Avalanche criticality during ferroic switching

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UNIVERSITY OF CAMBRIDGE

Avalanche 2022 Debrecen

1 september 2022

@blaicasals

Where I am

UNIVERSITY OF CAMBRIDGE

Prof. Ekhard Salje

ICN2 Institut Català de Nanociència i Nanotecnologia

EXCELENCIA SEVERO OCHOA

Prof. Gustau Catalan

Next month: Physics Faculty

LIBERTAS PERFVNDET

OMNIA LVCE

UNIVERSITAT DE BARCELONA

Outline

Domain walls properties

Domain motion on:

Ferroelectrics

Ferroelastics

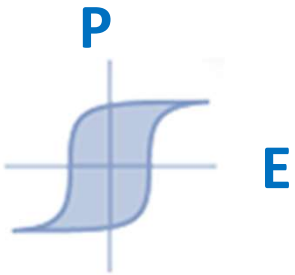
Ferrowrinkles

Ferromagnetics

Ferroic materials under field

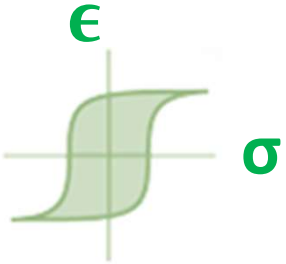
Ferroelectric

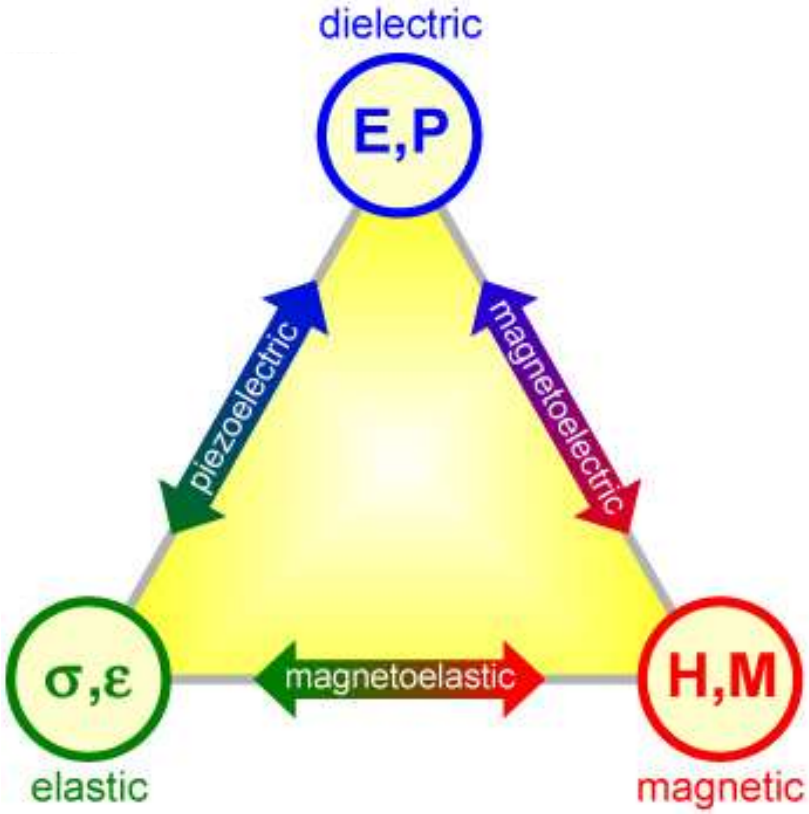
$\mathbf{P}(\mathbf{E} = 0) \neq 0 = \mathrm{P_r}$



Ferroelastic

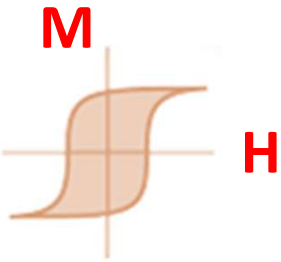
$\boldsymbol{\epsilon}(\boldsymbol{\sigma} = 0) \neq 0 = \boldsymbol{\epsilon}_\mathrm{r}$





Ferromagnetic

$\mathbf{M}(\mathbf{H} = 0) \neq 0 = \mathrm{M_r}$

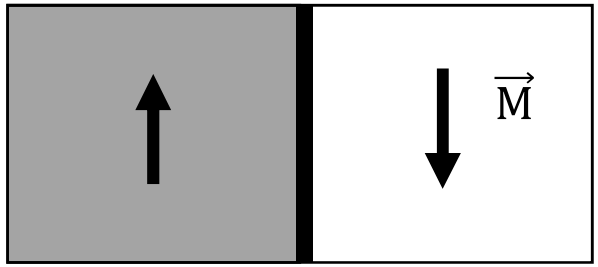


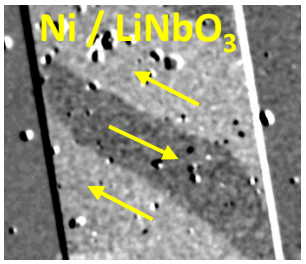
M. Oppel. J. Phys. D: Appl. Phys. 45 (2012)

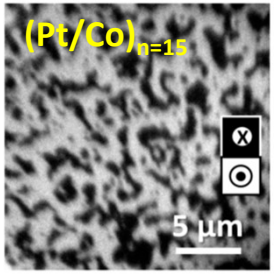
Domain walls (DW)

DW

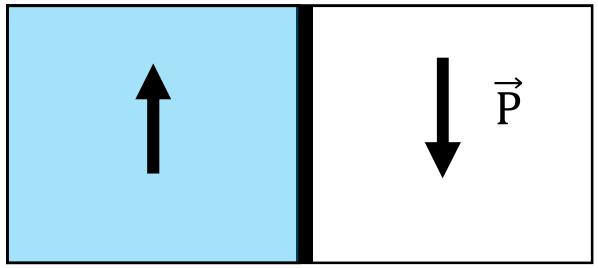
Ferromagnetic

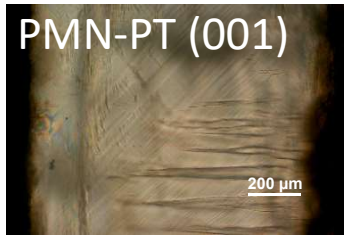




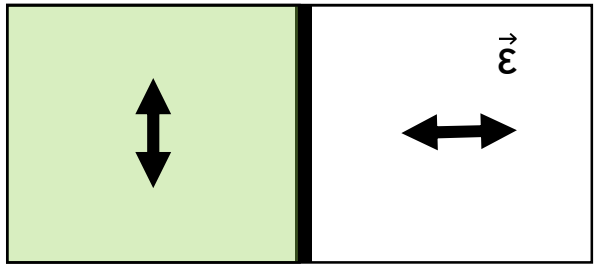


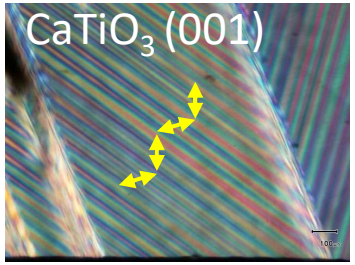
Ferroelectric

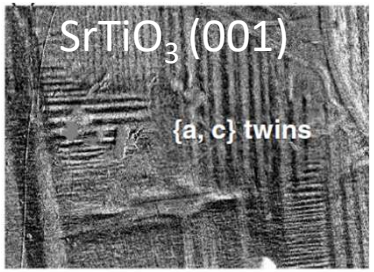




Ferroelastic







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Zoology of domain wall properties

Superconductivity

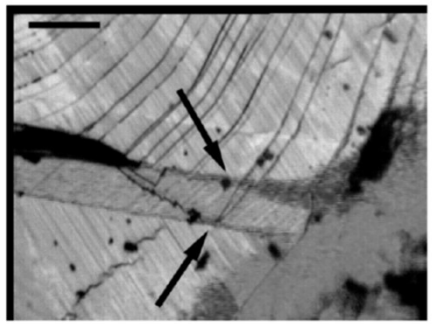
J. Phys.: Condens. Matter **10** (1998) L377–L380. Printed in the UK PII: S0953-8984(98)92366-9

LETTER TO THE EDITOR

Sheet superconductivity in twin walls: experimental evidence of WO_{3-x}

Alison Aird and Ekhard K H Salje

IRC in Superconductivity and Department of Earth Sciences, University of Cambridge, Downing Street, Cambridge CB2 3EQ, UK



Conduction

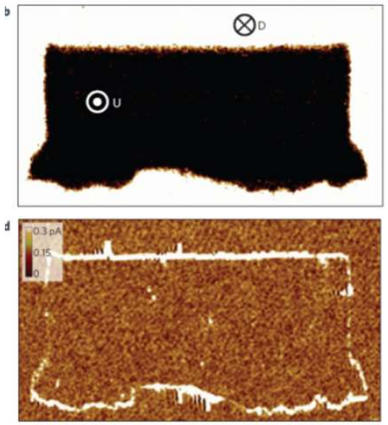
nature materials

ARTICLES

PUBLISHED ONLINE: 25 JANUARY 2009 | DOI: 10.1038/NMAT2373

Conduction at domain walls in oxide multiferroics

J. Seidel[1,2*†], L. W. Martin[2,3*], Q. He[1], Q. Zhan[2], Y.-H. Chu[2,3,4], A. Rother[5], M. E. Hawkridge[2], P. Maksymovych[6], P. Yu[1], M. Gajek[1], N. Balke[1], S. V. Kalinin[6], S. Gemming[7], F. Wang[1], G. Catalan[8], J. F. Scott[8], N. A. Spaldin[9], J. Orenstein[1,2] and R. Ramesh[1,2,3]



Polarity

$SrTiO_3$
$LaAlO_3$
$CaTiO_3$

PRL **111,** 247603 (2013) PHYSICAL REVIEW LETTERS week ending 13 DECEMBER 2013

Domains within Domains and Walls within Walls: Evidence for Polar Domains in Cryogenic $SrTiO_3$

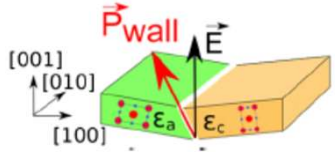
E. K. H. Salje,* O. Aktas, and M. A. Carpenter

Department of Earth Sciences, University of Cambridge, Downing Street, Cambridge CB2 3EQ, United Kingdom

V. V. Laguta

Institute of Physics AS CR, Cukrovarnicka 10, 16200 Prague, Czech Republic

J. F. Scott



Blai Casals et al, PRL 2018

Zoology of domain wall properties

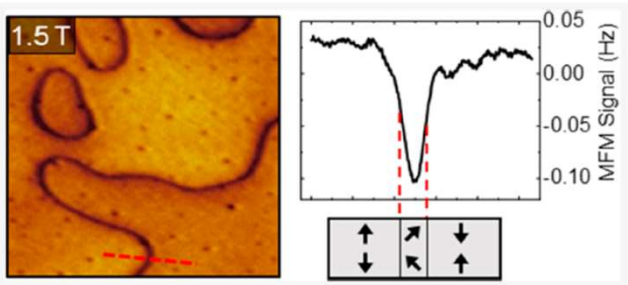
Magnetism

NANO LETTERS

pubs.acs.org/NanoLett Letter

Magnetic Imaging of Domain Walls in the Antiferromagnetic Topological Insulator $MnBi_2Te_4$

Paul M. Sass, Wenbo Ge, Jiaqiang Yan, D. Obeysekera, J. J. Yang, and Weida Wu*

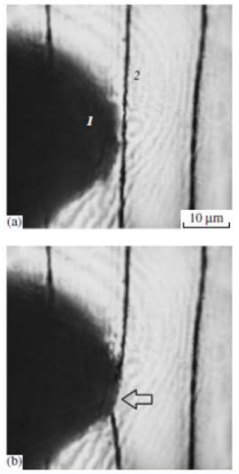


Polarity on ferromagnet

ISSN 0021-3640, JETP Letters, 2007, Vol. 86, No. 2, pp. 115–118. © Pleiades Publishing, Ltd., 2007.
Original Russian Text © A.S. Logginov, G.A. Meshkov, A.V. Nikolaev, A.P. Pyatakov, 2007, published in Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2007, Vol. 86, No. 2, pp. 124–127.

Magnetoelectric Control of Domain Walls in a Ferrite Garnet Film

A. S. Logginov, G. A. Meshkov, A. V. Nikolaev, and A. P. Pyatakov



Switch Polar

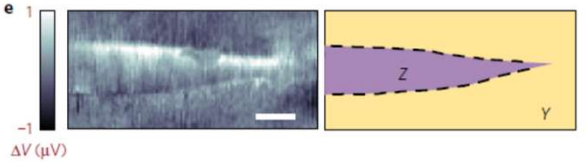
nature materials

ARTICLES

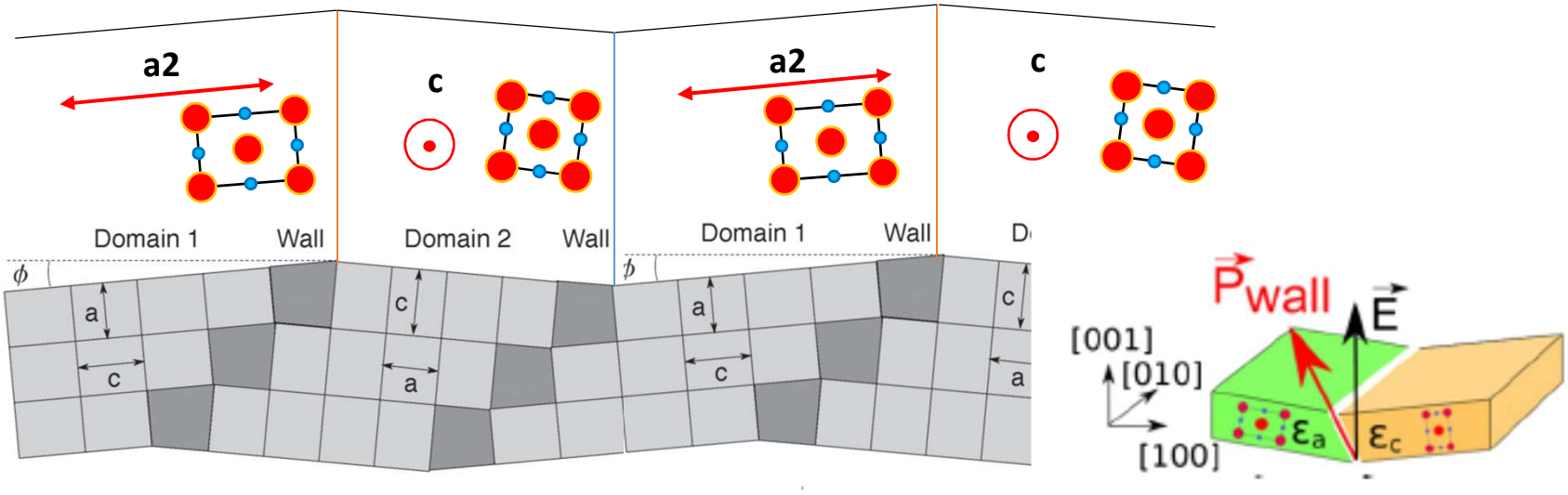
PUBLISHED ONLINE: 18 SEPTEMBER 2017 | DOI: 10.1038/NMAT4966

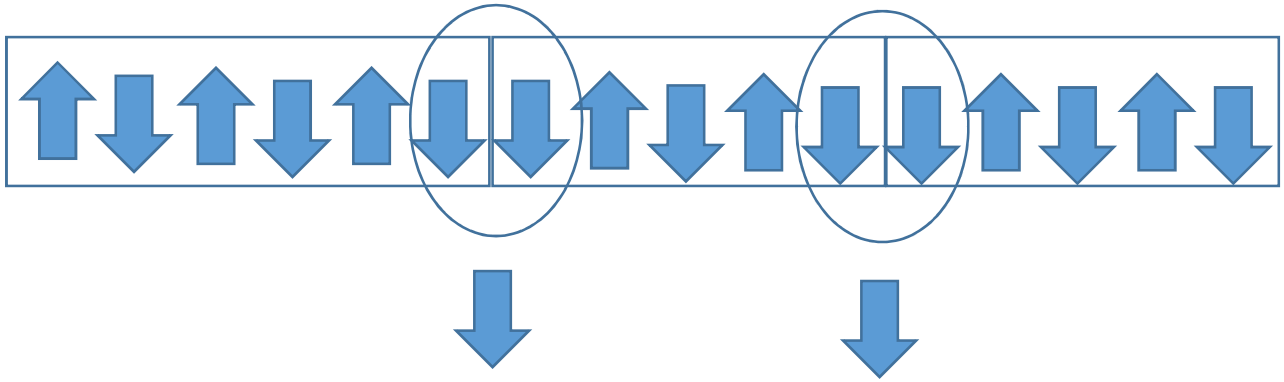
Imaging and tuning polarity at $SrTiO_3$ domain walls

Yiftach Frenkel[1], Noam Haham[1], Yishai Shperber[1], Christopher Bell[2], Yanwu Xie[3,4,5], Zhuoyu Chen[5], Yasuyuki Hikita[3], Harold Y. Hwang[3,5], Ekhard K. H. Salje[6,7] and Beena Kalisky[1]*

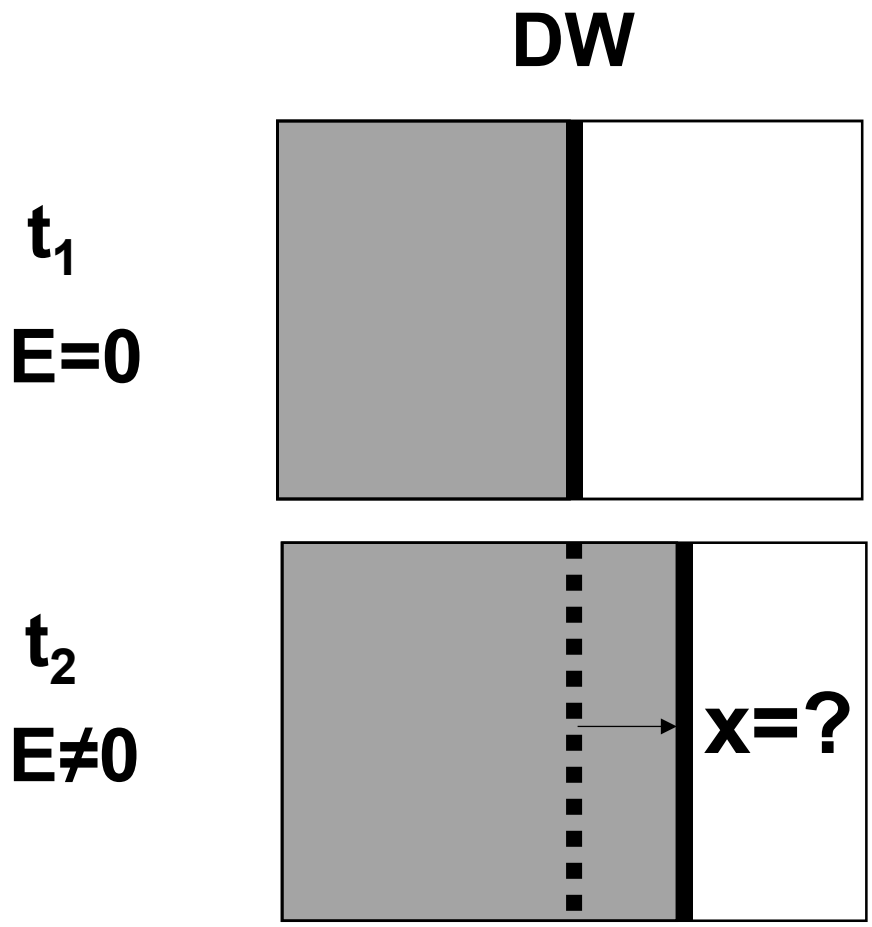


Origin flavour of the DW properties

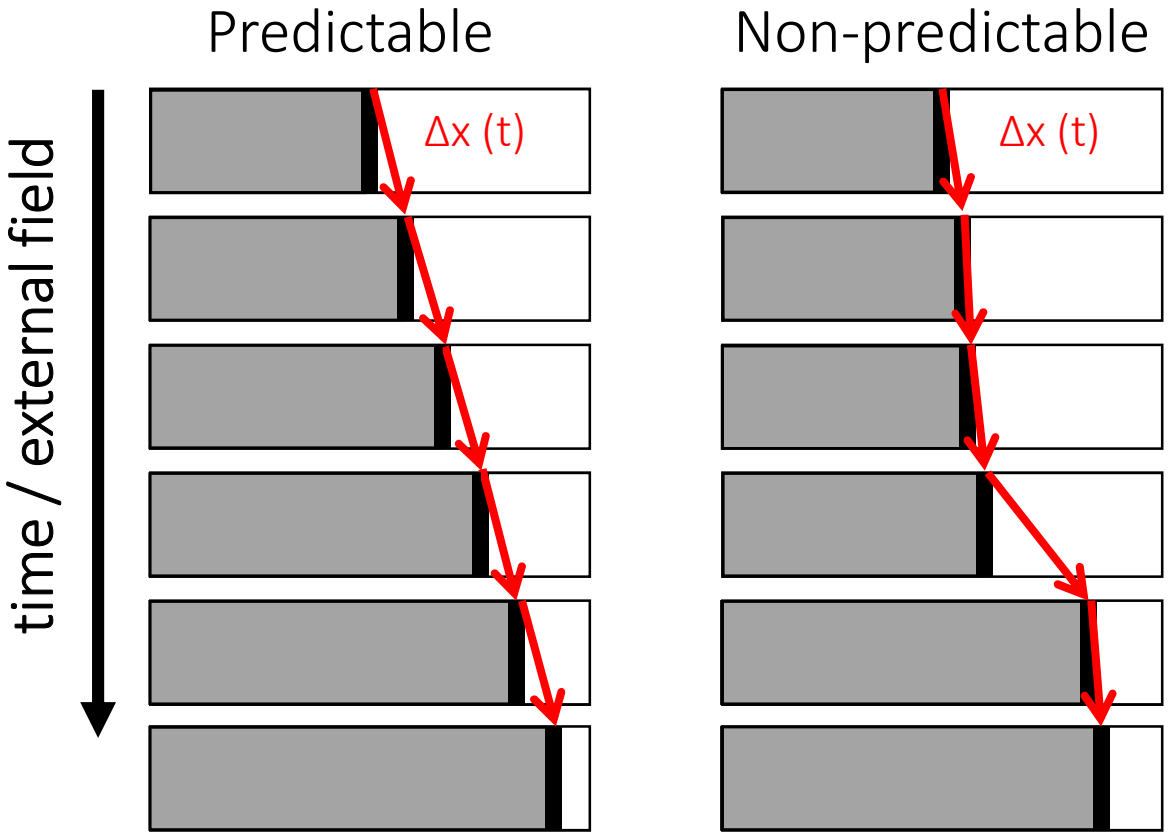


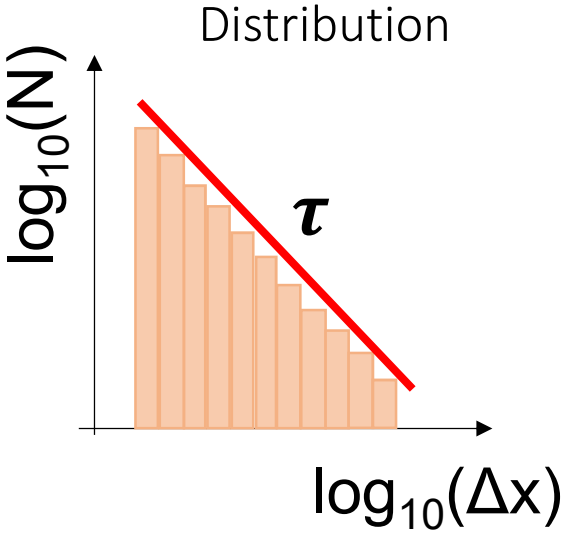


How a domain wall move?



