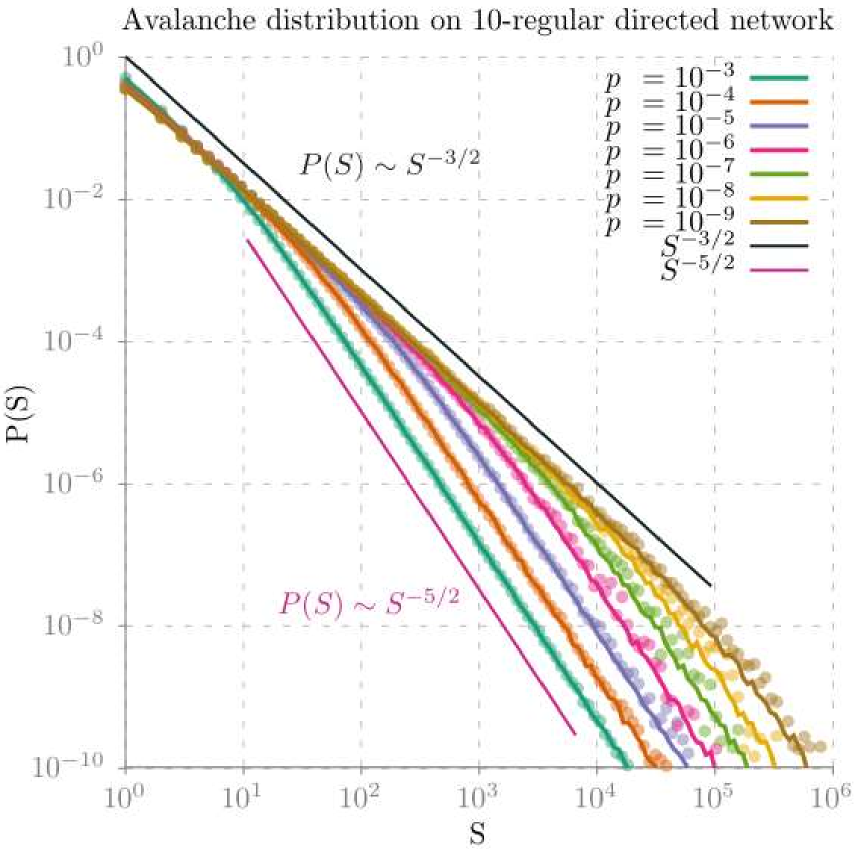
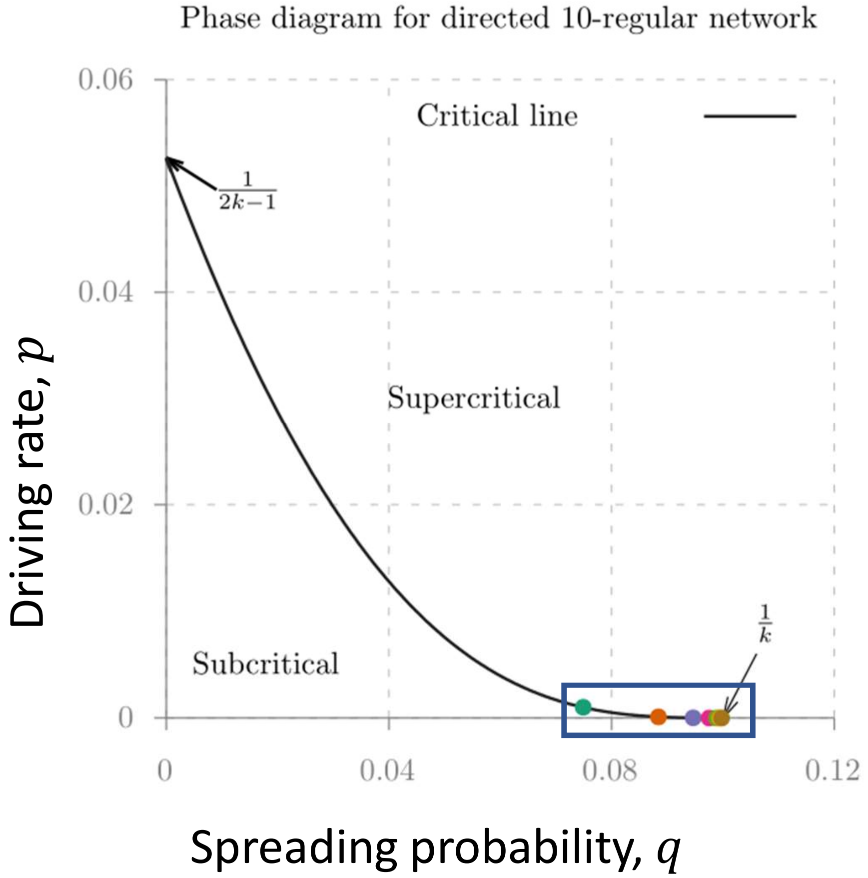
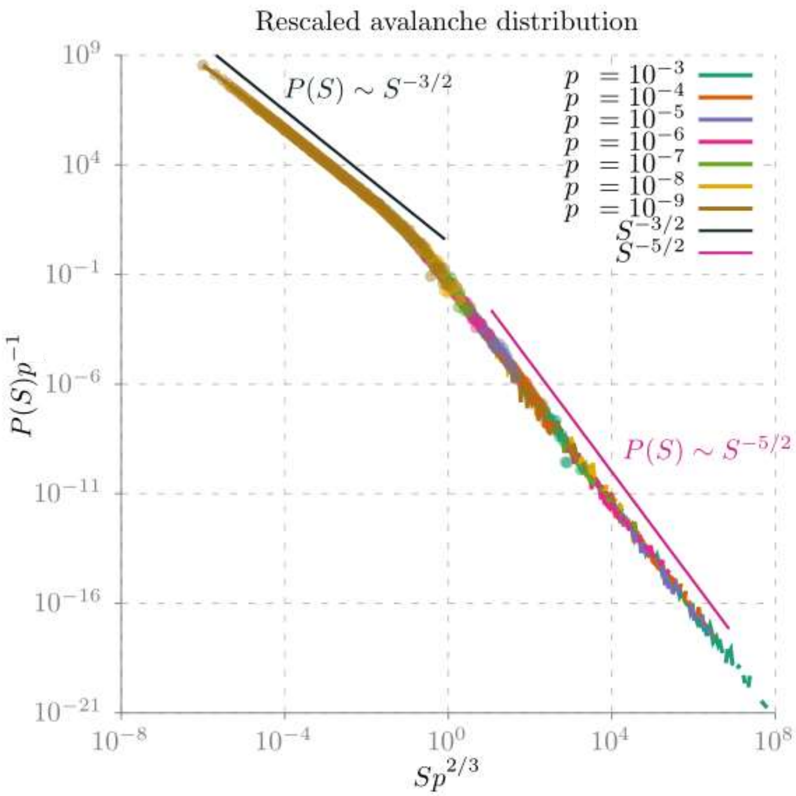