Domain wall motion





Avalanches on domain motion.

$$\mathbf{PDF(\Delta x) \sim \Delta x^{-\tau}}$$

Strategies to control the DW motion / position

Coupling/pinning with the topography

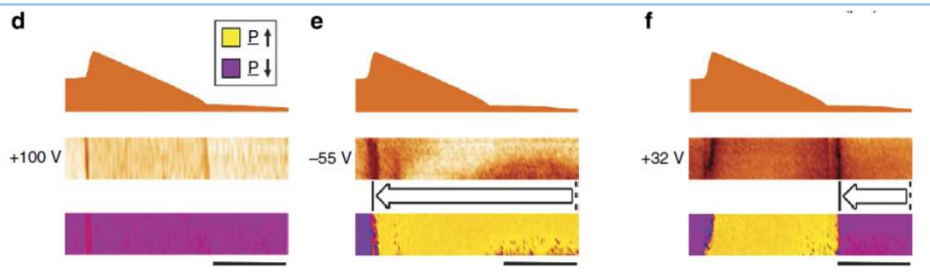
nature COMMUNICATIONS

ARTICLE

Received 15 Jan 2015 | Accepted 29 Apr 2015 | Published 10 Jun 2015 DOI: 10.1038/ncomms8361 OPEN

A diode for ferroelectric domain-wall motion

J.R. Whyte[1] & J.M. Gregg[1]



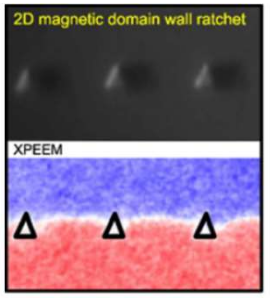
ELSEVIER

Materials and Design

journal homepage: www.elsevier.com/locate/matdes

2D magnetic domain wall ratchet: The limit of submicrometric holes

J. Herrero-Albillos [a,b,c,*], C. Castán-Guerrero [b,c], F. Valdés-Bango [d,e], J. Bartolomé [b,c], F. Bartolomé [b,c], F. Kronast [f], A. Hierro-Rodriguez [g], L.M. Álvarez Prado [d,e], J.I. Martín [d,e], M. Vélez [d,e], J.M. Alameda [d,e], J. Sesé [b,c], L.M. García [b,c]

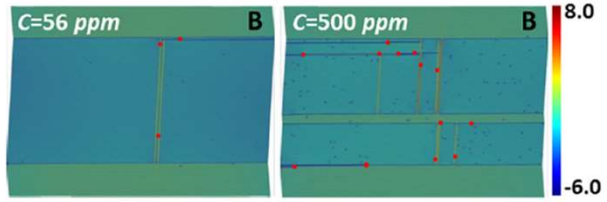


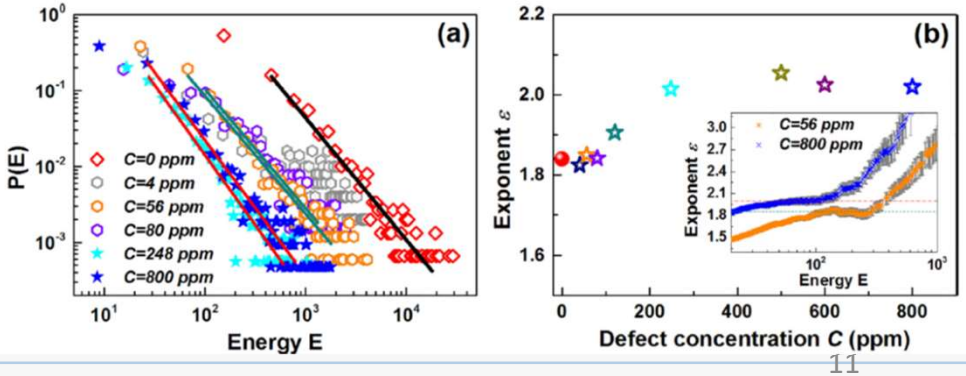
Defects density

APPLIED PHYSICS LETTERS 112, 092904 (2018)

Immobile defects in ferroelastic walls: Wall nucleation at defect sites

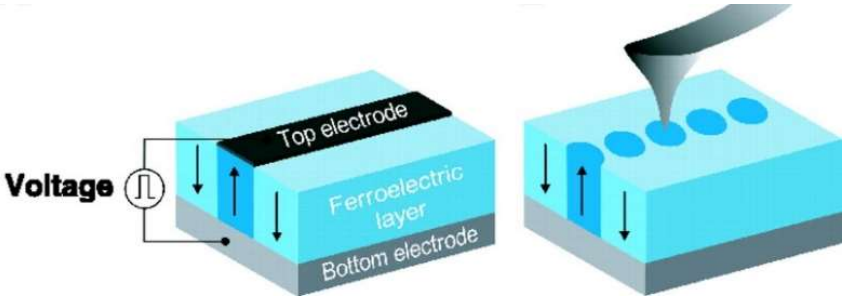
X. He,[1] E. K. H. Salje,[1,2,a)] X. Ding,[1,a)] and J. Sun[1]





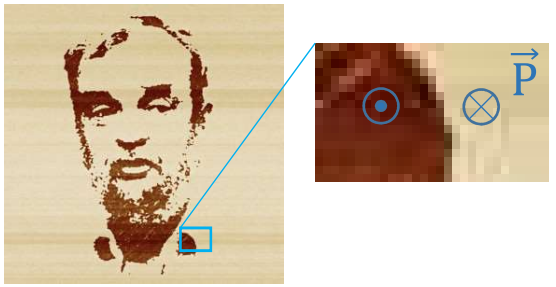
Strategies to control the DW motion / position

Writing (contacts, AFM, ...)



C. H. Ahn et al., Science 23, 303, 488-491 (2004)

James F. Scott (1942 - 2020)



bismuth ferrite, 30 µm × 30 µm.

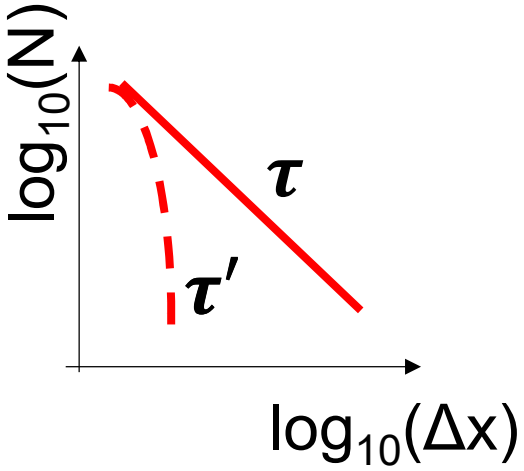
G. Catalan et al., Nature Materials volume 19, 580 (2020)

Non-invasive and reversible strategies?

Predictable

Non-predictable

Study the Dynamics + perturbation



Outline

Domain motion on:

Ferroelectrics

Ferroelastics

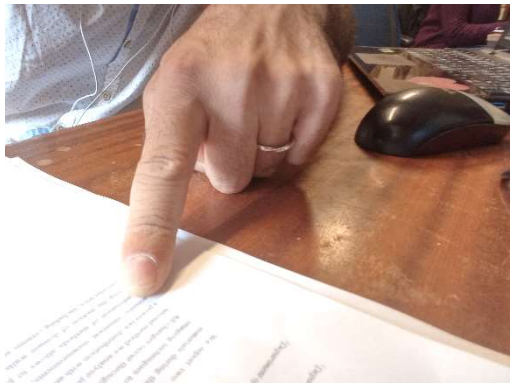
Ferrowrinkles

Ferromagnetics

Measuring avalanches on ferroelectrics

"Listen"

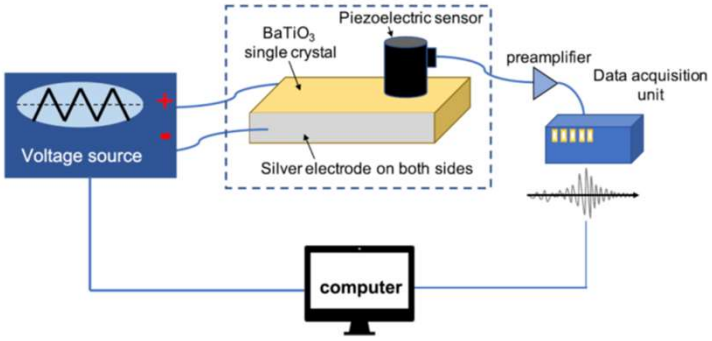
"Touch"



"Watch"

Acoustic Emission

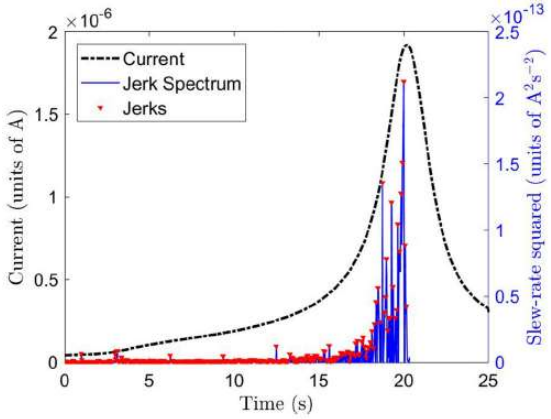
Strain changes



E. K. H. Salje et al., Phys Rev Mat (2019)

Displacement current

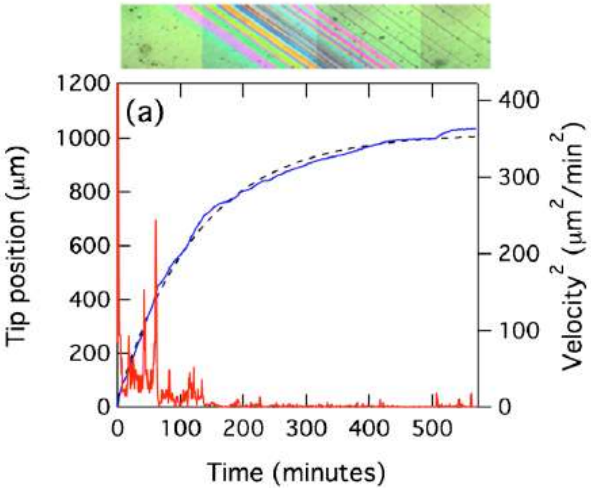
Polarization changes



Tan et al, Phys Rev Mat (2019)

Imaging patern changes

Polarization and Strain

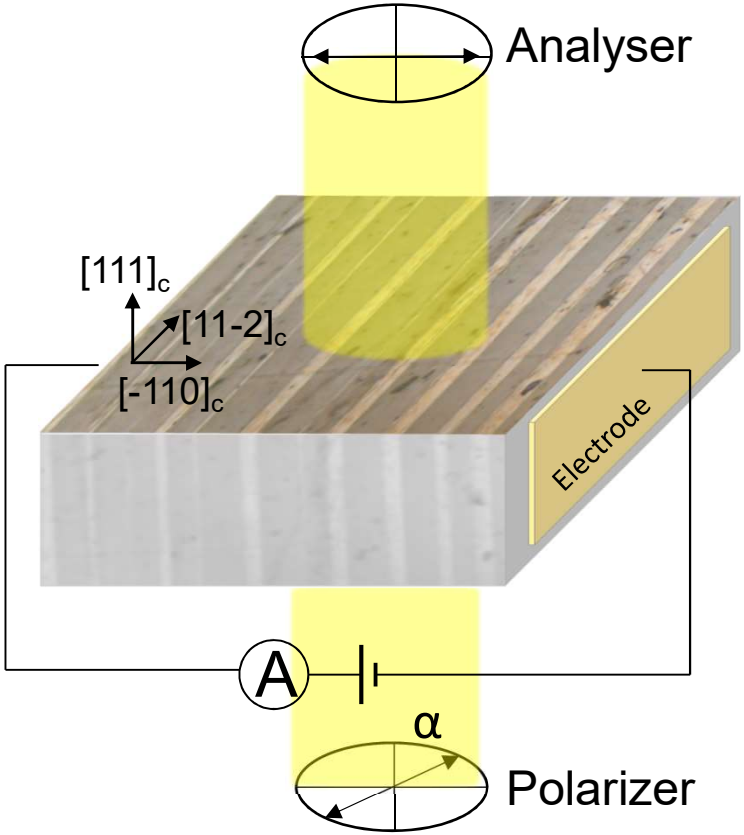


R. Harrison, E. K. H. Salje. Appl. Phys. Lett. (2010)

B. Casals et al. , APL Mater. 8, 011105 (2020)

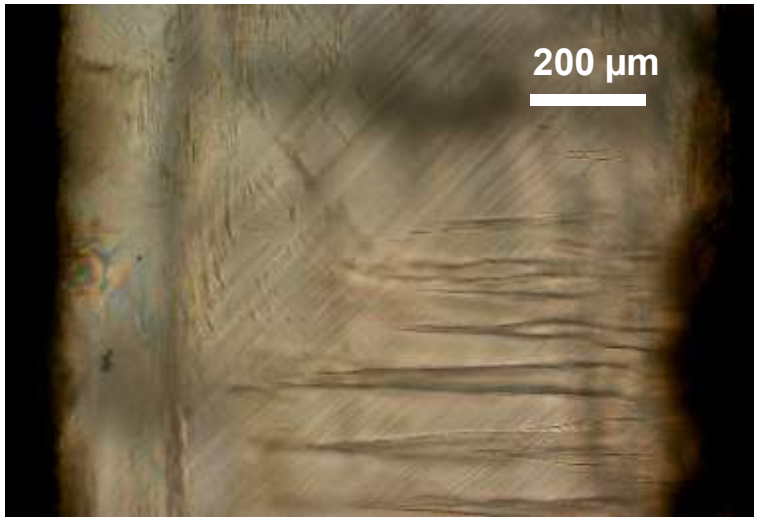
The Experiment, imaging pattern changes

Simultaneous measurement:
Birefringence images and displacement current



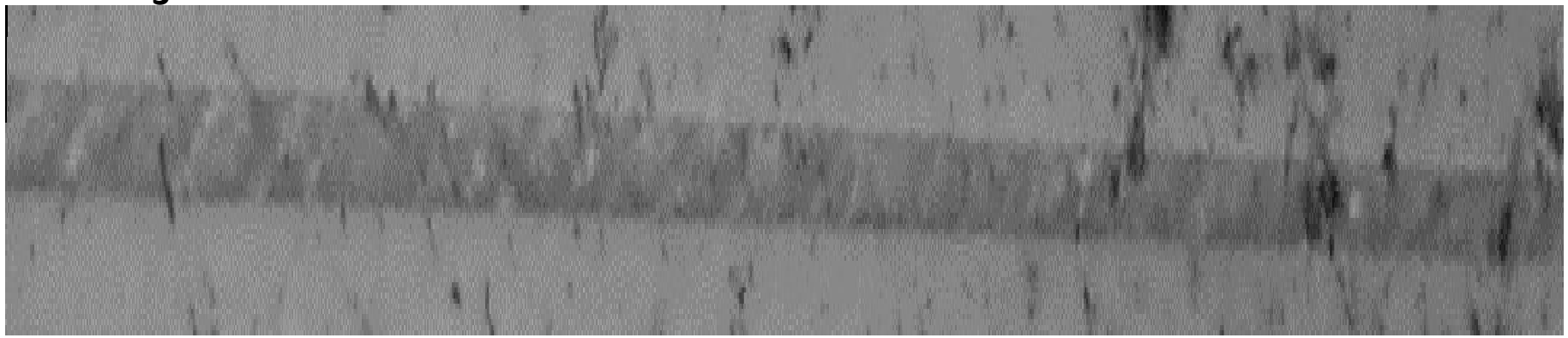
$BaTiO_3$ (111)

PMN-PT (001) (1-x)[Pb(Mg1/3Nb2/3)O3]-x[PbTiO3], x=0.32



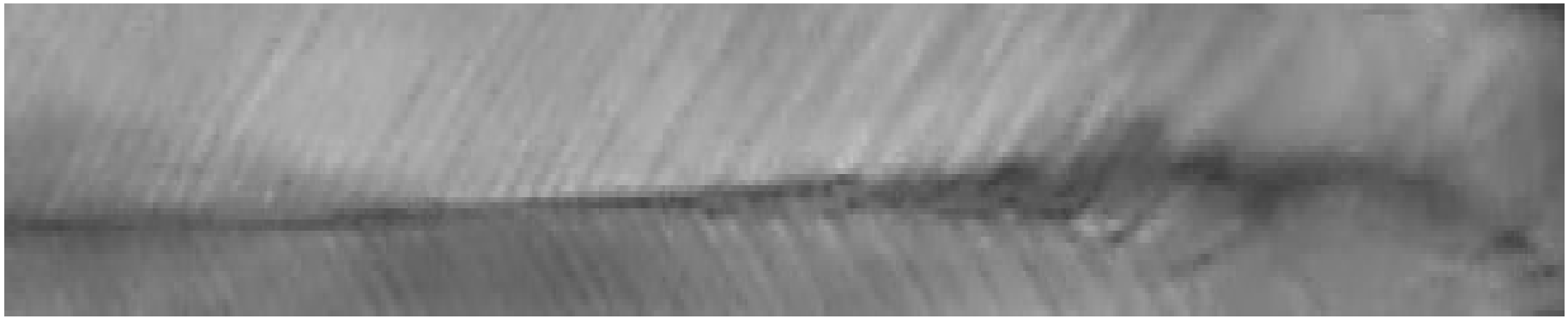
Two Ferroelectrics

$BaTiO_3$ (111)



Simple domain pattern with parallel DWs

PMN-PT (001) **$(1-x)[Pb(Mg_{1/3}Nb_{2/3})O_3]-x[PbTiO_3]$, x=0.32**



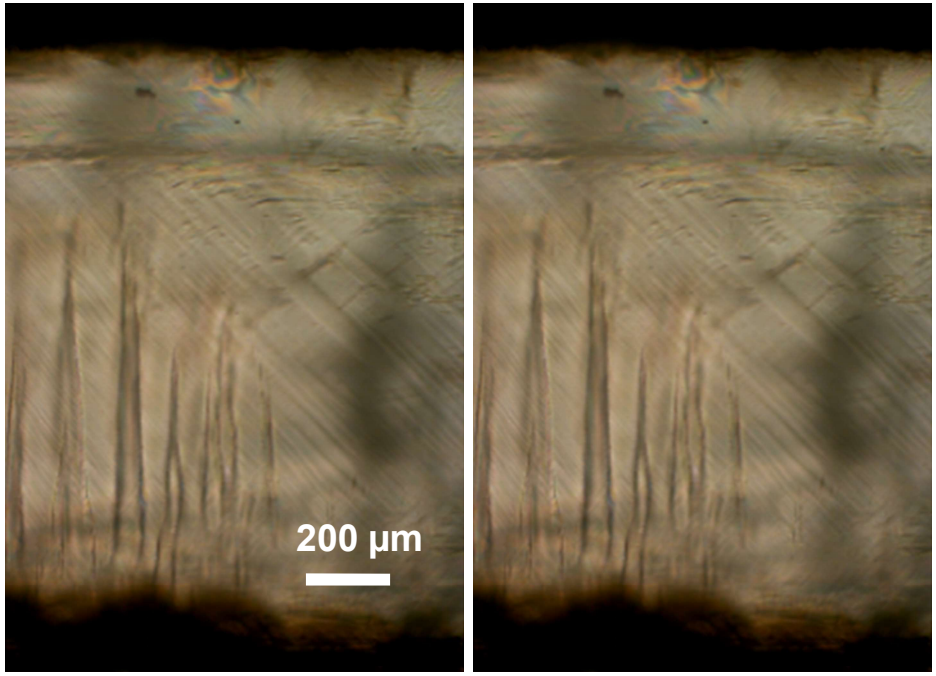
Complex domain pattern with junctions of DWs