Circles are simulations on $N=10^7$ graphs, solid lines are on loop-free ("infinite") lattices

Avalanches without timescale separation





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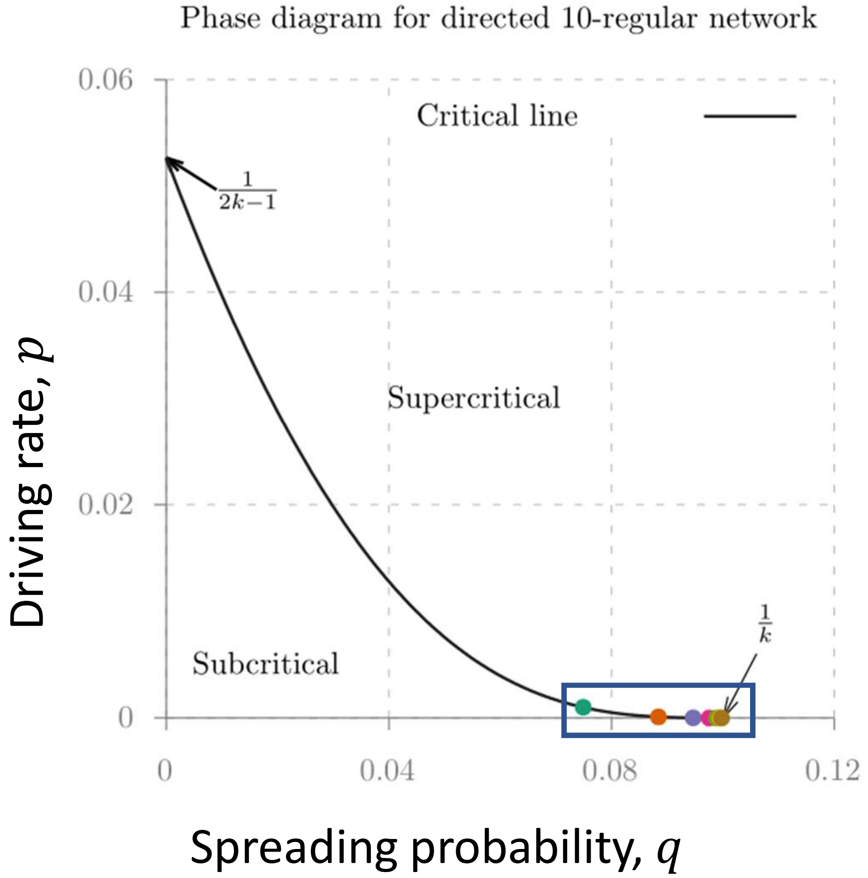
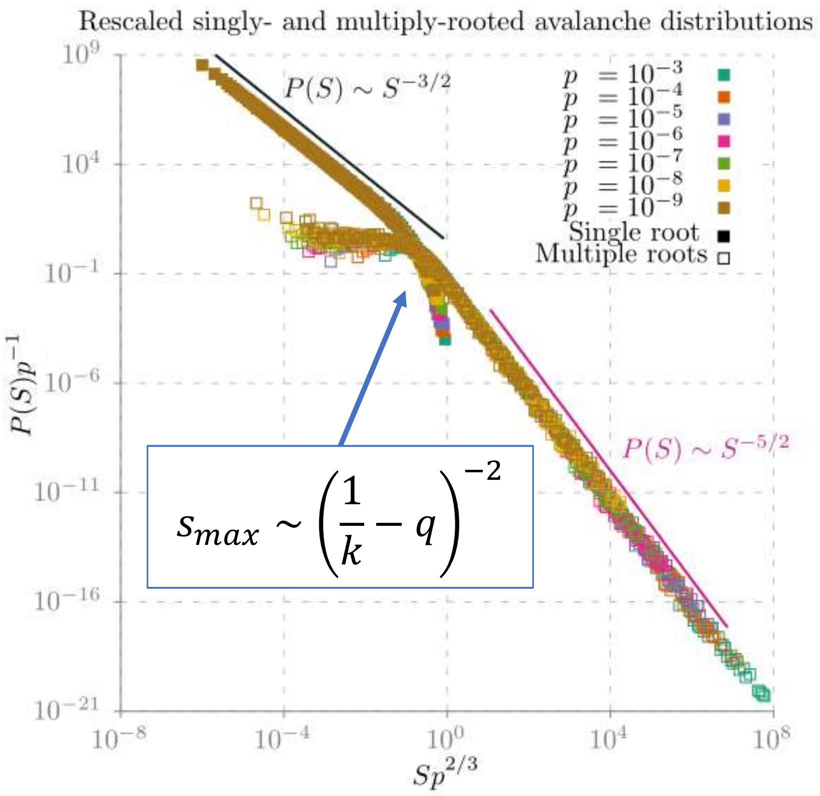
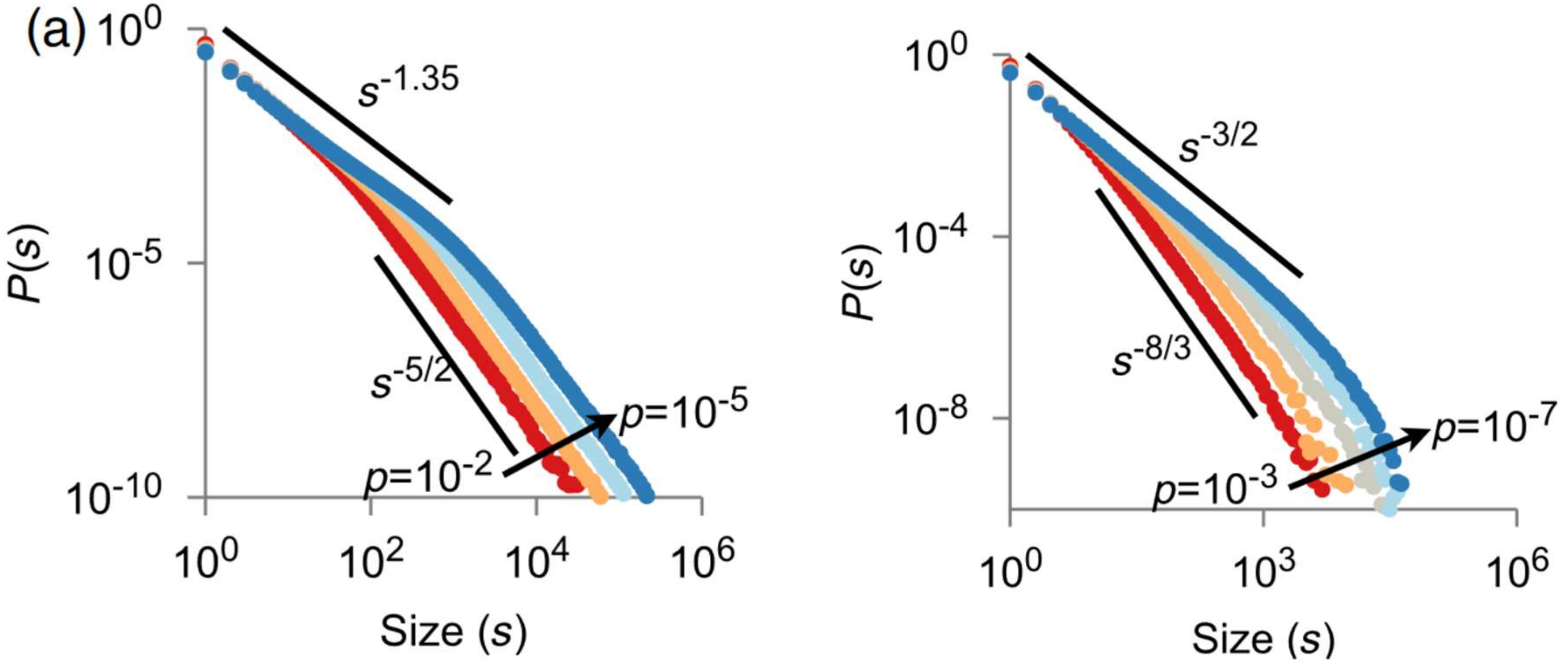


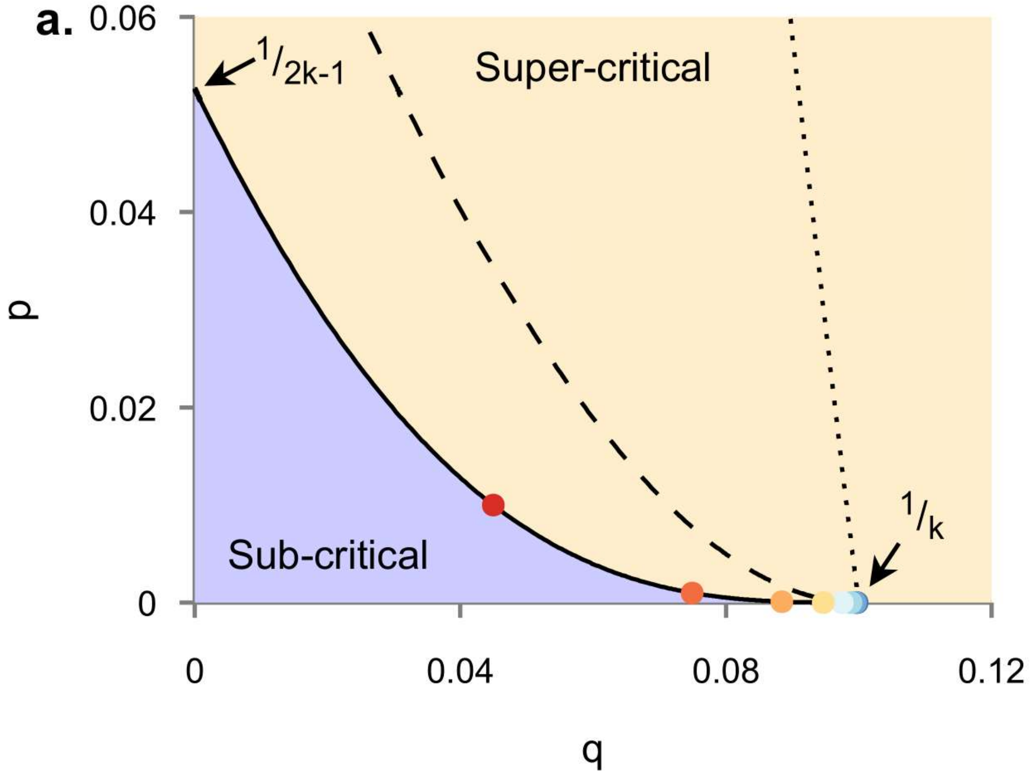


Avalanches without timescale separation

- Exponents are from directed and undirected percolation respectively
- Tested numerically on random networks with different τ exponents