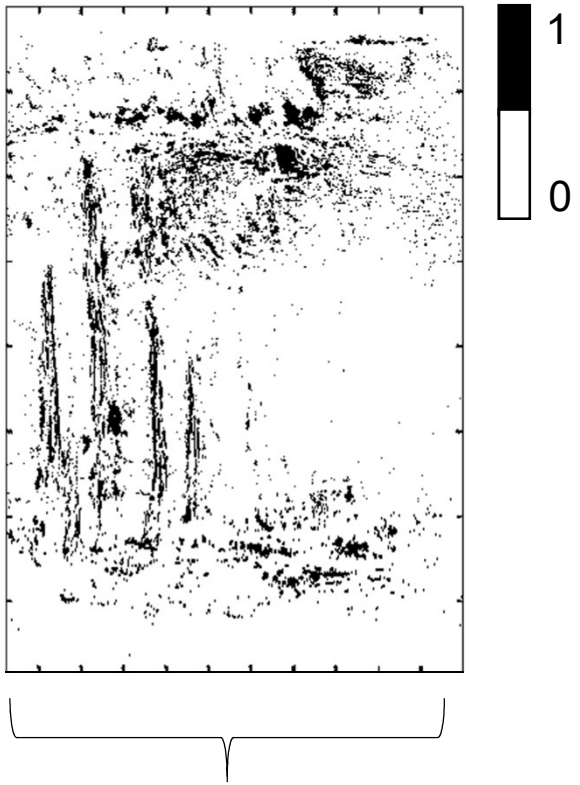
Pixel by Pixel analysis

$B(t_n)$ $B(t_{n+1})$

$$J_{ij} = \left(\frac{dB_{ij}}{dt}\right)^2$$

J_{ij} > threshold





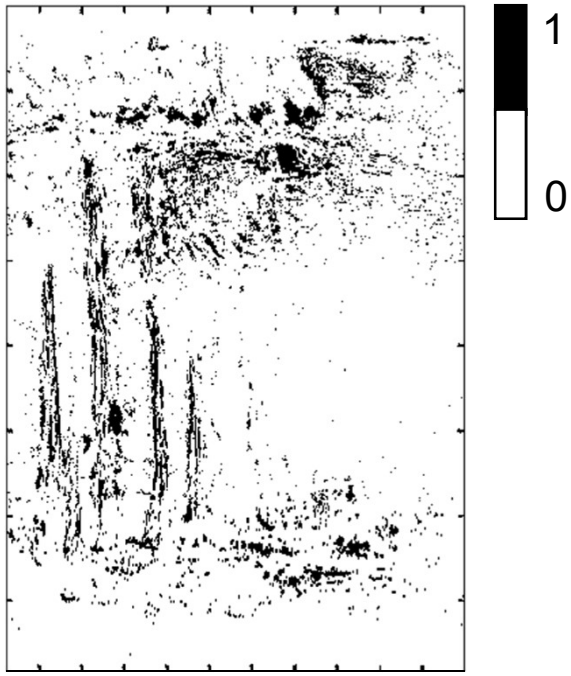
N, Activity (# areas)

A, Areas

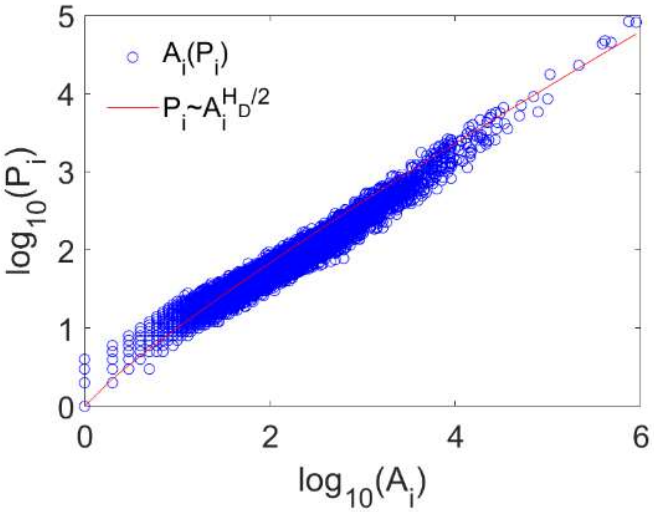
P, Perimeters

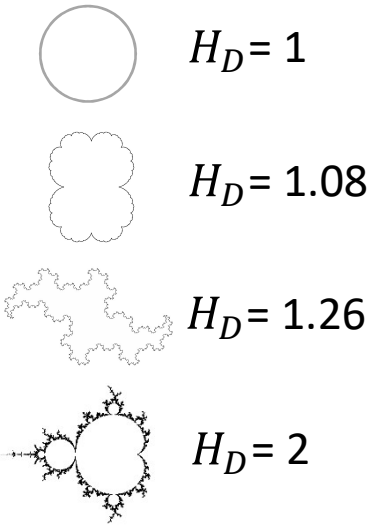
Pixel by Pixel analysis

$$J_{ij} = \left(\frac{dB_{ij}}{dt}\right)^2$$ J_{ij} > threshold



Fractal dimension (Hausdorff dimension) $\boldsymbol{P \sim A^{H_D/2}}$





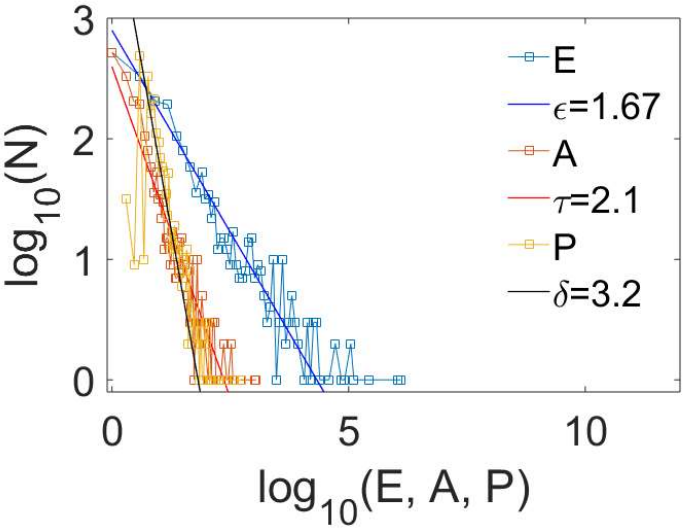
$H_D = 1$

$H_D = 1.08$

$H_D = 1.26$

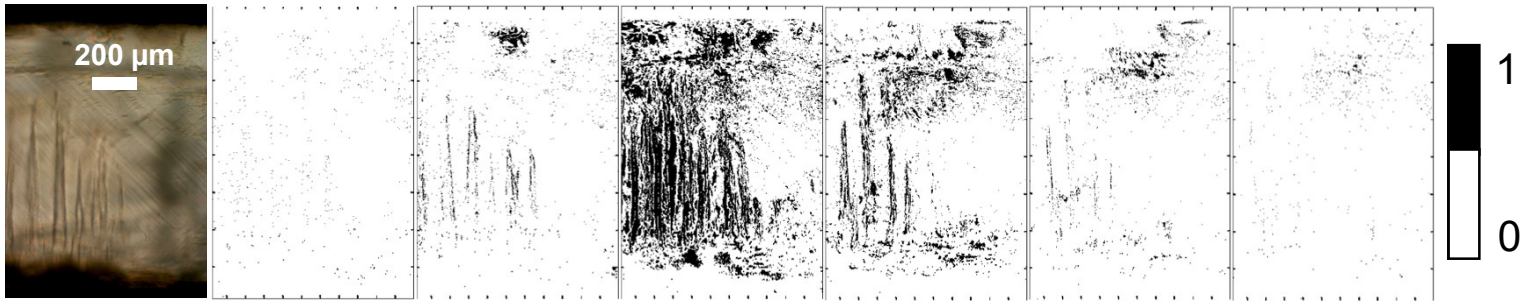
$H_D = 2$

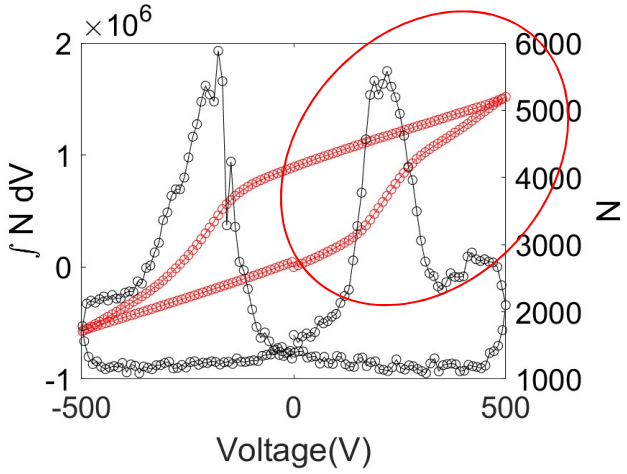
Distribution of Areas A, Perimeters P and Energies E.



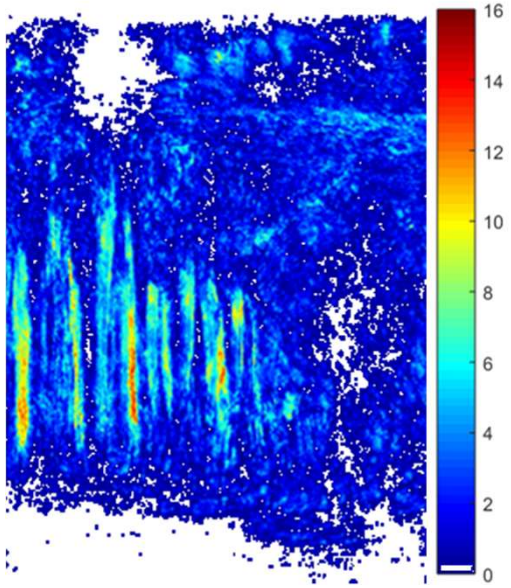
$\mathrm{PDF(A)} \sim \mathrm{A}^{-\tau}$

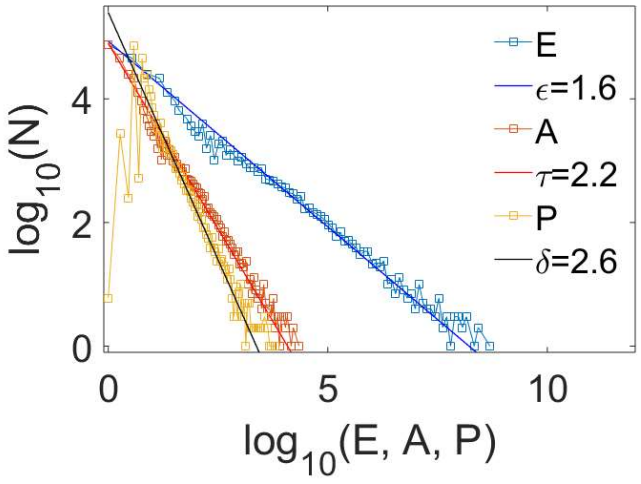
Ferroelectric switching



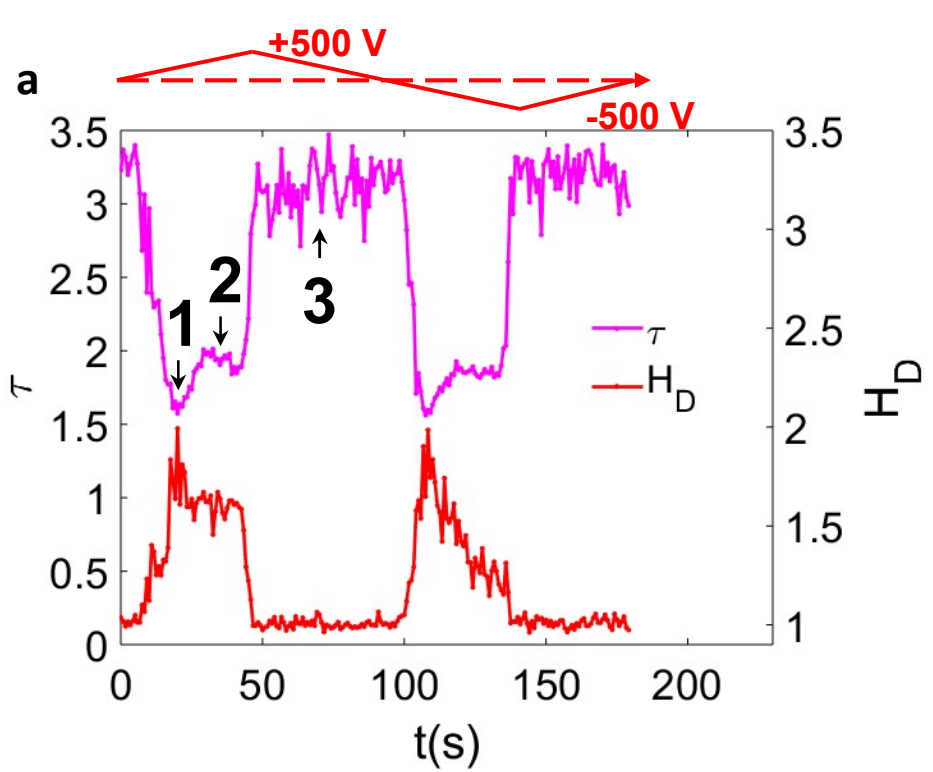


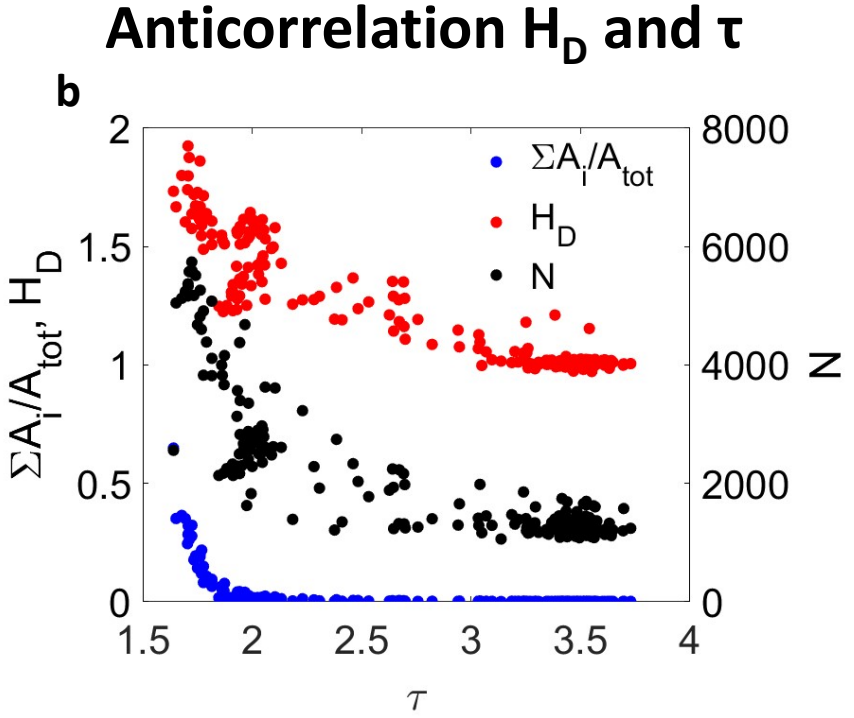
Avalanches map N

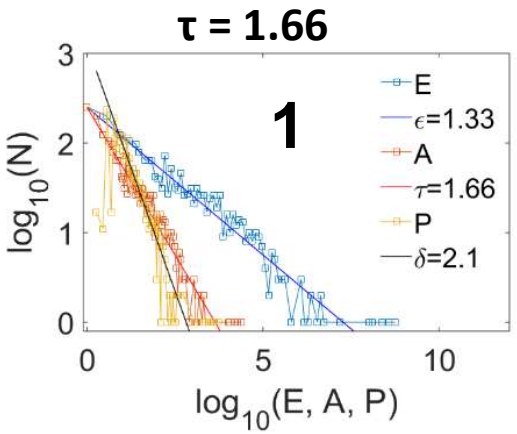


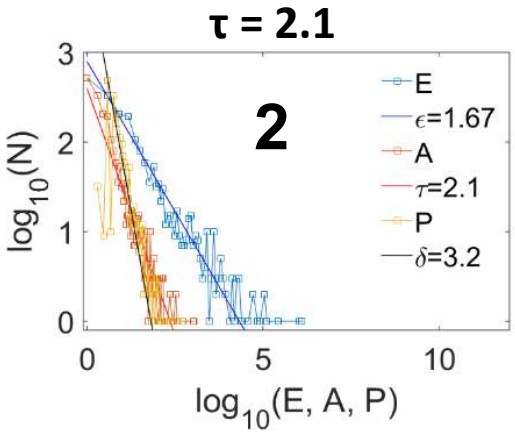


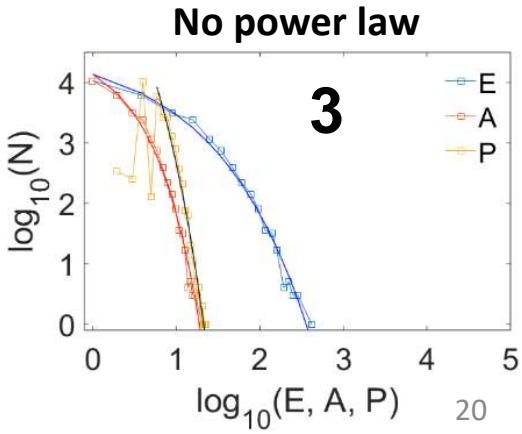
Avalanches during FE switching, PMN-PT



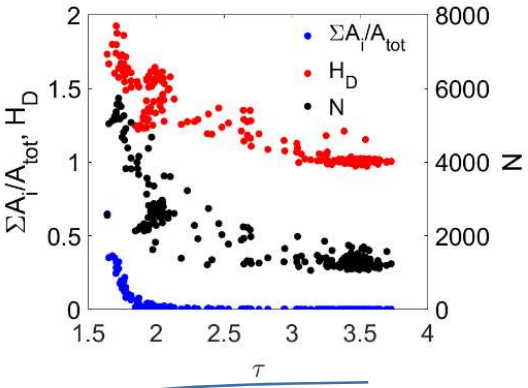








H_D and τ during FE switching



E_C

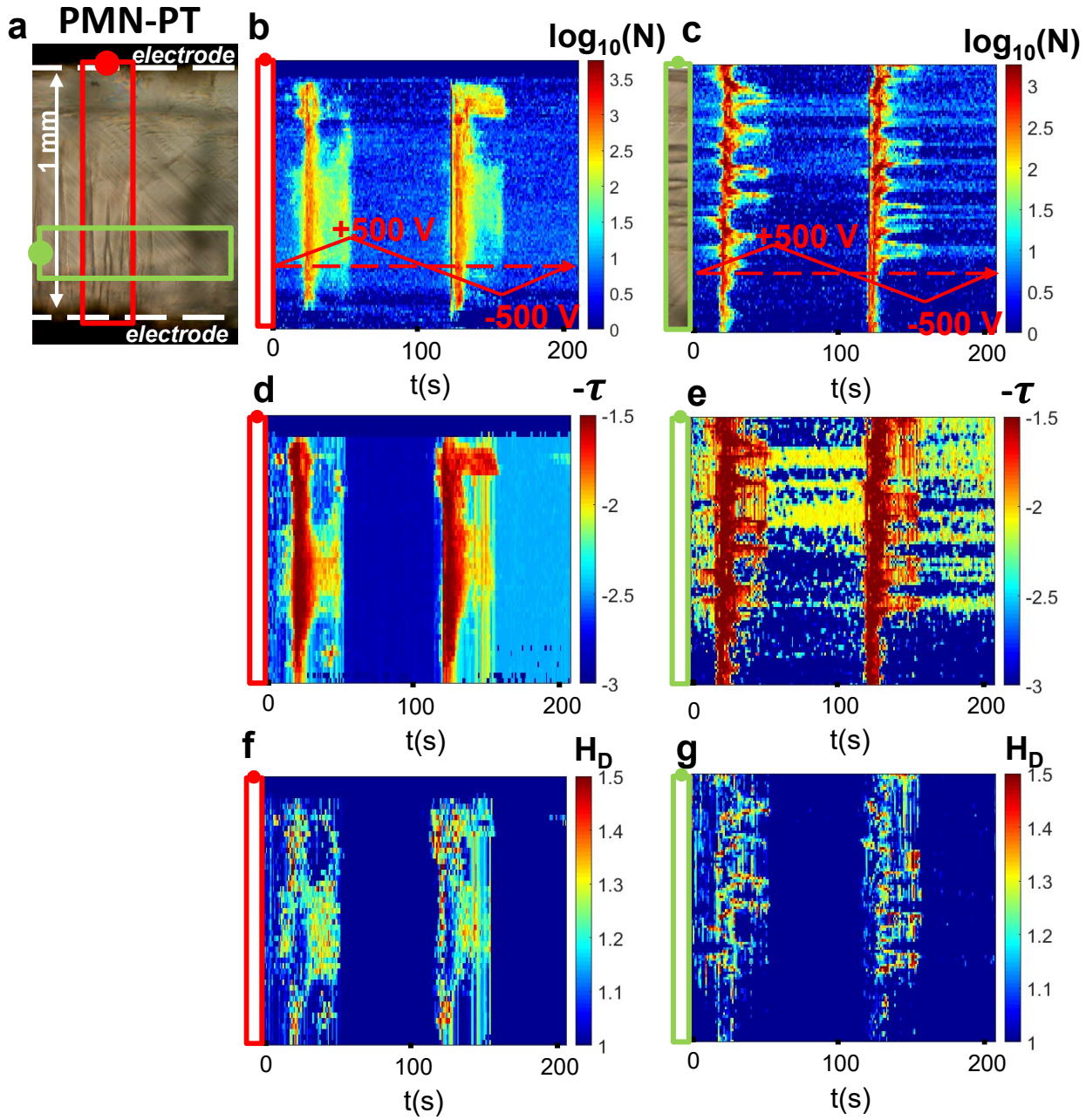
τ

N , H_D

τ = 2.2 (ε=1.66)

τ = 1.66 (ε=4/3)