Critical phase diagram



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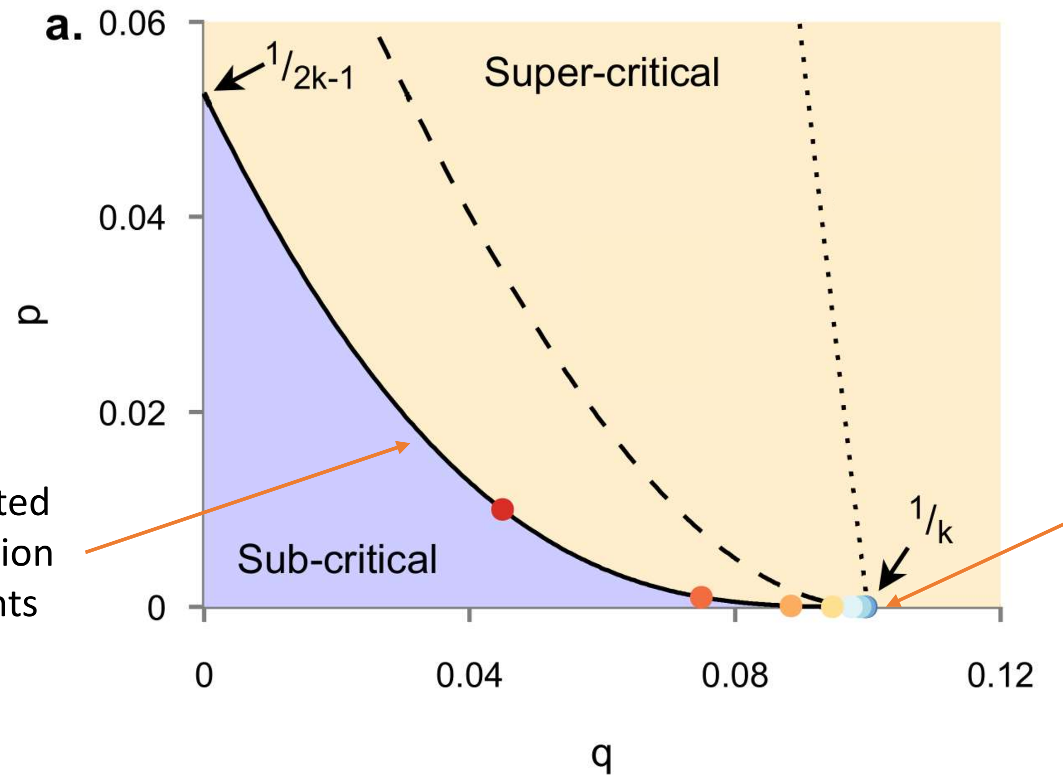
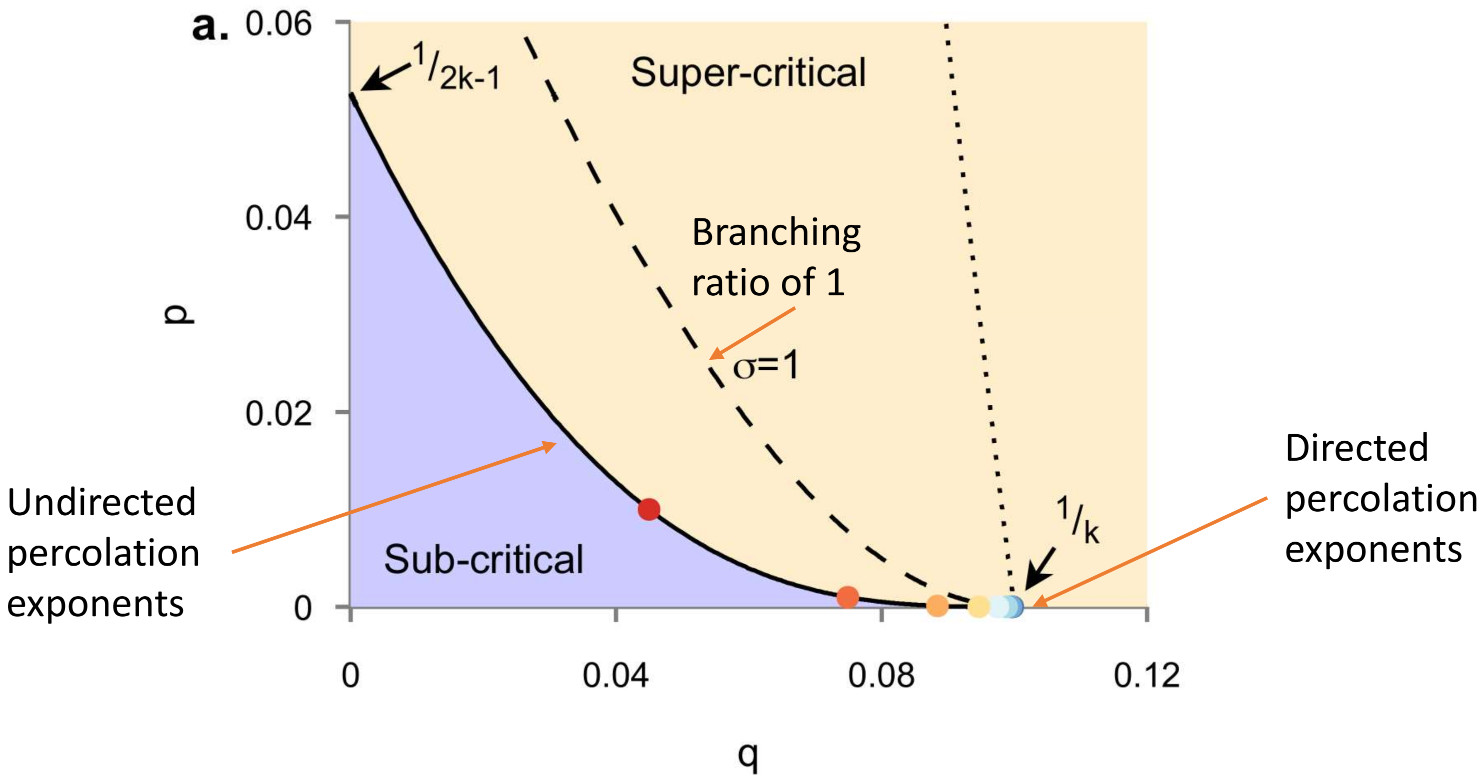


Undirected percolation exponents

Directed percolation exponents

Critical phase diagram



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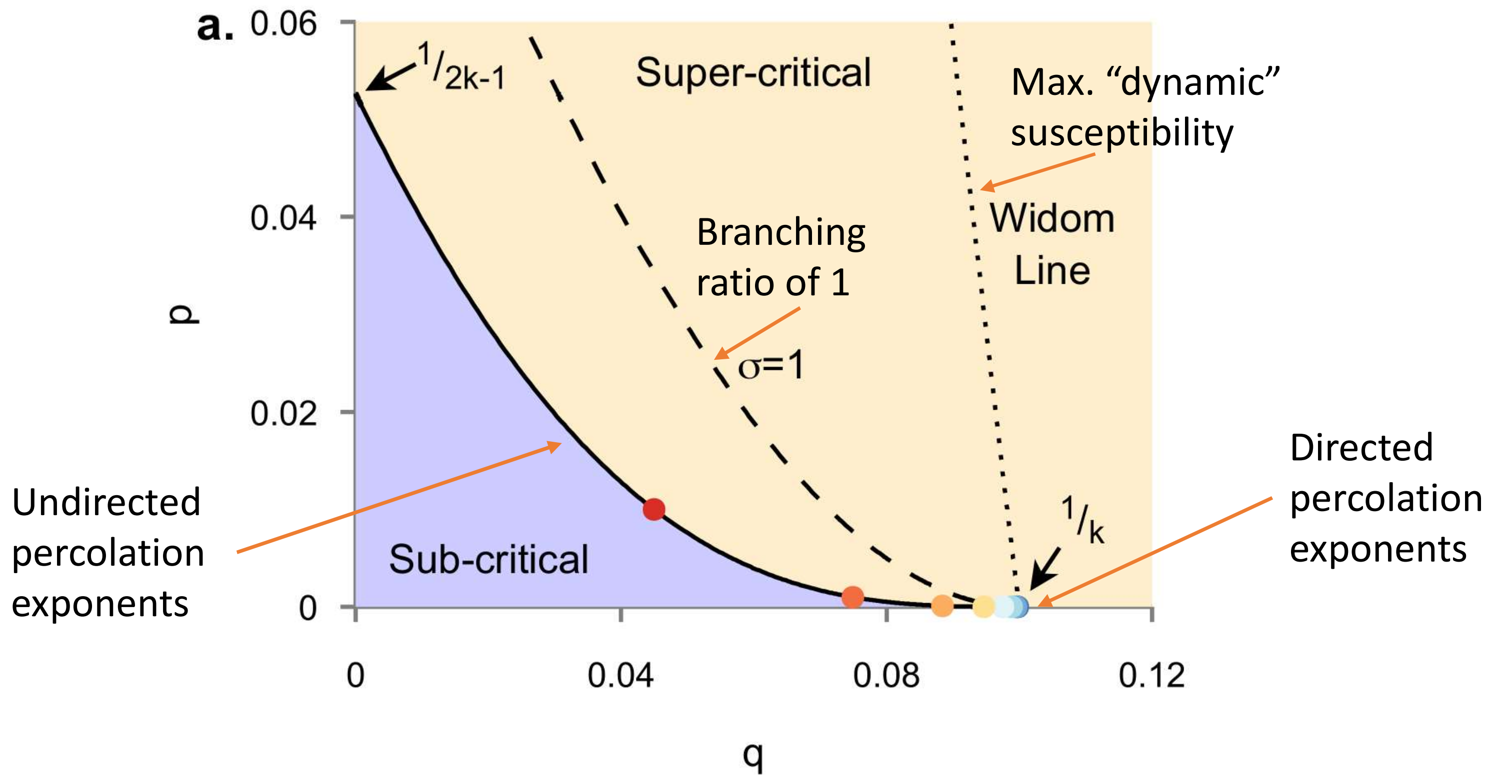