Spatiotemporal mapping, PMN-PT



Activity

N

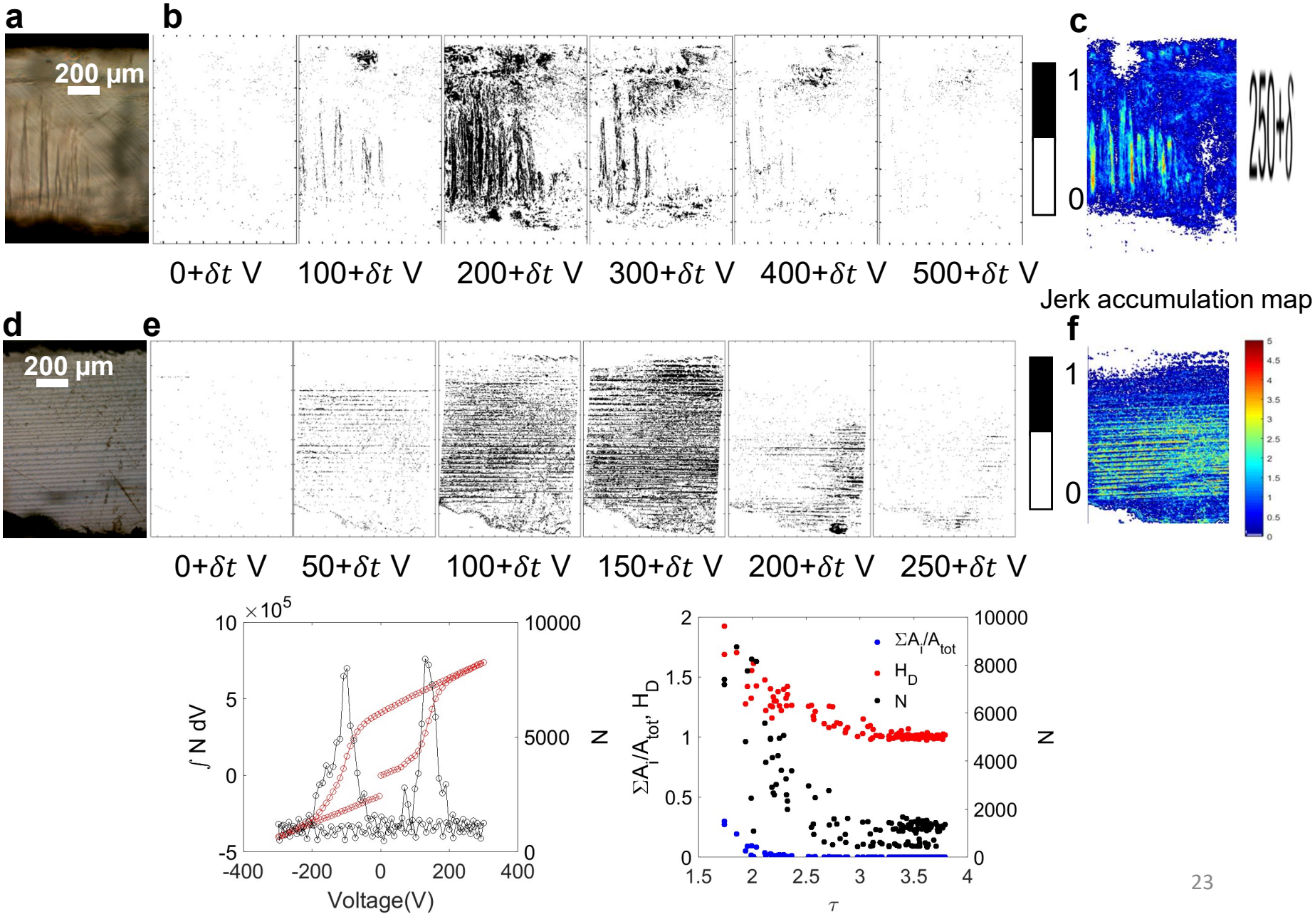
Criticality

$PDF(A) \sim A^{-\tau}$

Hausdorff dimension

H_D

PMN-PT, $BaTiO_3$



Avalanche criticality in ferroelectrics switching

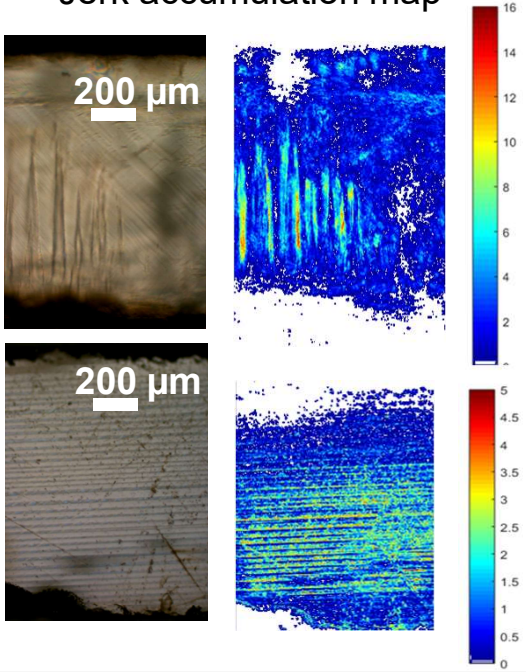
ARTICLE

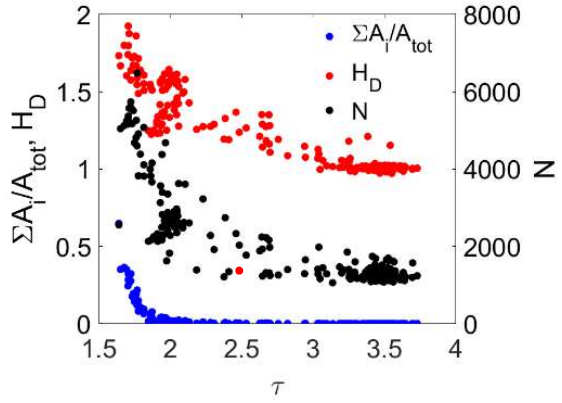
https://doi.org/10.1038/s41467-020-20477-6 OPEN

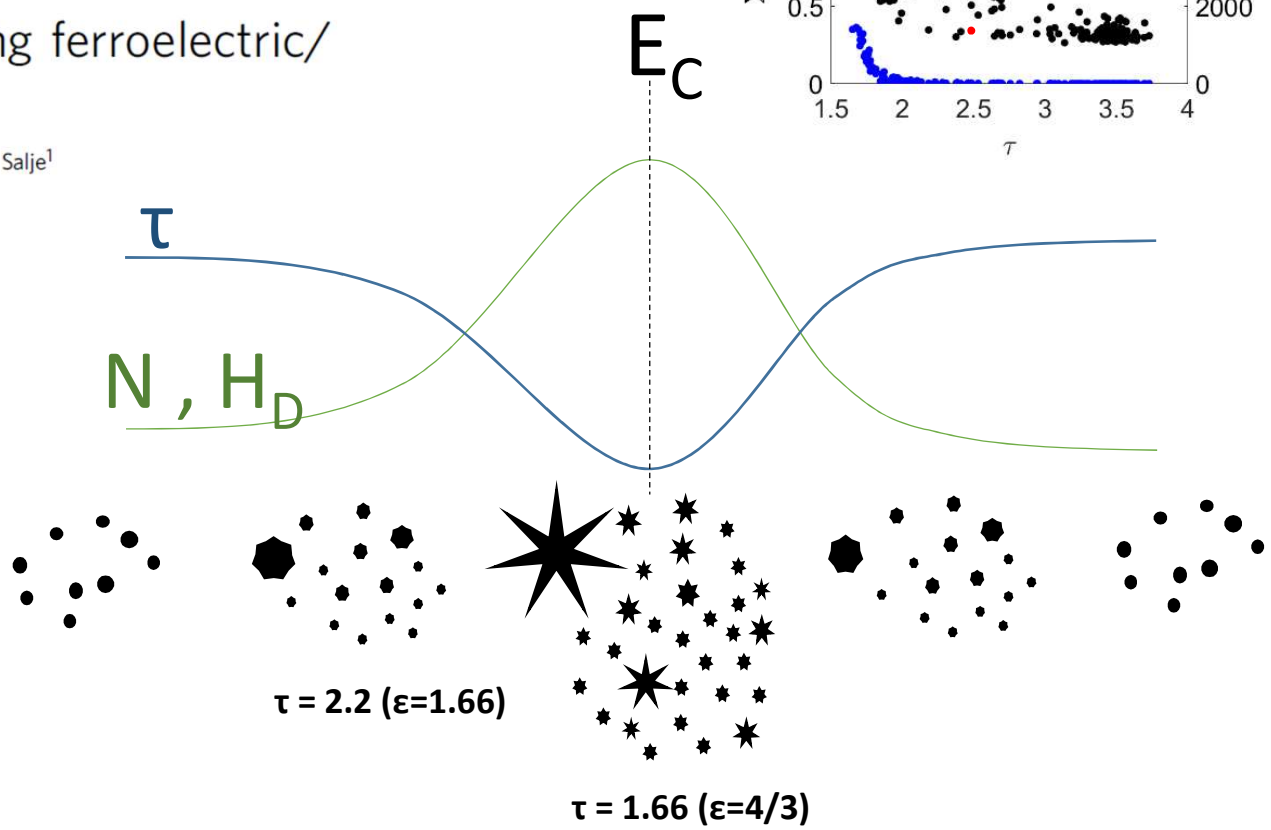
Avalanche criticality during ferroelectric/ ferroelastic switching

Blai Casals[1✉], Guillaume F. Nataf[2] & Ekhard K. H. Salje[1]

Jerk accumulation map







Outline

Domain motion on:

Ferroelectrics

Ferroelastics

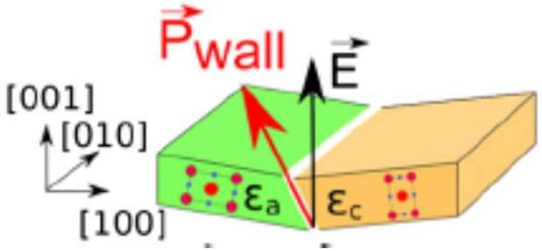
Ferrowrinkles

Ferromagnetics

Domain wall interaction change the dynamics

$SrTiO_3$

Ferroelastic



PHYSICAL REVIEW RESEARCH **1**, 032025(R) (2019)

Rapid Communications

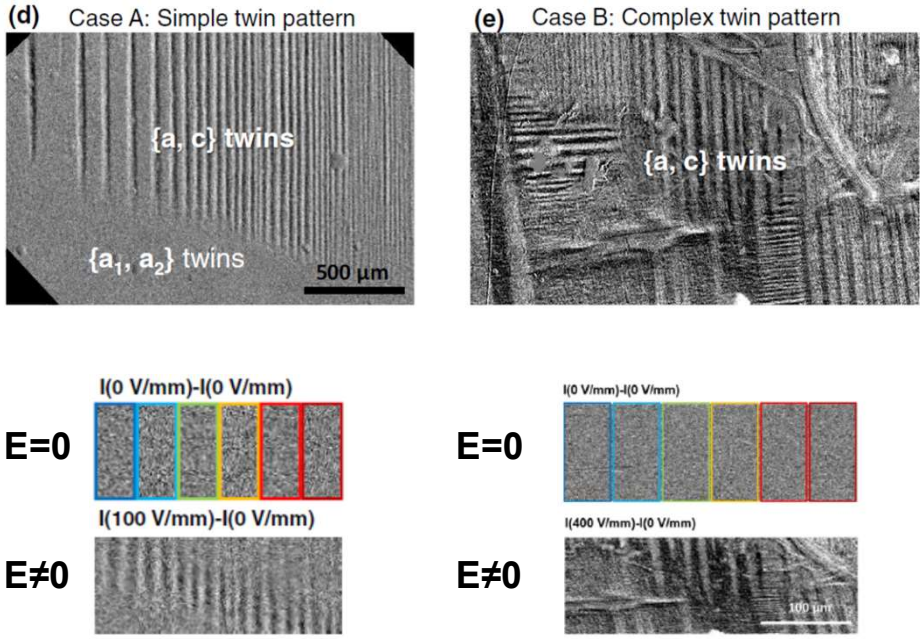
Electric-field-induced avalanches and glassiness of mobile ferroelastic twin domains in cryogenic $SrTiO_3$

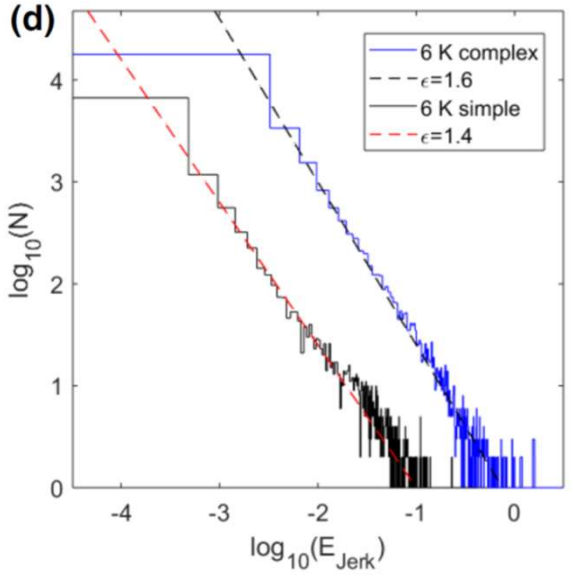
Blai Casals,[1] Sebastiaan van Dijken,[2] Gervasi Herranz,[3] and Ekhard K. H. Salje[1]

PHYSICAL REVIEW LETTERS **120,** 217601 (2018)

Low-Temperature Dielectric Anisotropy Driven by an Antiferroelectric Mode in $SrTiO_3$

Blai Casals,[1] Andrea Schiaffino,[1] Arianna Casiraghi,[2] Sampo J. Hämäläinen,[2] Diego López González,[2] Sebastiaan van Dijken,[2] Massimiliano Stengel,[1,3] and Gervasi Herranz[1]

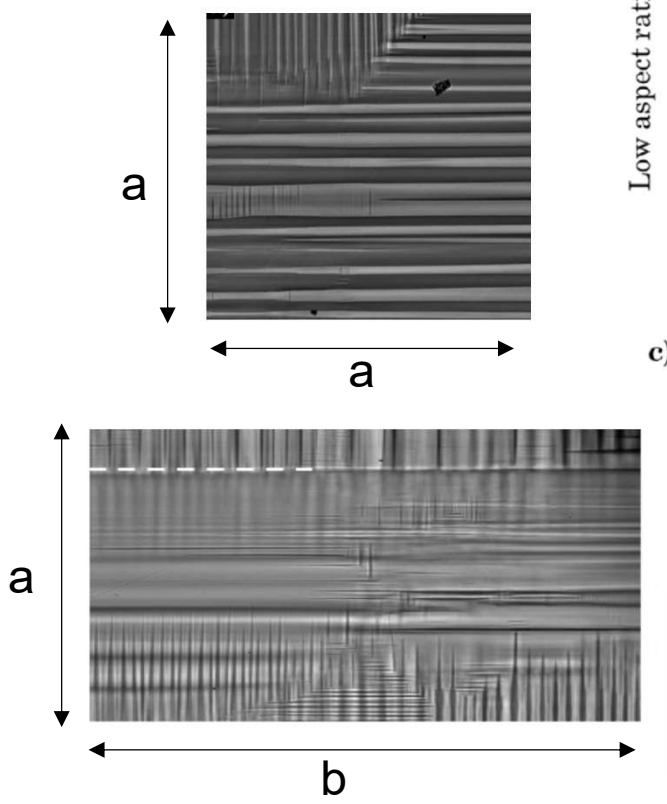




Aspect ratio, domain pattern

$LaAlO_3$

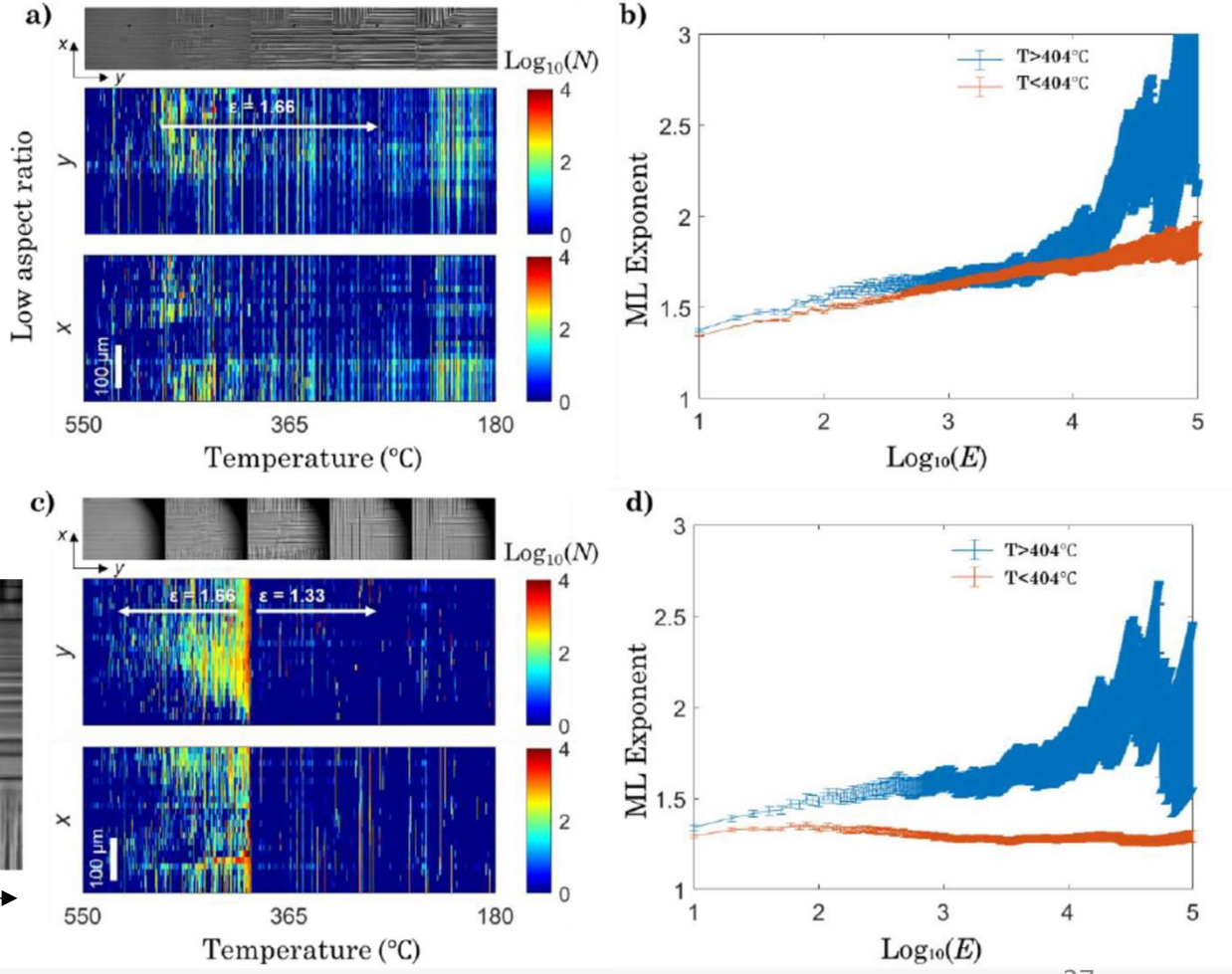
Ferroelastic



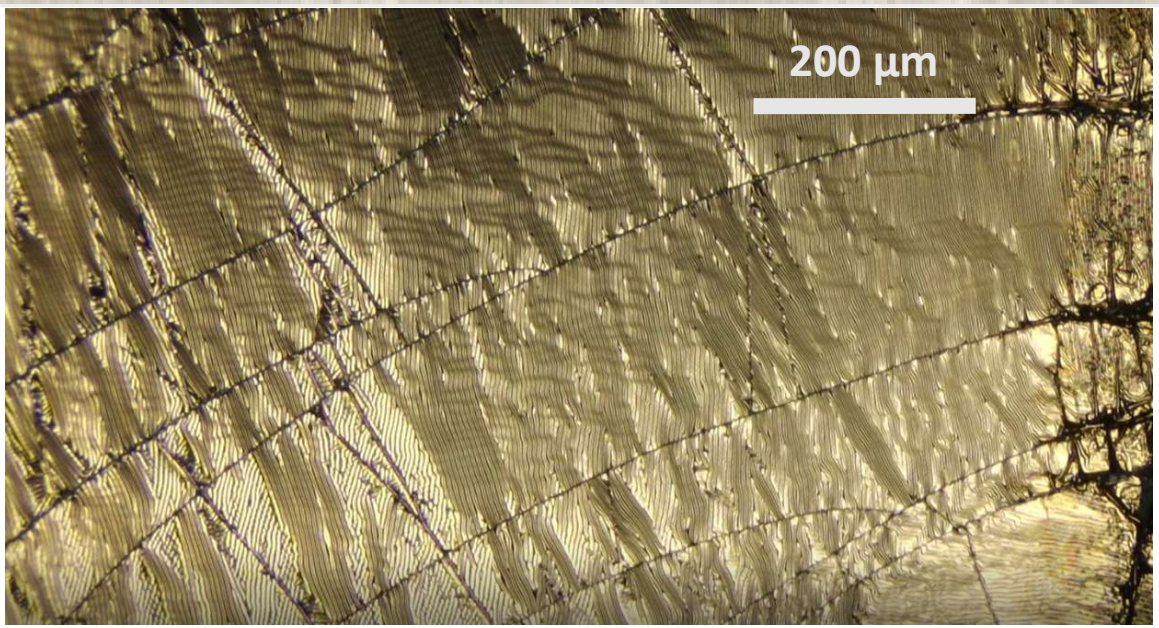
Avalanche Criticality in $LaAlO_3$: The Effect of Aspect Ratio

Sci. Rep. 2022

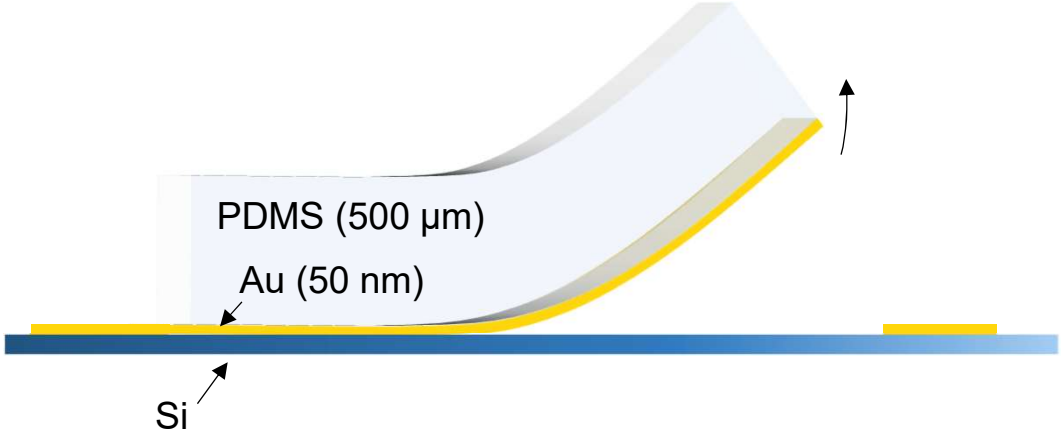
John J. R. Scott[1,*], Blai Casals[2], King-Fa Luo[1], Atta Haq[3], Davide Mariotti[3], Ekhard K. H. Salje[2], and Miryam Arredondo[1]