Conclusions

- Criticality is still possible, even with strong driving
- External input changes the universality class to undirected percolation
- Other markers of directed percolation (dynamic susceptibility, branching ratio=1) don’t capture criticality

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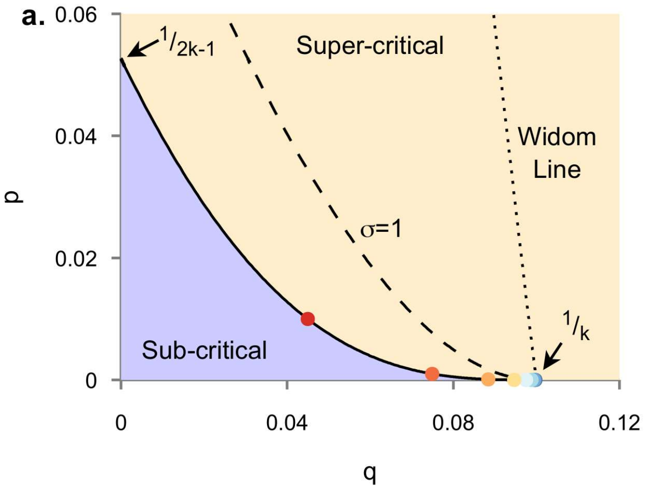
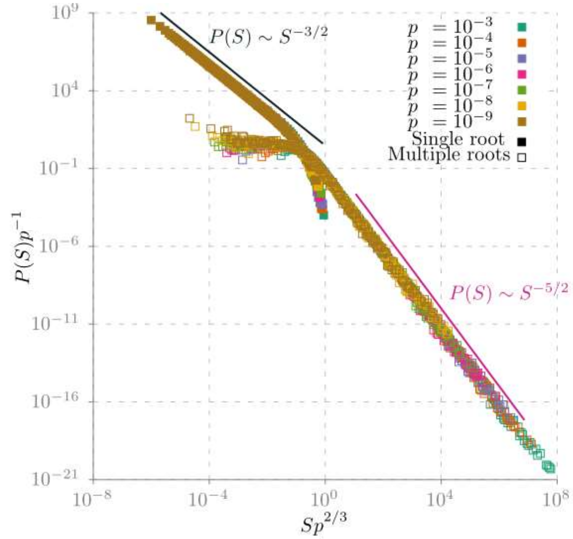
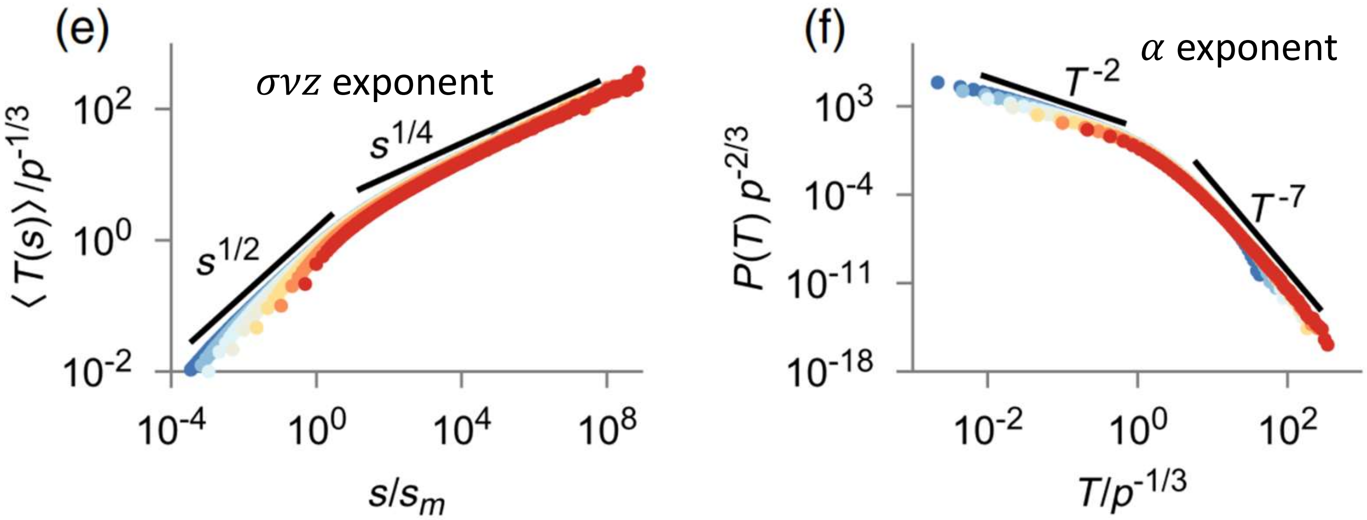


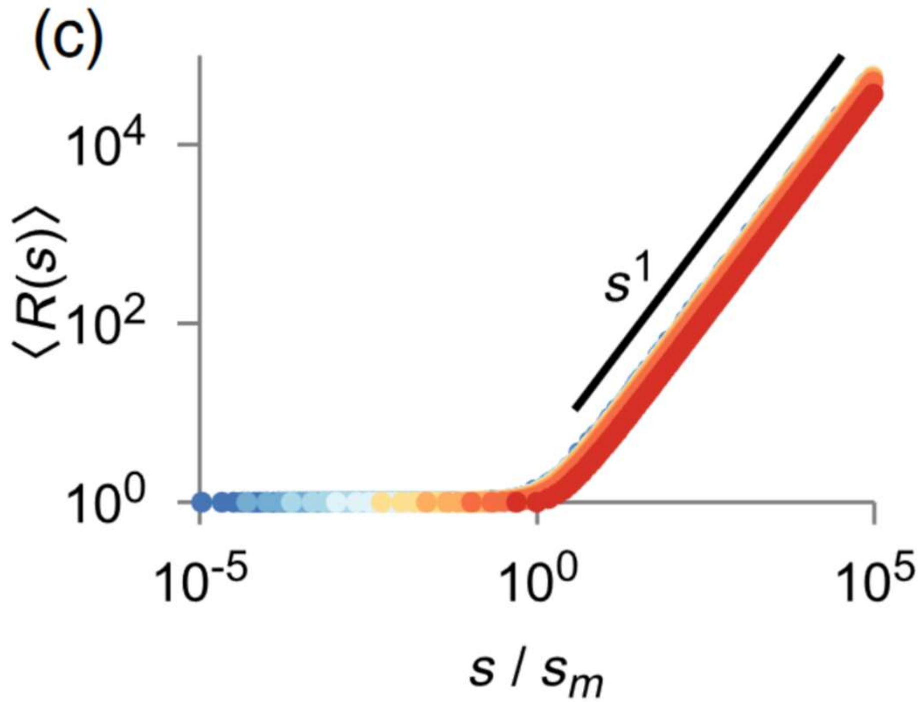


Scaling relation for size and time

- $\frac{\tau-1}{\alpha-1} = \sigma\nu z$ satisfied for directed and undirected regimes respectively with $(\tau, \alpha, \sigma\nu z)$ being $\left(\frac{3}{2}, 2, \frac{1}{2}\right)$ and $\left(\frac{5}{2}, 7, \frac{1}{4}\right)$



Clusters grow by merging on large scales



Self-consistency equations

The active fraction self-consistency equation:

$$\Phi = \sum_{m=0}^{k} \binom{k}{m} \Phi^m \bar{\Phi}^{k-m} (1 - \bar{p}\bar{q}^m)$$
$$= 1 - \bar{p}\overline{q\Phi}^k.$$

$$s_d = 1 + \sum_{l=1}^{\tilde{n}_p} s_{p,l} + \sum_{m=1}^{n_d} s_{d,m},$$

Generating functions

$$H_p(x) = xA_o(H_p(x))A_i(H_d(x)).$$