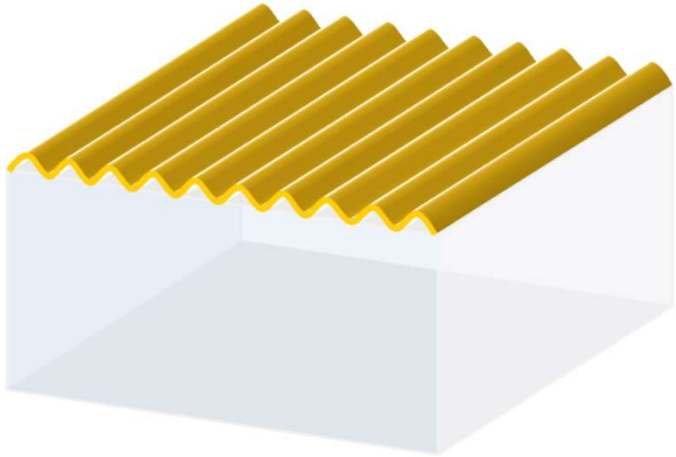
Wrinkle, film on a viscoelastic



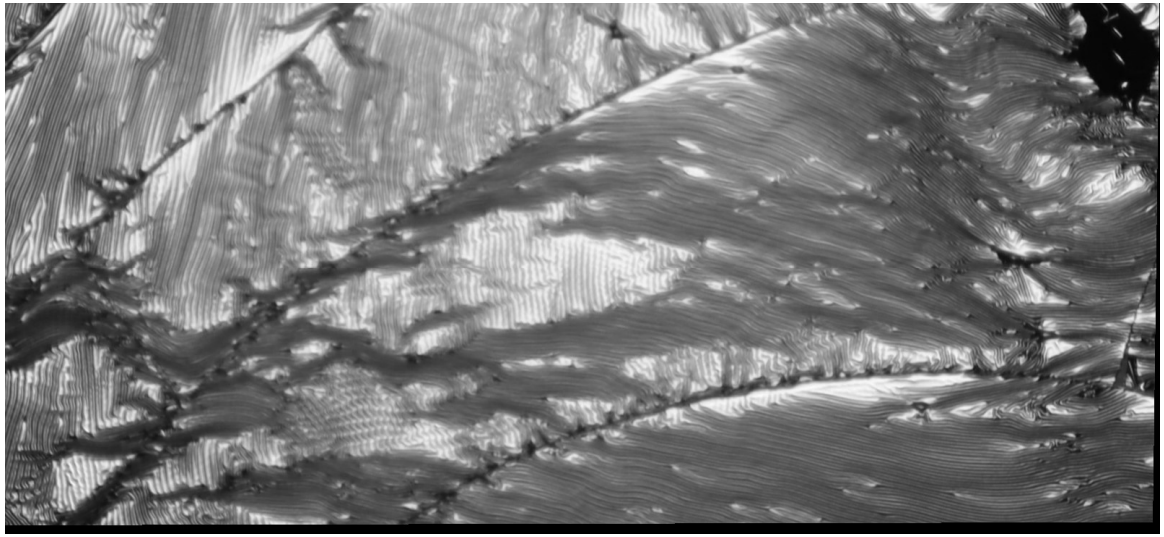
Peeling

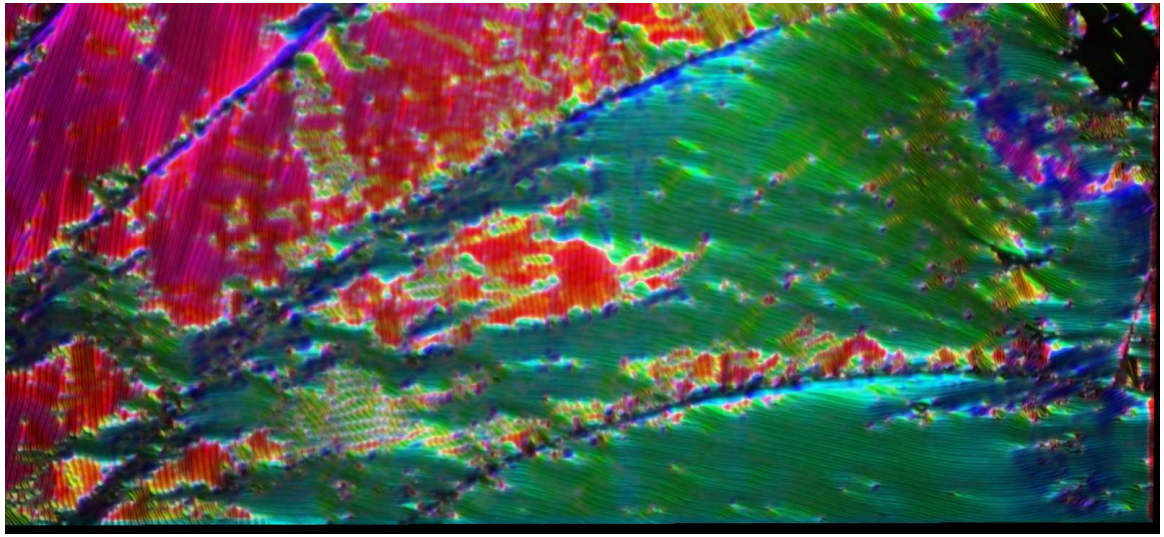


Wrinkled surface

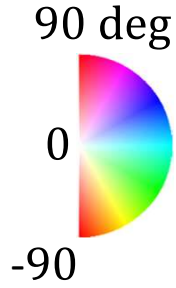


Wrinkle domains?



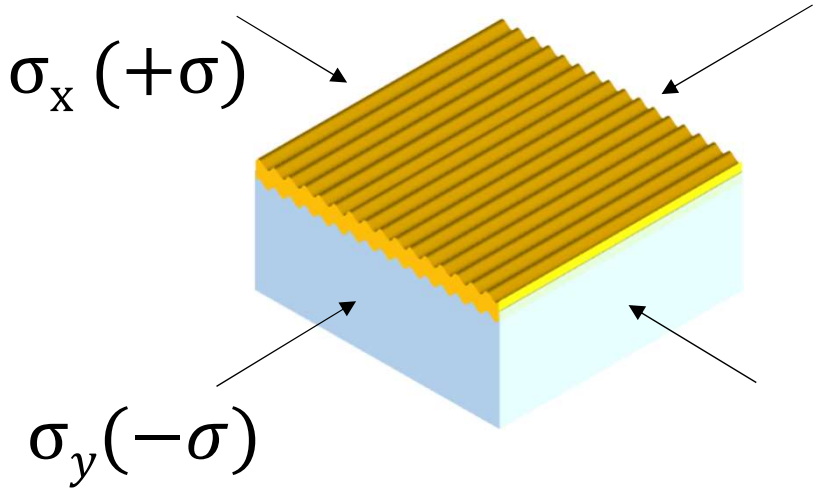


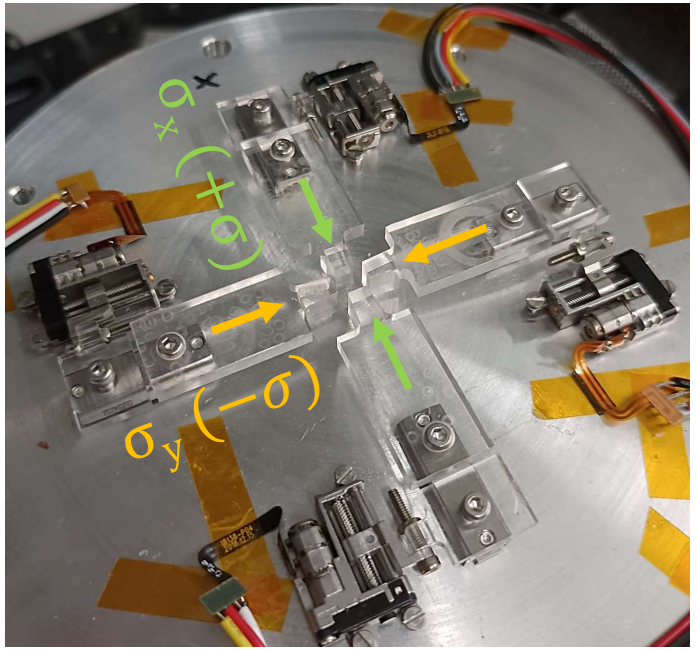
Wrinkle angle



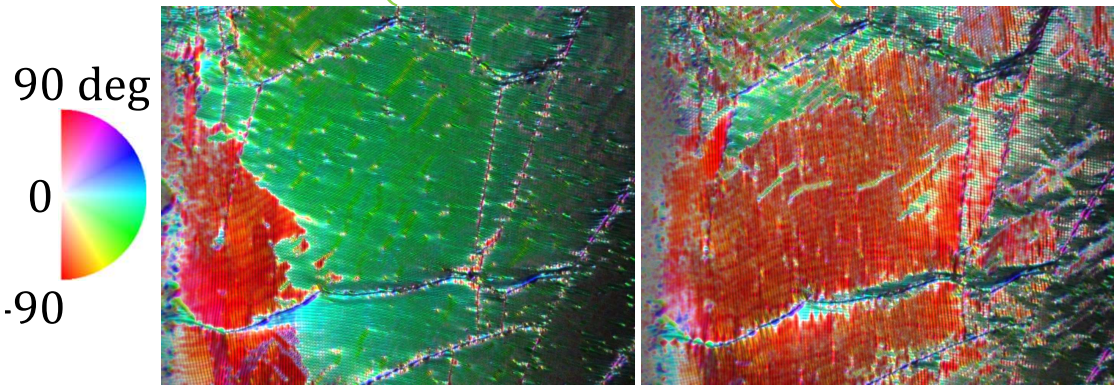
Ferrowrinkle, wrinkles as ferroelastics

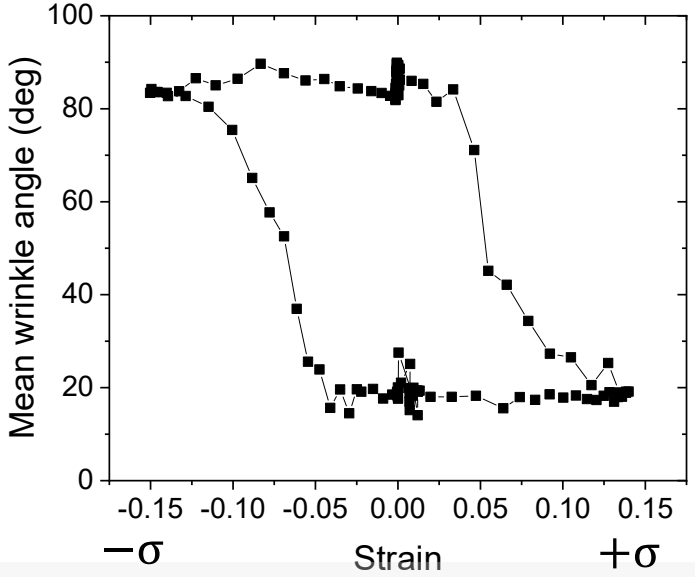
Scheme of compressin device



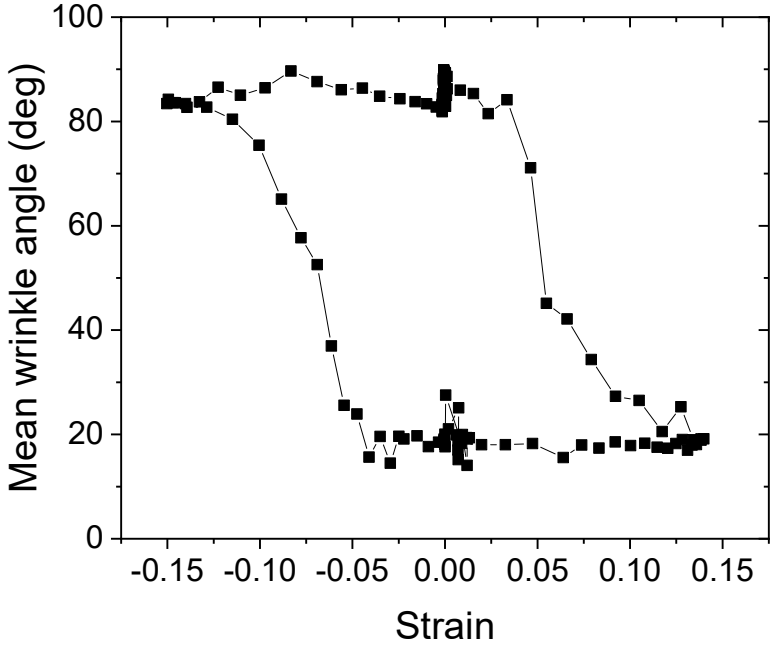


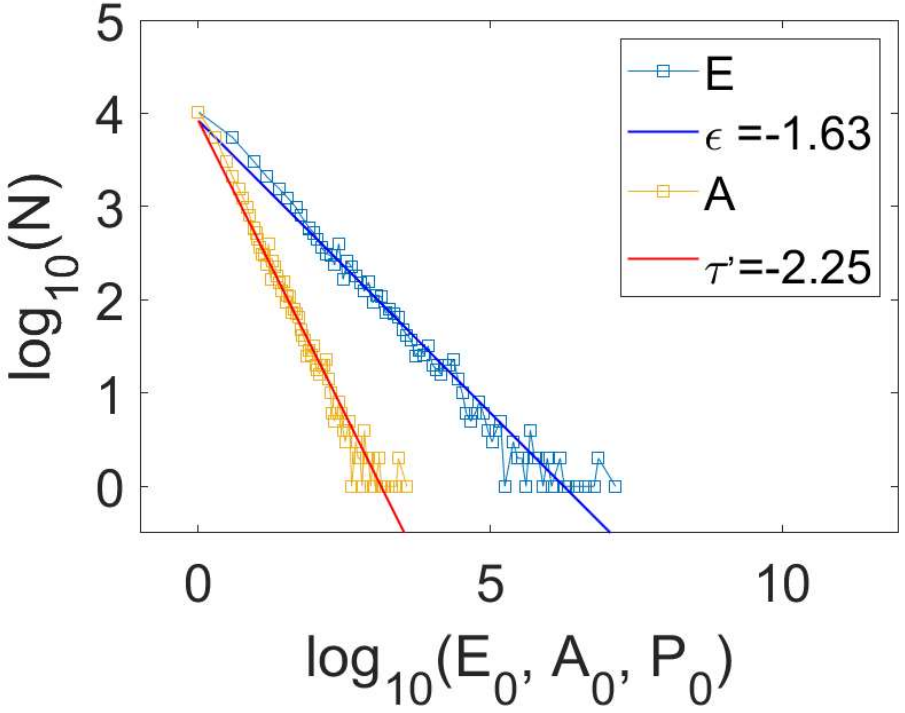
$\sigma_{x\ (+}{}^{0)}$ $\sigma_{y\ (-}{}^{0)}$





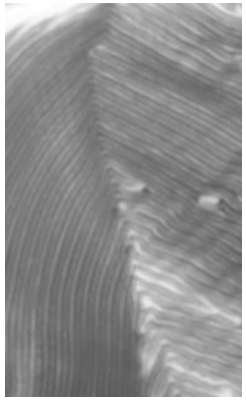
Wrinkles as ferroelastics



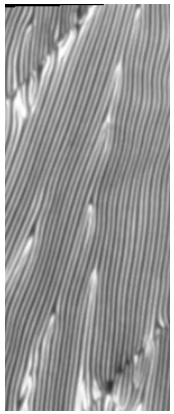


Wrinkle defects zoology

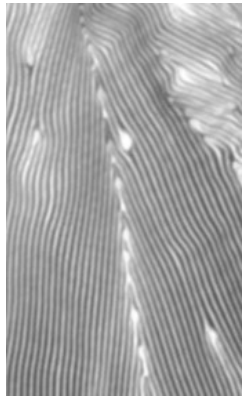
Folds



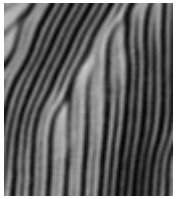
Grain boundary



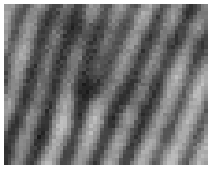
Grain boundary or Fold + dislocation



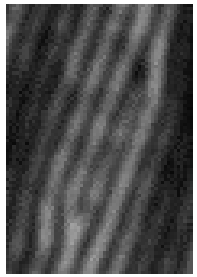
Disclination



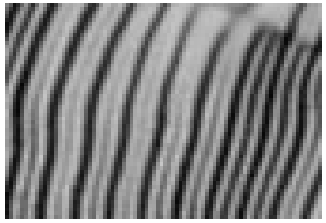
Dislocacions



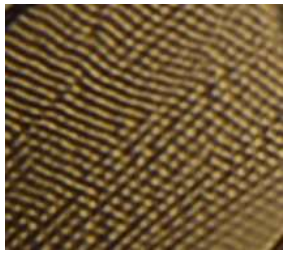
Stacking fault



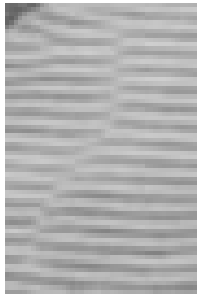
Period doubling

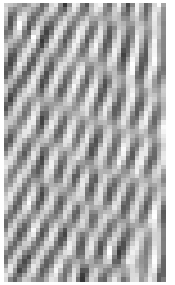


Tweed



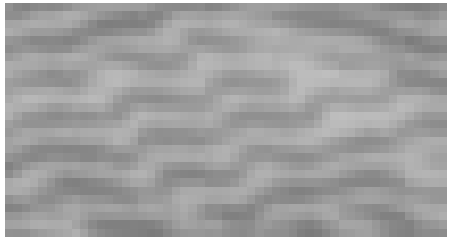
Antiphase Boundary



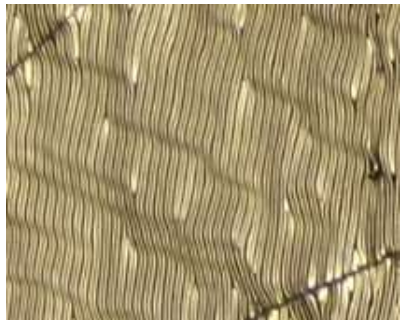


Miura-ori pattern

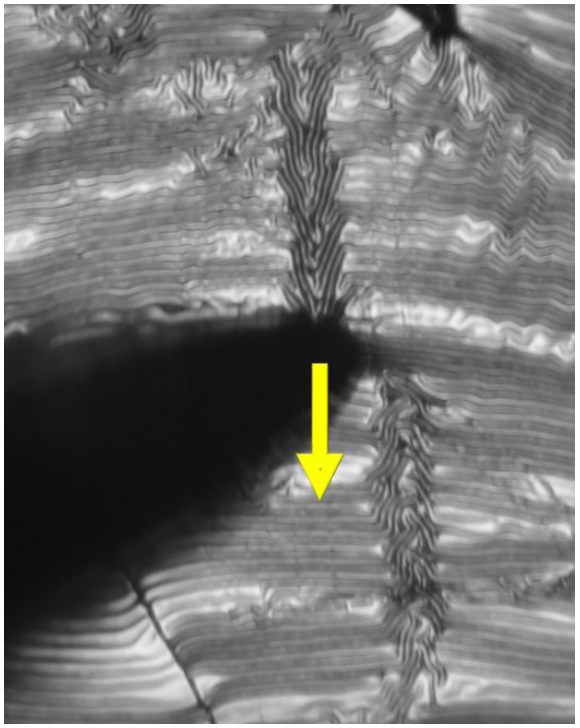
Herringbone pattern



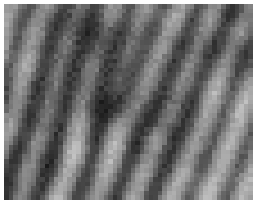
Shear bands, leading dislocation

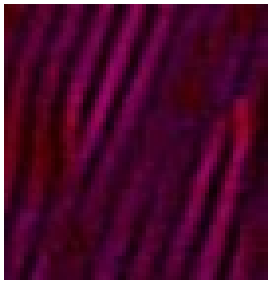


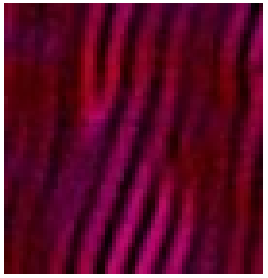
All animals in the same field of view

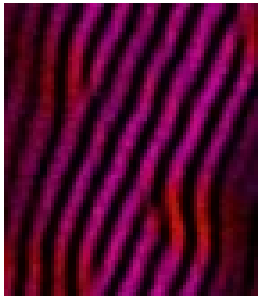


Dislocacions









Outline

Domain motion on:

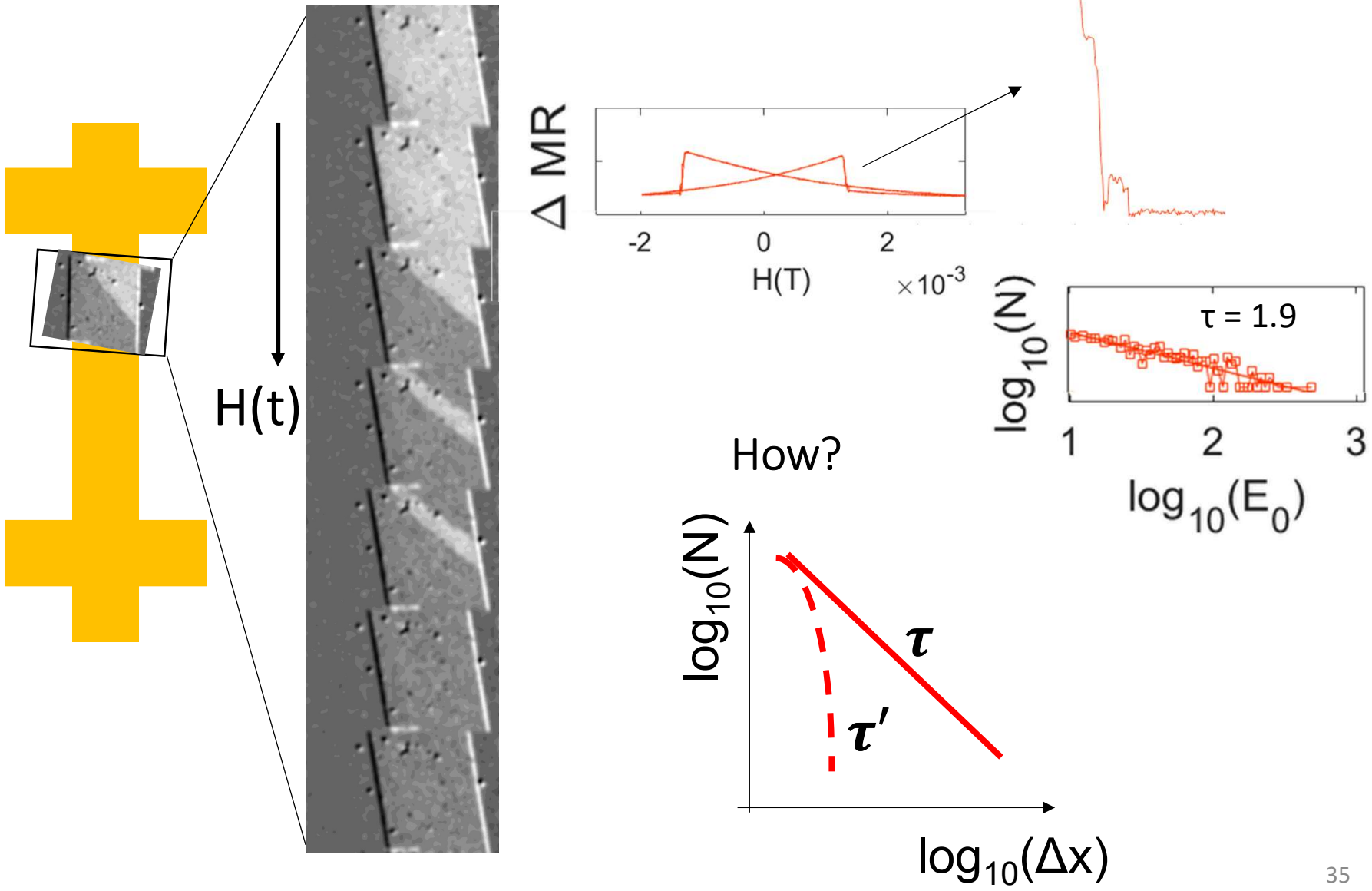
Ferroelectrics

Ferroelastics

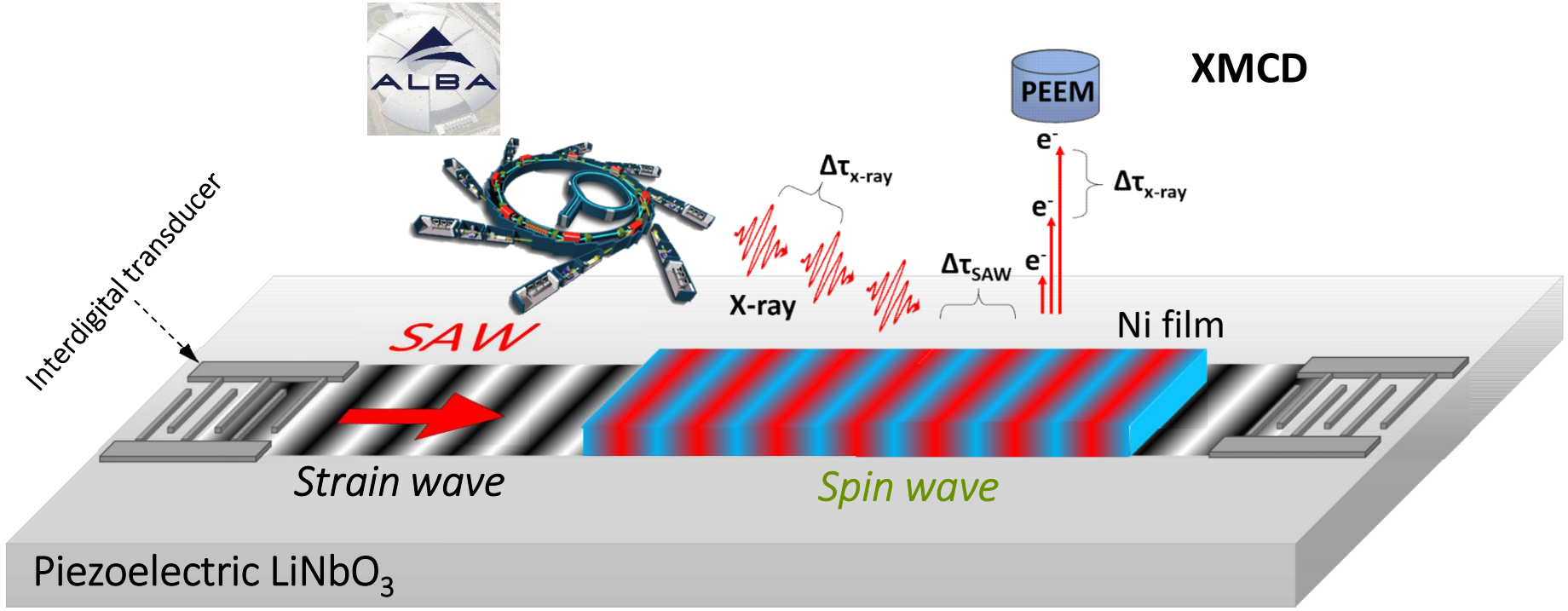
Ferrowrinkles

Ferromagnetics

Ni thin film


H(t)
Δ MR
-2
0
2
H(T)
×10^-3
τ = 1.9
log10(N)
1
2
3
log10(E0)
How?
log10(N)
τ
τ′
log10(Δx)

Magnetic waves, magnetoelastic coupling



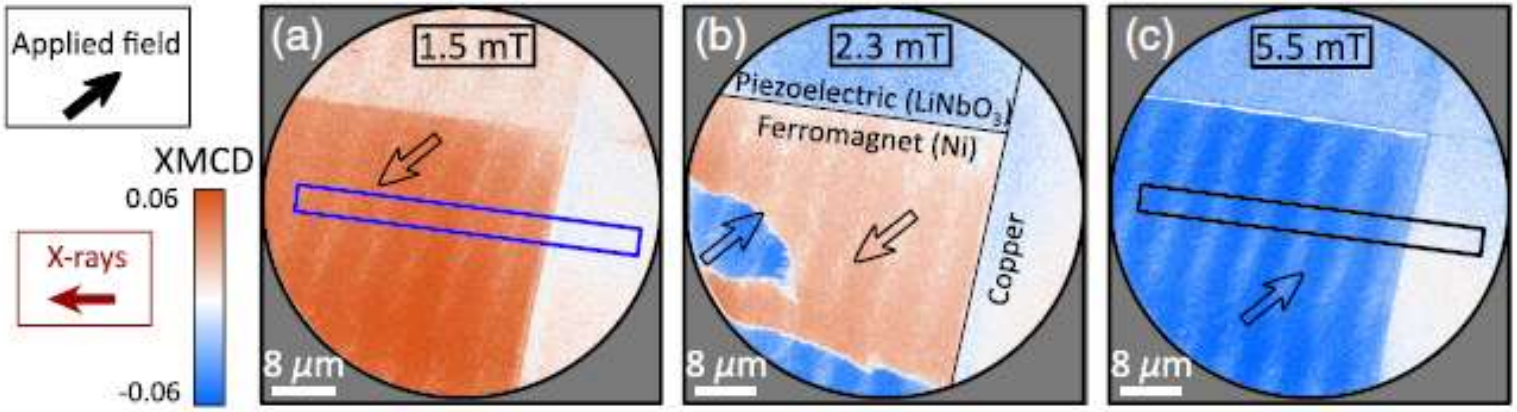
PHYSICAL REVIEW LETTERS **124,** 137202 (2020)

Editors' Suggestion | Featured in Physics

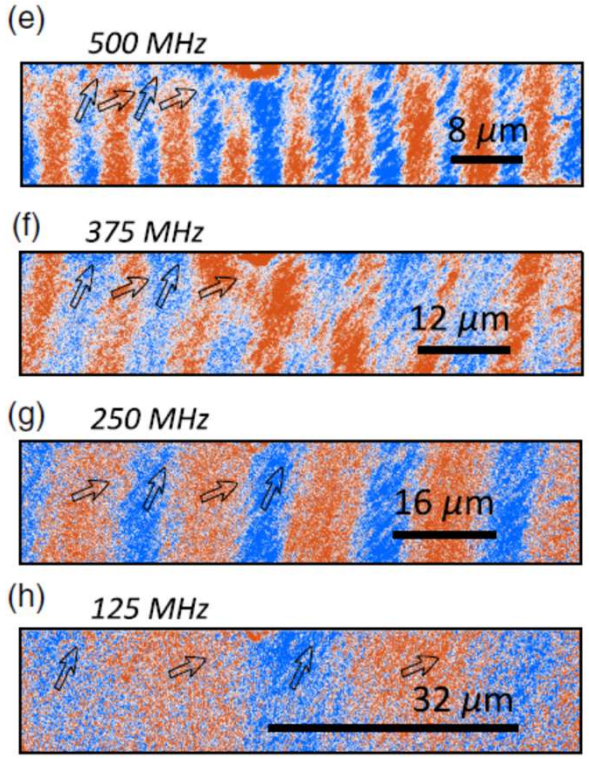
Generation and Imaging of Magnetoacoustic Waves over Millimeter Distances

Blai Casals,[1,‡] Nahuel Statuto,[2,§] Michael Foerster,[3] Alberto Hernández-Mínguez,[4] Rafael Cichelero,[1,†] Peter Manshausen,[1,‖] Ania Mandziak,[3,5] Lucía Aballe,[3] Joan Manel Hernàndez,[2,6] and Ferran Macià[1,2,6,*]

Magnetoacustic waves

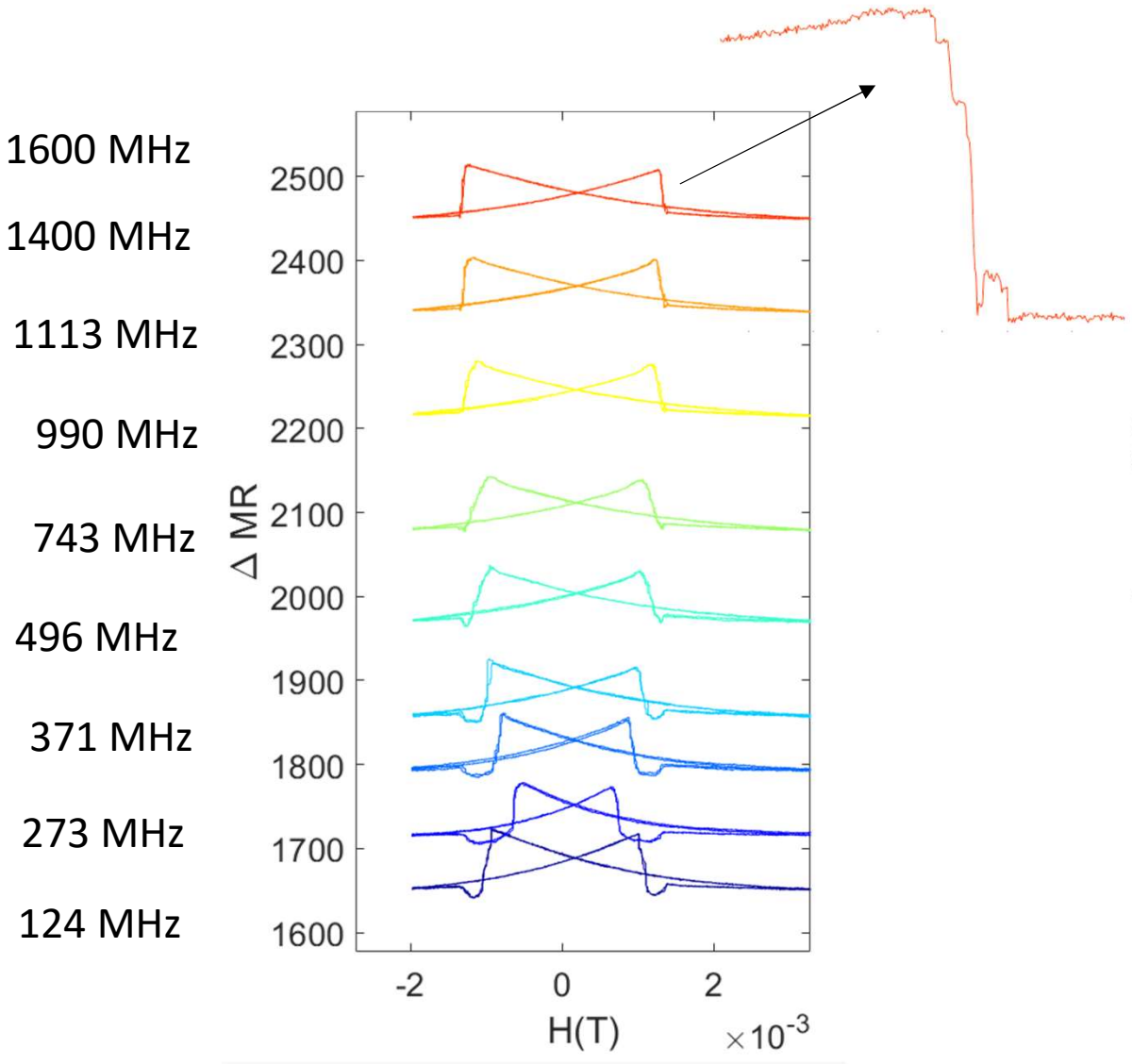


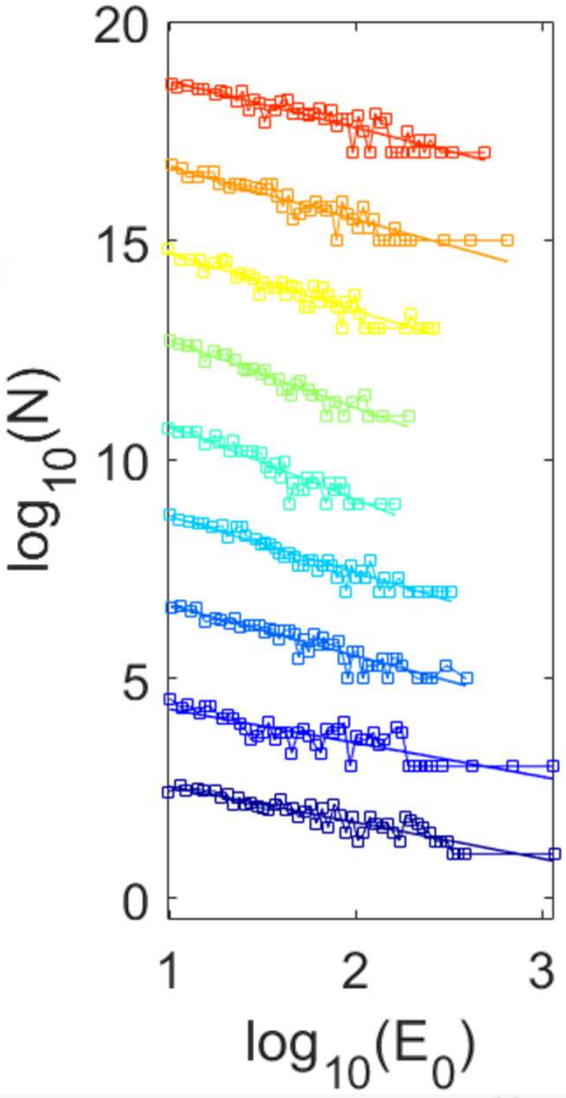
XMCD



B. Casals et al, PRL 124 (2020)

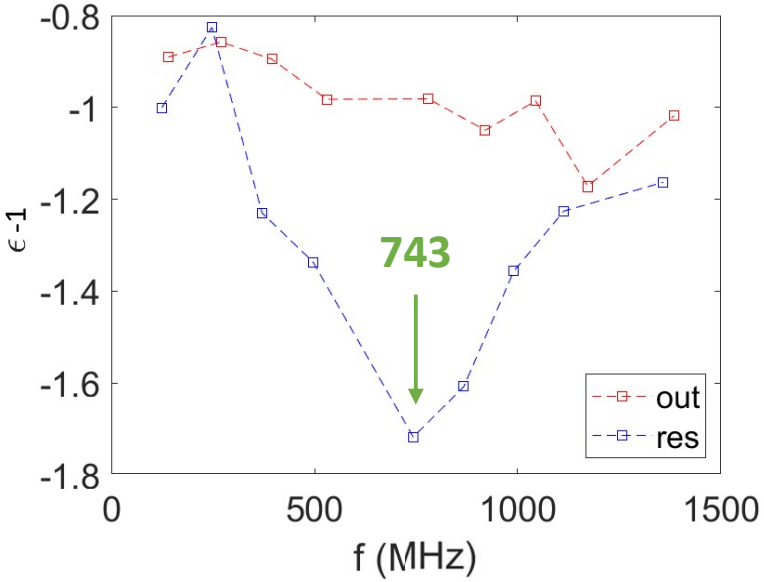
Magnetoresistance under SAW

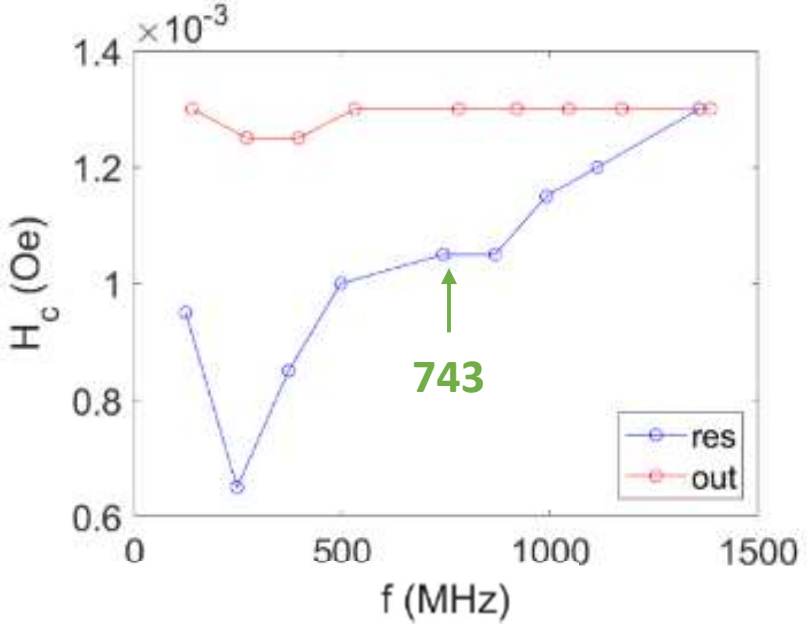


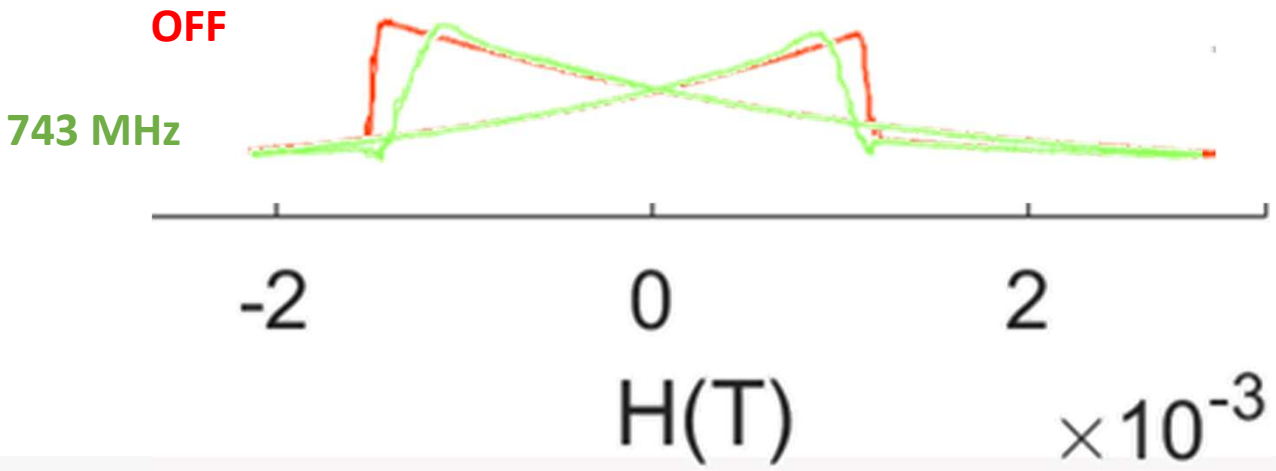


Change of dynamics, wave-wall interaction

Exponent from magnetoresistance measurements

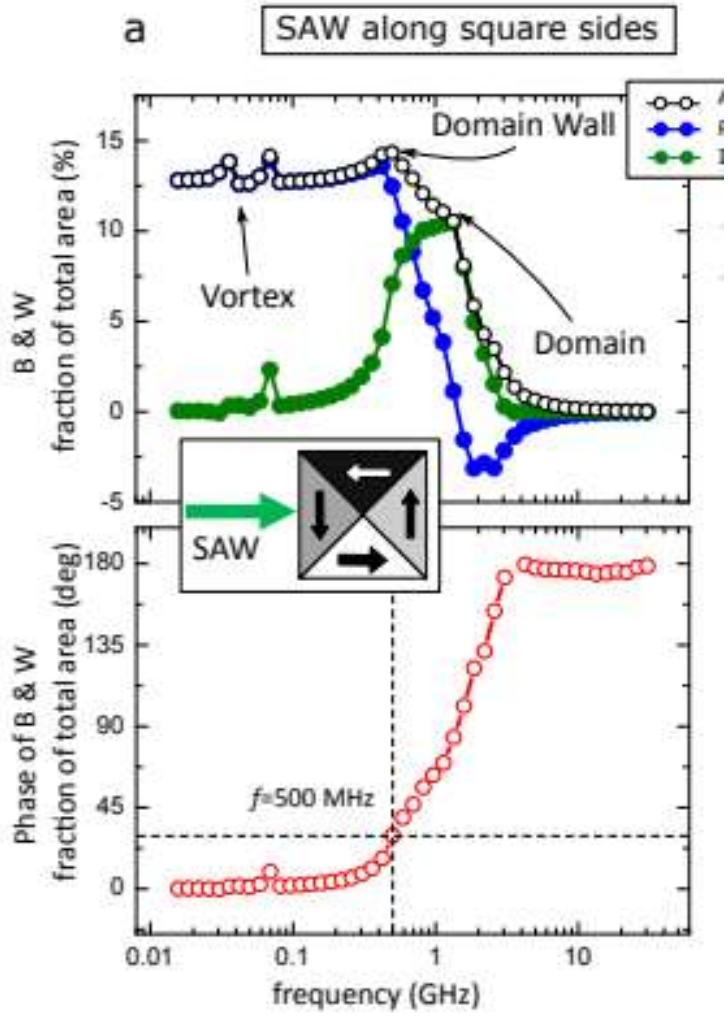






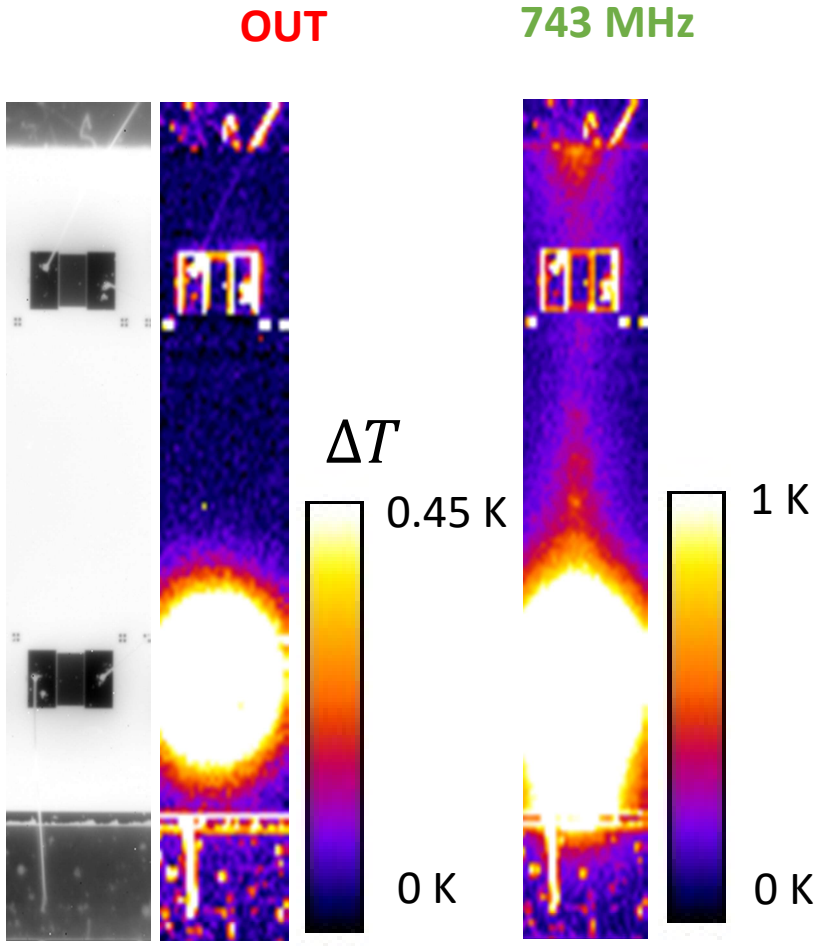
Domain wall ressonance

Micromagnetic simulations

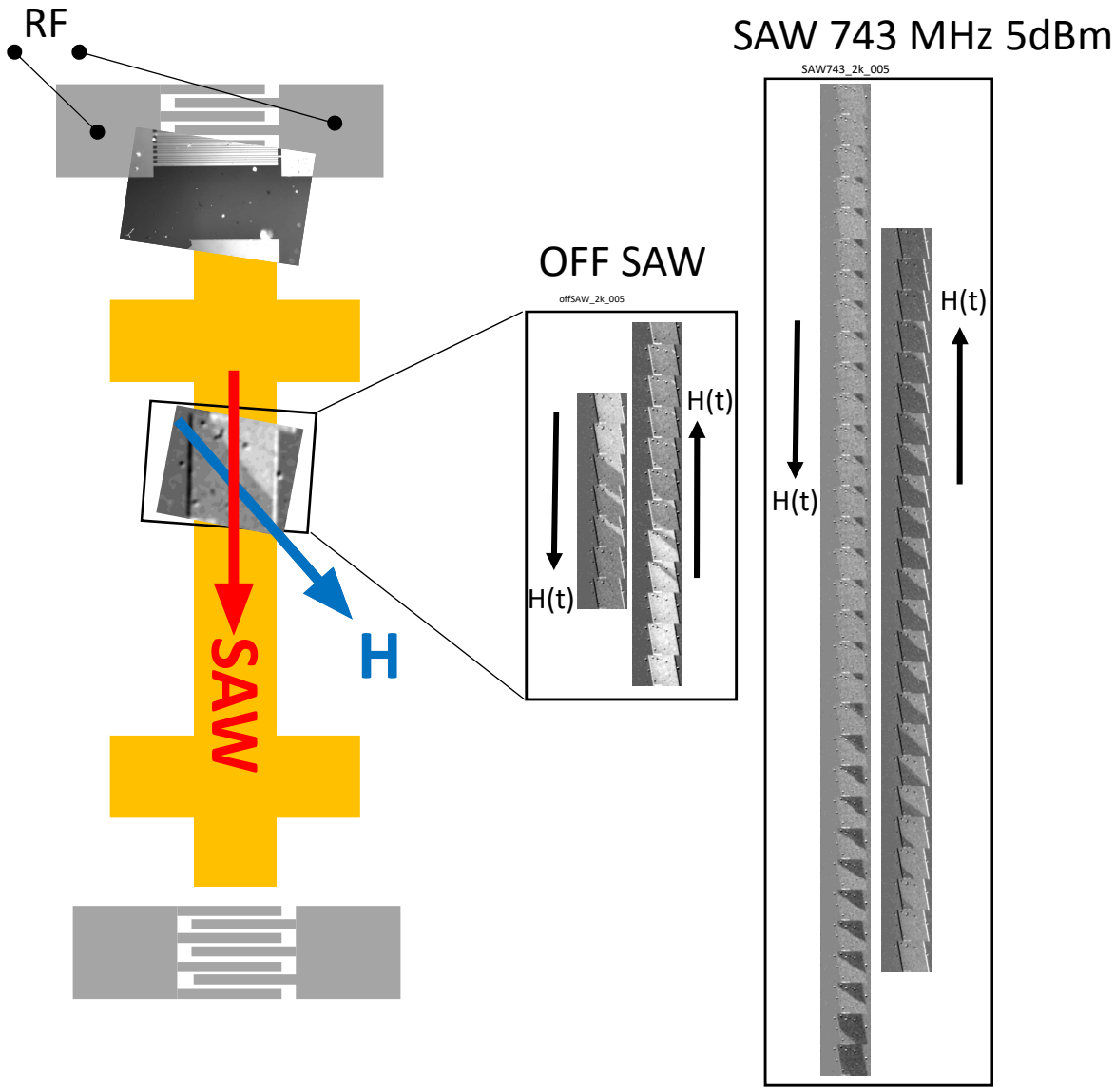


Temperature changes?

Eduard Vives, Michela Romanini

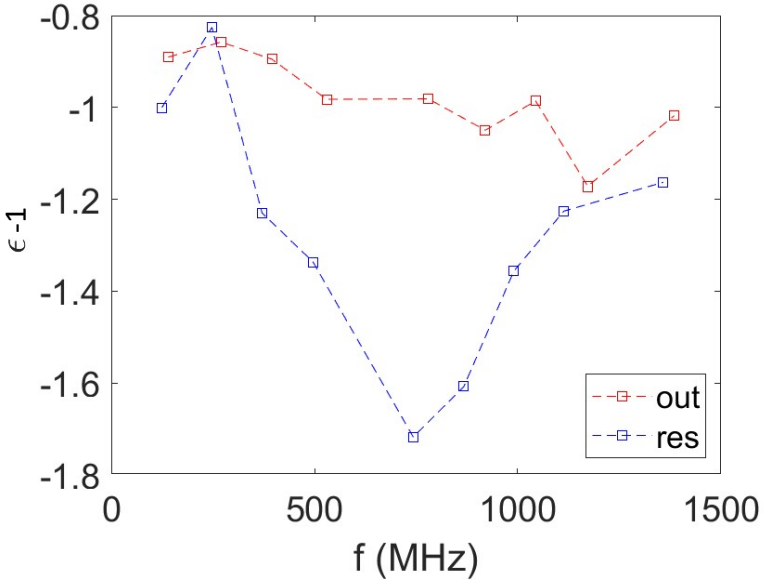


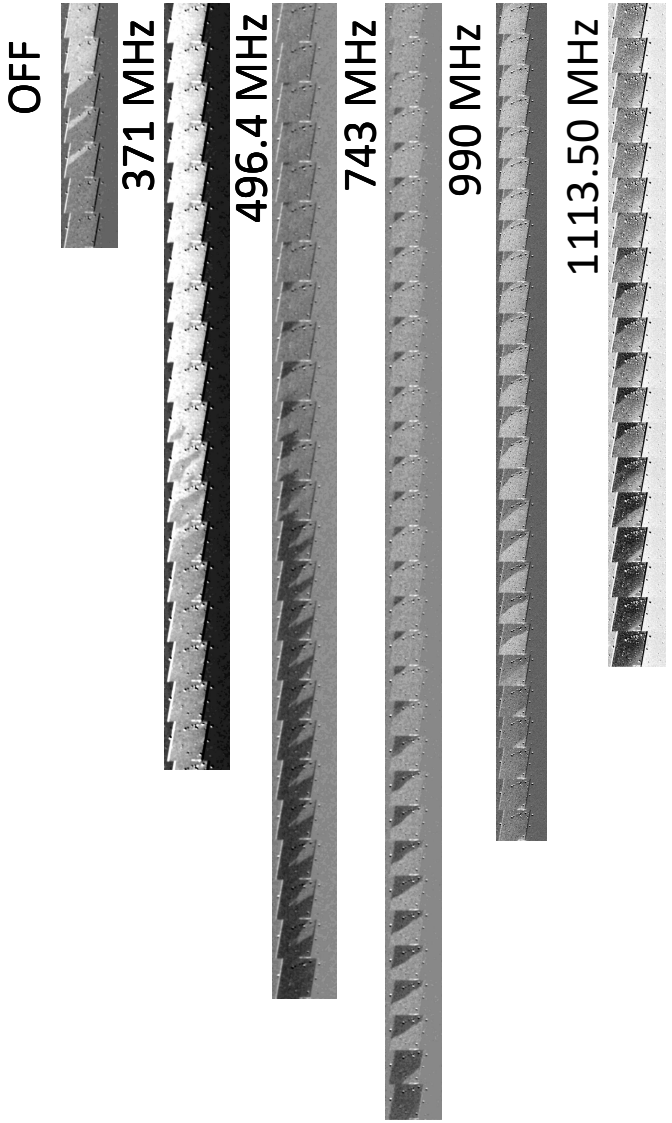
It can be observed with MOKE



Change of dynamics, wave-wall interaction

Exponent from magnetoresistance measurements





Köszönöm a figyelmet!

Thanks to:
Ekhard Salje,
Guillaume Nataf,
David Pesquera,
Gustau Catalan,
Jordi Baró

Funding: EPSRC

Juan de la Cierva Incorporación 2021

Blai Casals

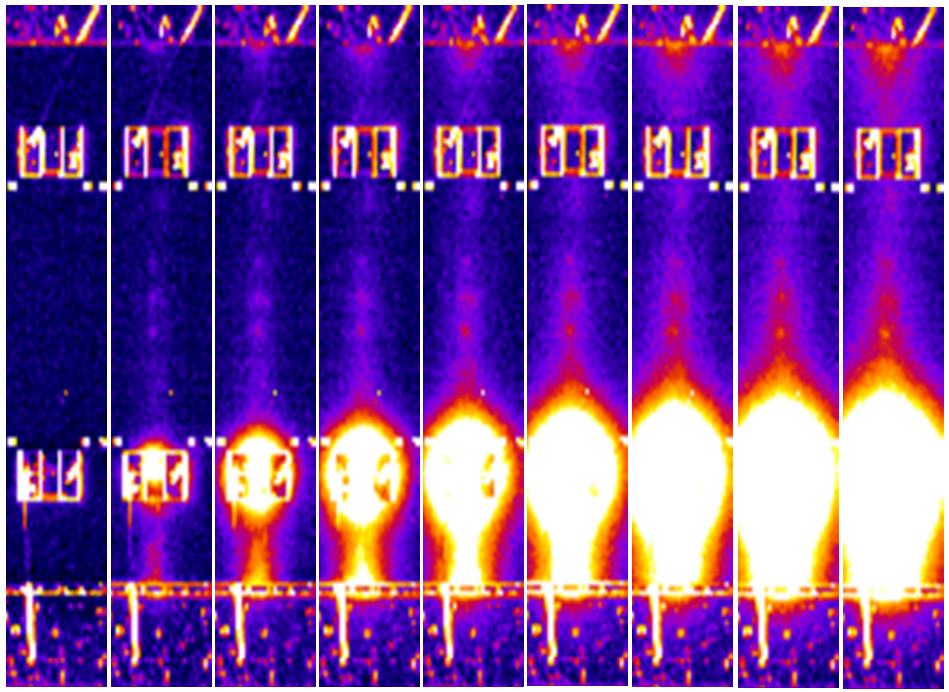
Avalanche 2022 Debrecen

1 september 2022

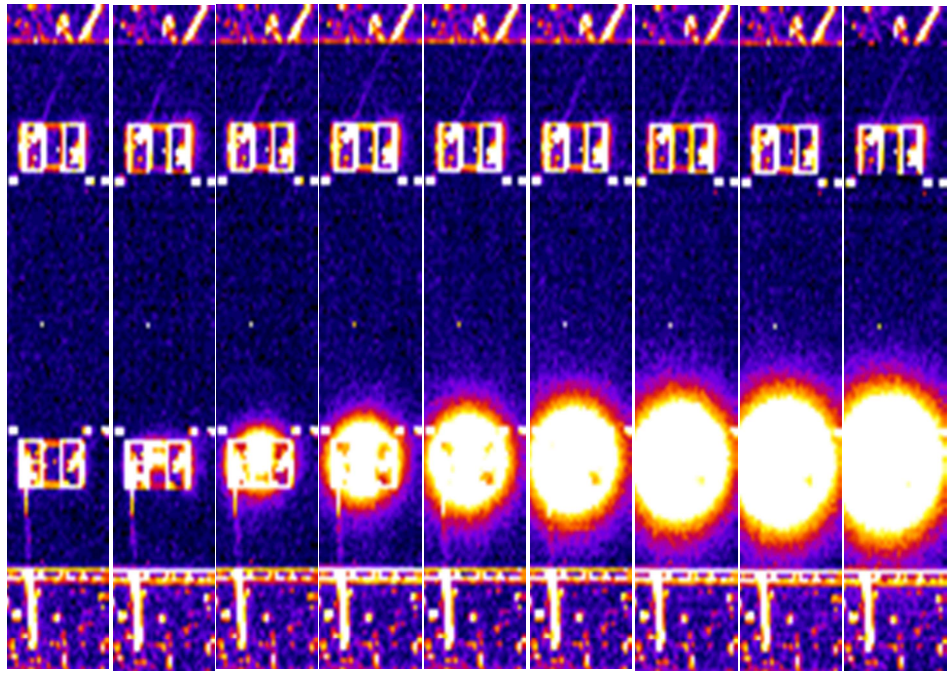
@blaicasals

SAW

Sequence 10 **(498 MHz)**



Sequence 11 **(450 MHz)**

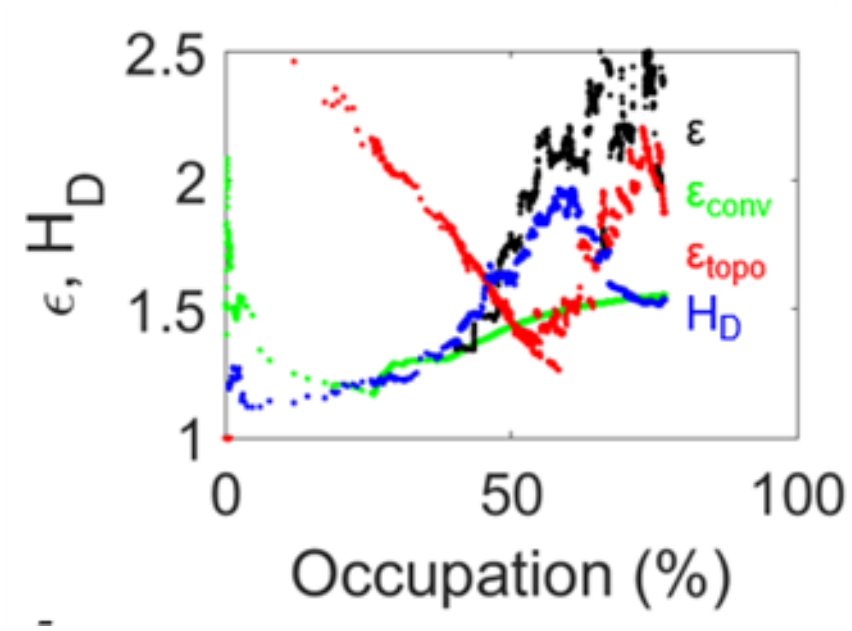


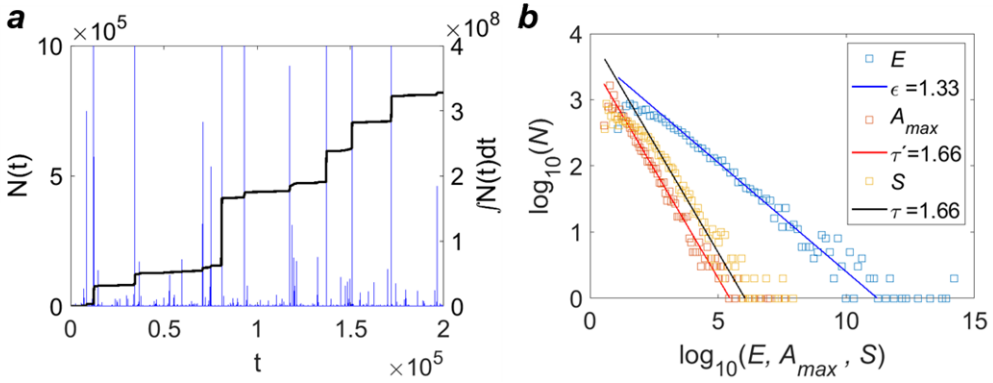
Eduard Vives,
Michela Romanini
UB

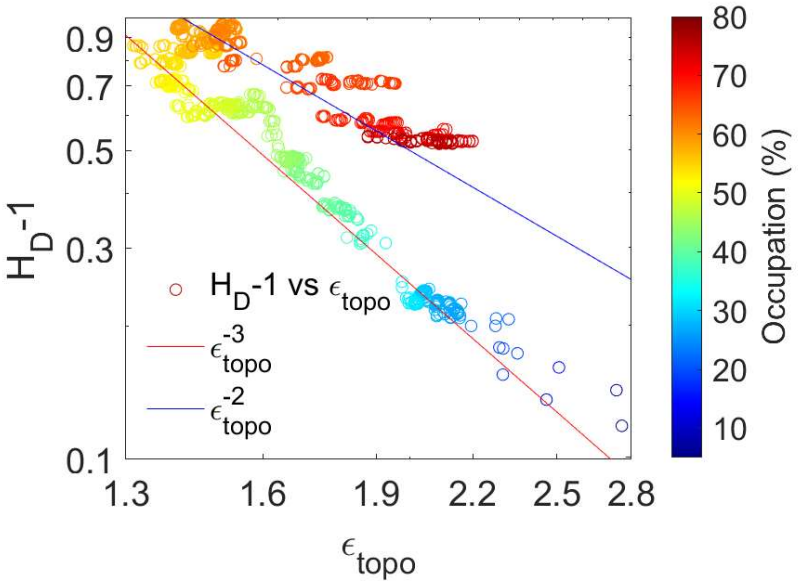
Correlation betwen criticallity and fractality

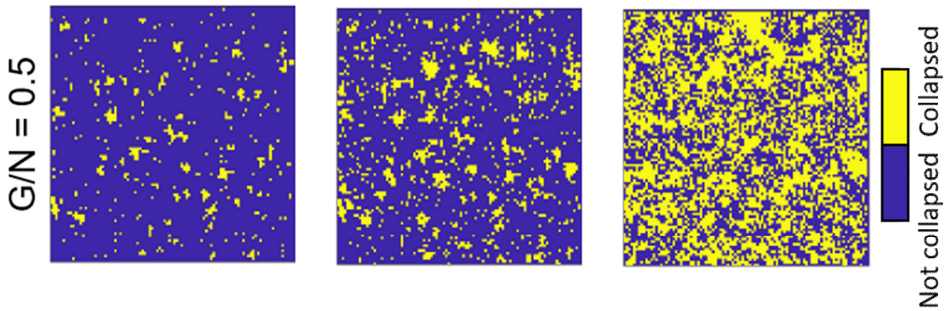
Statistical model (collapse model)

$$N_a(t) = f(t)N_s(t-1) + g(t)$$





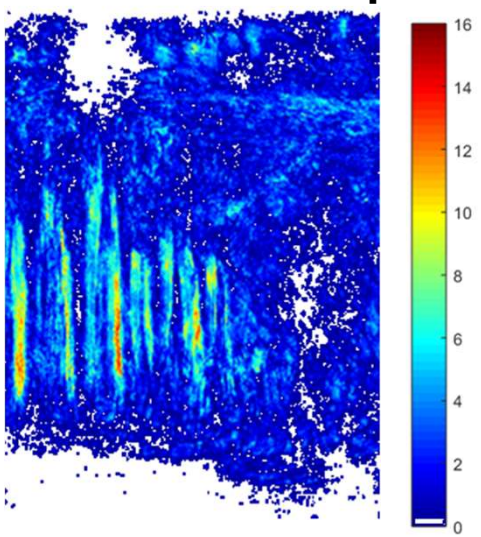




B. Casals, E. K.H. Salje, PRE 2021

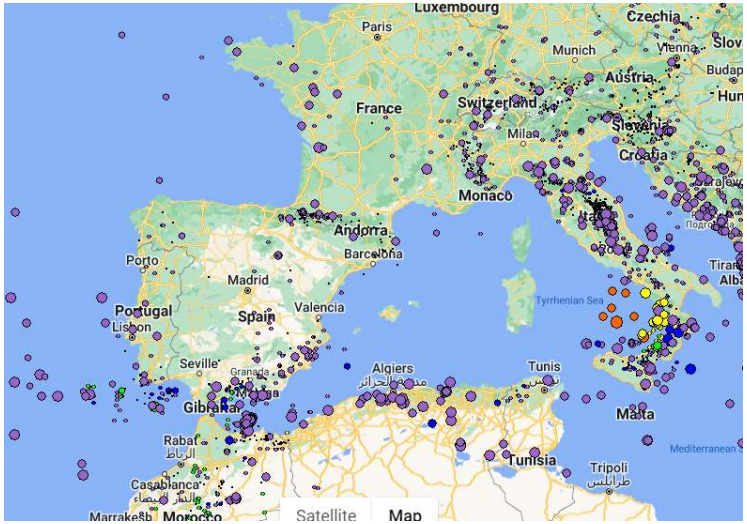
Ferroelectrics, ferroquakes

Avalanches map

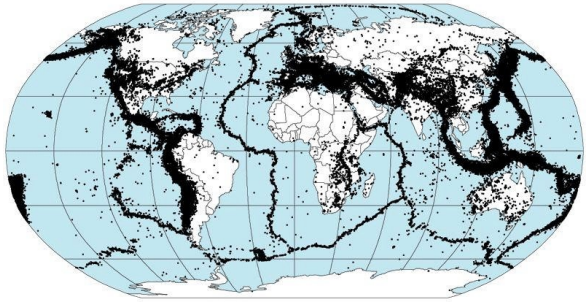


10^5 events on a cycle

Earthquakes since 1970 in mediterranean united nations



10^5 events



Ferroelectrics, neuromorphics

Neuromorphic computing

Ferroelectric materials for neuromorphic computing

Cite as: APL Mater. 7, 091109 (2019); doi: 10.1063/1.5108562
Submitted: 30 April 2019 • Accepted: 5 September 2019 •
Published Online: 19 September 2019

S. Oh, H. Hwang, and I. K. Yoo[a)]

Neuronal Avalanches Differ from Wakefulness to Deep Sleep – Evidence from Intracranial Depth Recordings in Humans

Viola Priesemann, Mario Valderrama, Michael Wibral, Michel Le Van Quyen

Published: March 21, 2013 • https://doi.org/10.1371/journal.pcbi.1002985

nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-020-16548-3 OPEN

Check for updates

Control of criticality and computation in spiking neuromorphic networks with plasticity

Benjamin Cramer[1✉], David Stöckel[1], Markus Kreft[1], Michael Wibral[2], Johannes Schemmel[1], Karlheinz Meier[1] & Viola Priesemann[3,4,5✉]

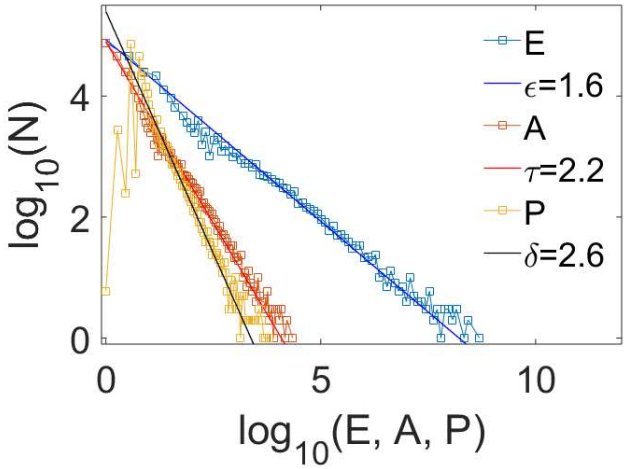
Relation between criticality, task-performance

From unit cell to the sky

Same ferroelectric sample

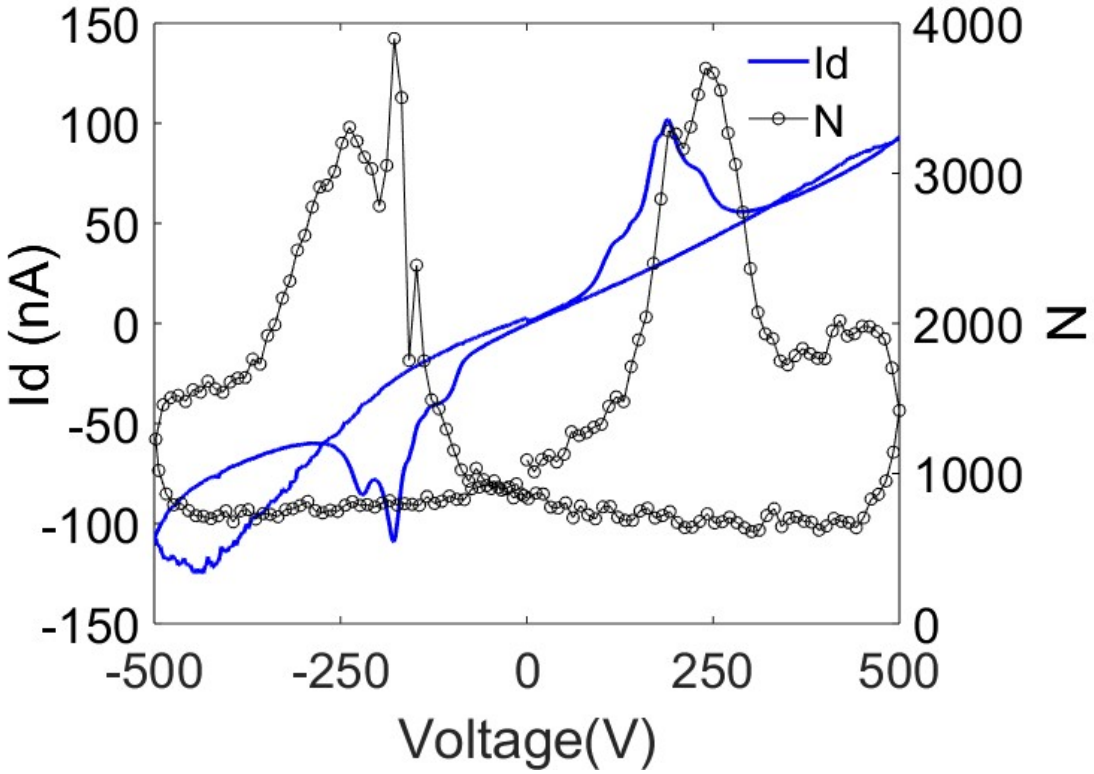
TEM nm	PFM nm - µm	Optics µm - mm
Shelly Conroy data (?)	Thin film PZT (1.98–2.87)	Bulk BTO PMNPT (2.2)

P. Tückmantel et al. PRL 126, 117601 (2021)



B. Casals et al. , APL Mater. 8, 011105 (2020)

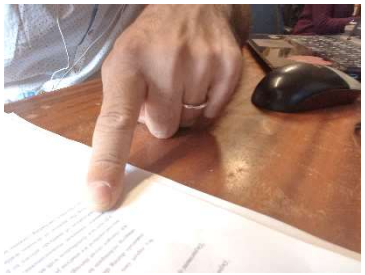
B. Casals et al. , Nat. Commun. (2021)


150
100
50
0
-50
-100
-150
Id (nA)
4000
3000
2000
1000
0
N
Id
N
-500
-250
0
250
500
Voltage(V)

Same material, different measurements

"Listen"

"Touch"



"Watch"

Acoustic Emission

Strain changes

Displacement current

Polarization changes

Imaging patern changes

Polarization and Strain

References:

Phase transition	$BaTiO_3$,... ε=1.35	$BaTiO_3$,PMNPT, ... ε=1.3 ε=1.5	STO, LAO (Ferroelastic) ε=1.4 – 1.6
Ferroelectric Switching	**$BaTiO_3$,... ε=1.65**	**PZT ε=1.61**	

Same sample for all measurements, same energy exponent?

$$PDF(E) \sim E^{-\varepsilon}$$

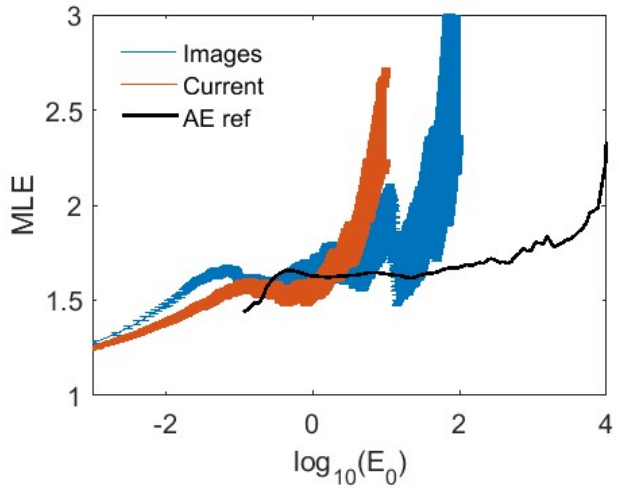
Same dynamics

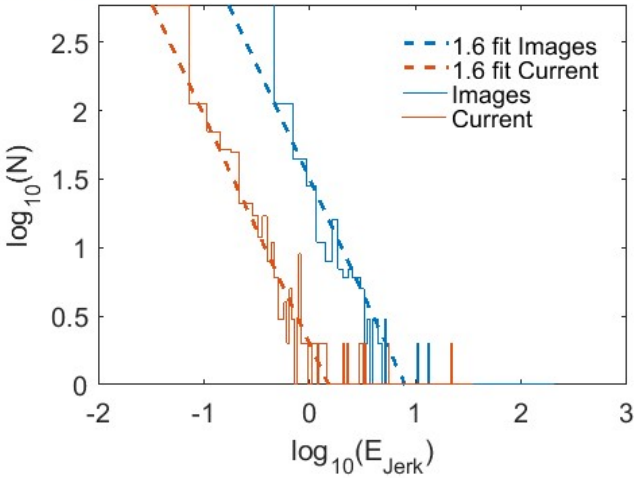
APL Materials ARTICLE scitation.org/journal/apm

Avalanches from charged domain wall motion in $BaTiO_3$ during ferroelectric switching

Cite as: APL Mater. 8, 011105 (2020); doi: 10.1063/1.5128892
Submitted: 23 September 2019 • Accepted: 17 December 2019 •
Published Online: 10 January 2020

Blai Casals,[1] Guillaume F. Nataf,[2] David Pesquera,[1] and Ekhard K. H. Salje[1]





$$PDF(E) \sim E^{-\varepsilon}$$

"Listen"

Acoustic Emission

Strain changes

ε=1.6

"Touch"

Displacement current

Polarization changes

ε=1.6

"Watch"

Imaging patern changes

Polarization and Strain

ε=1.6

$BaTiO_3$ under electric field

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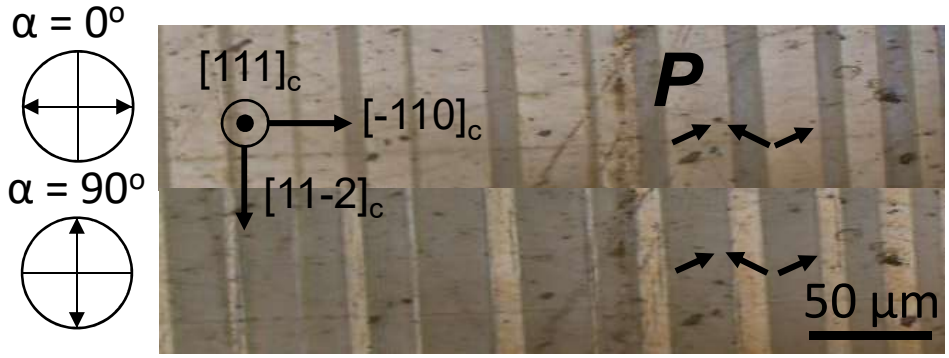
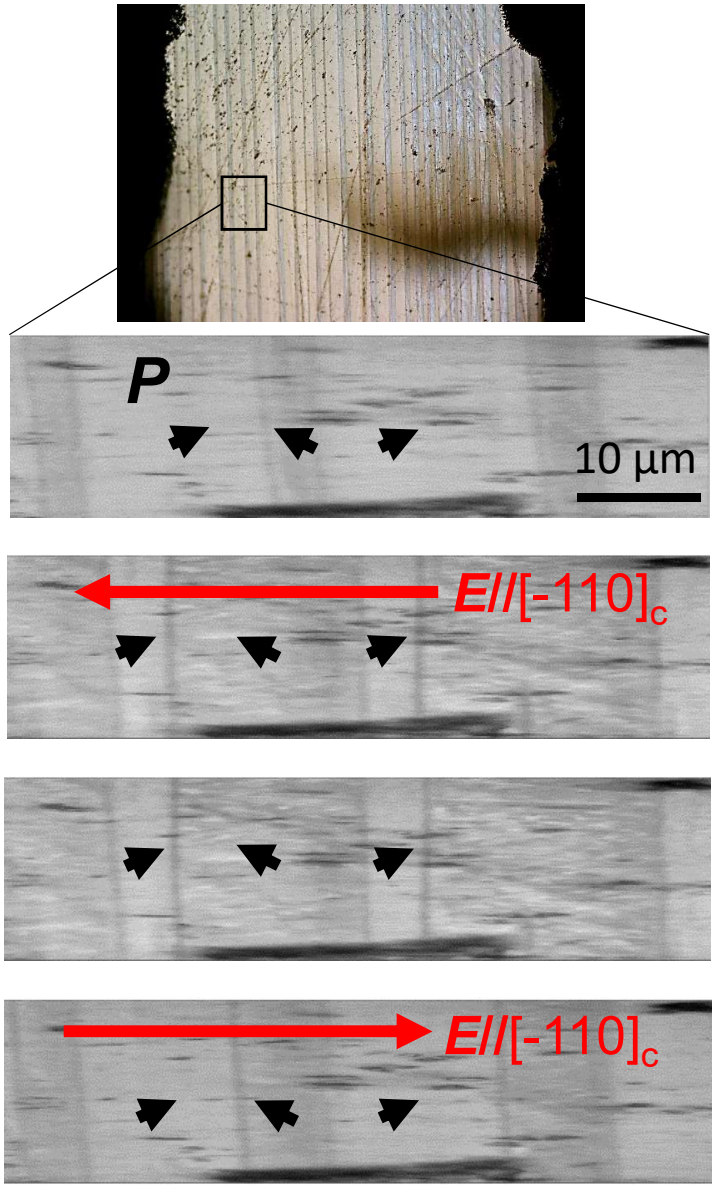
a **$BaTiO_3$ (111)**



b



PMN-PT under electric field

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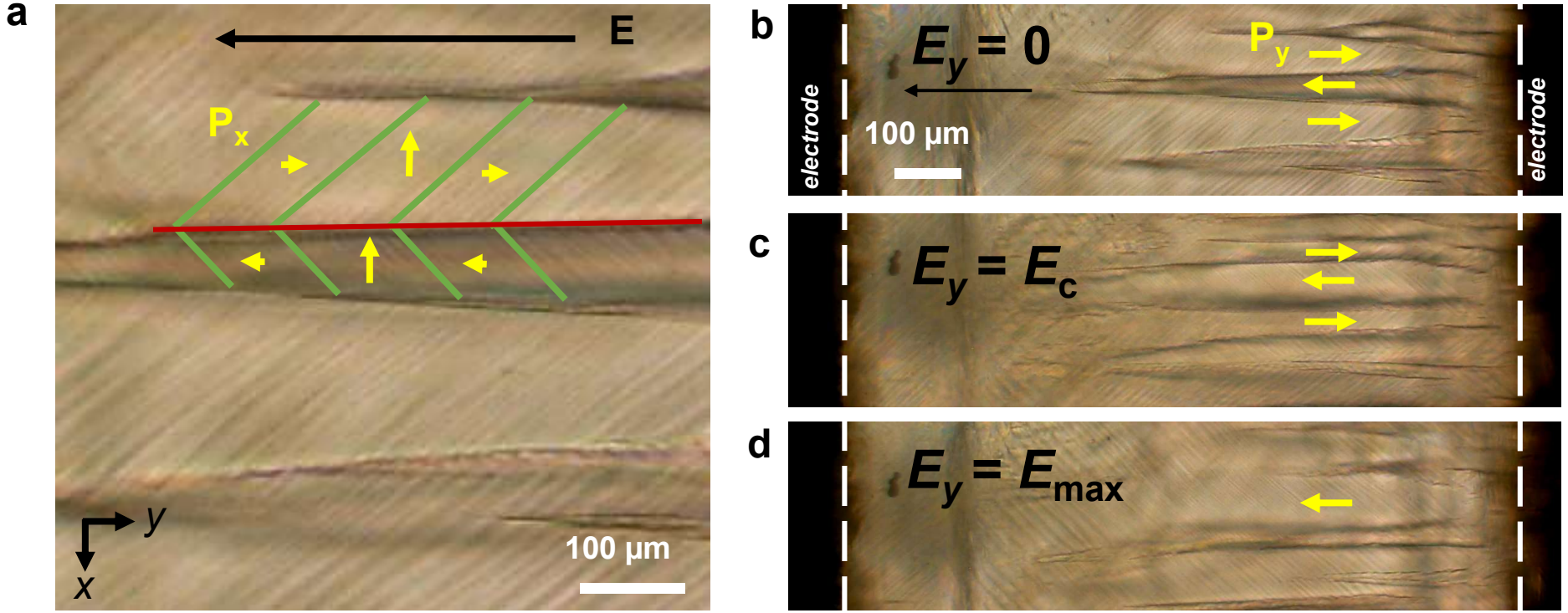

Single domain analysis, $BaTiO_3$

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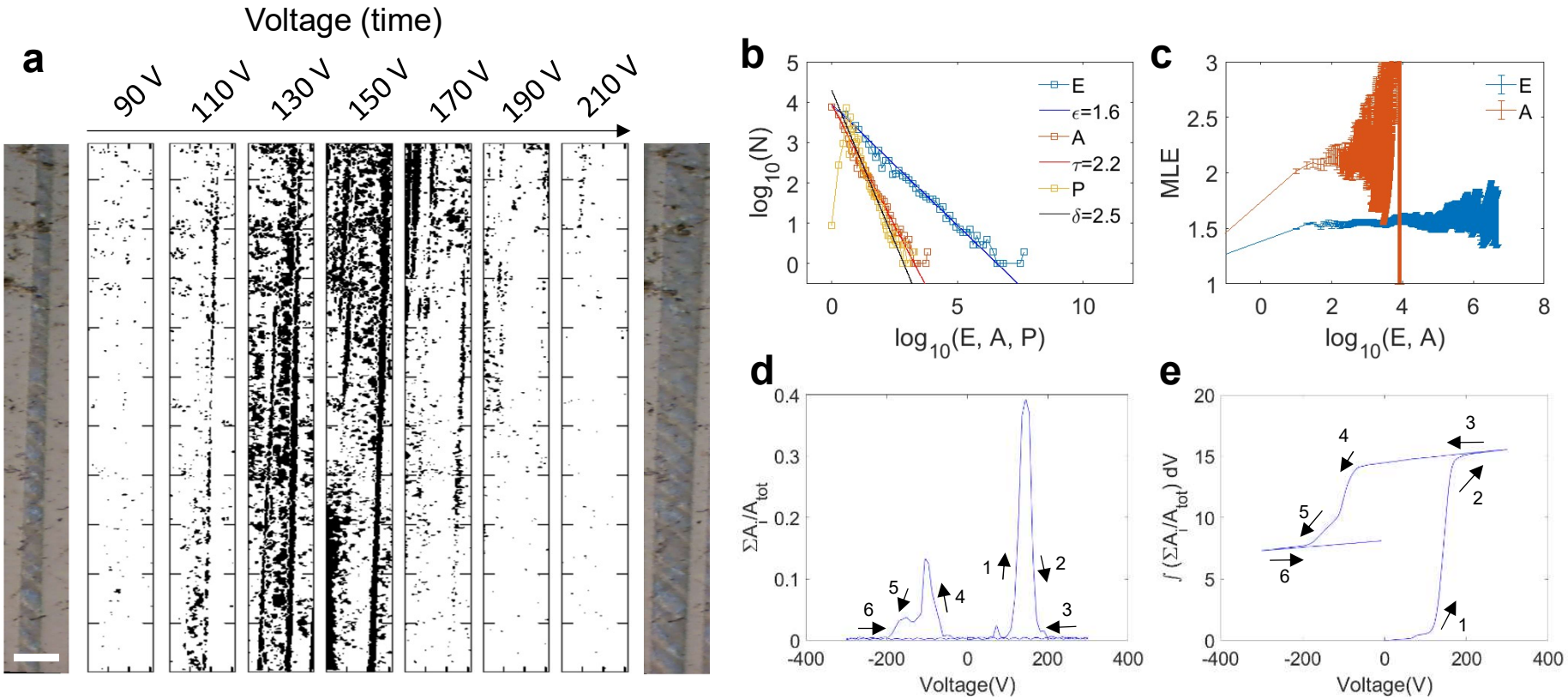
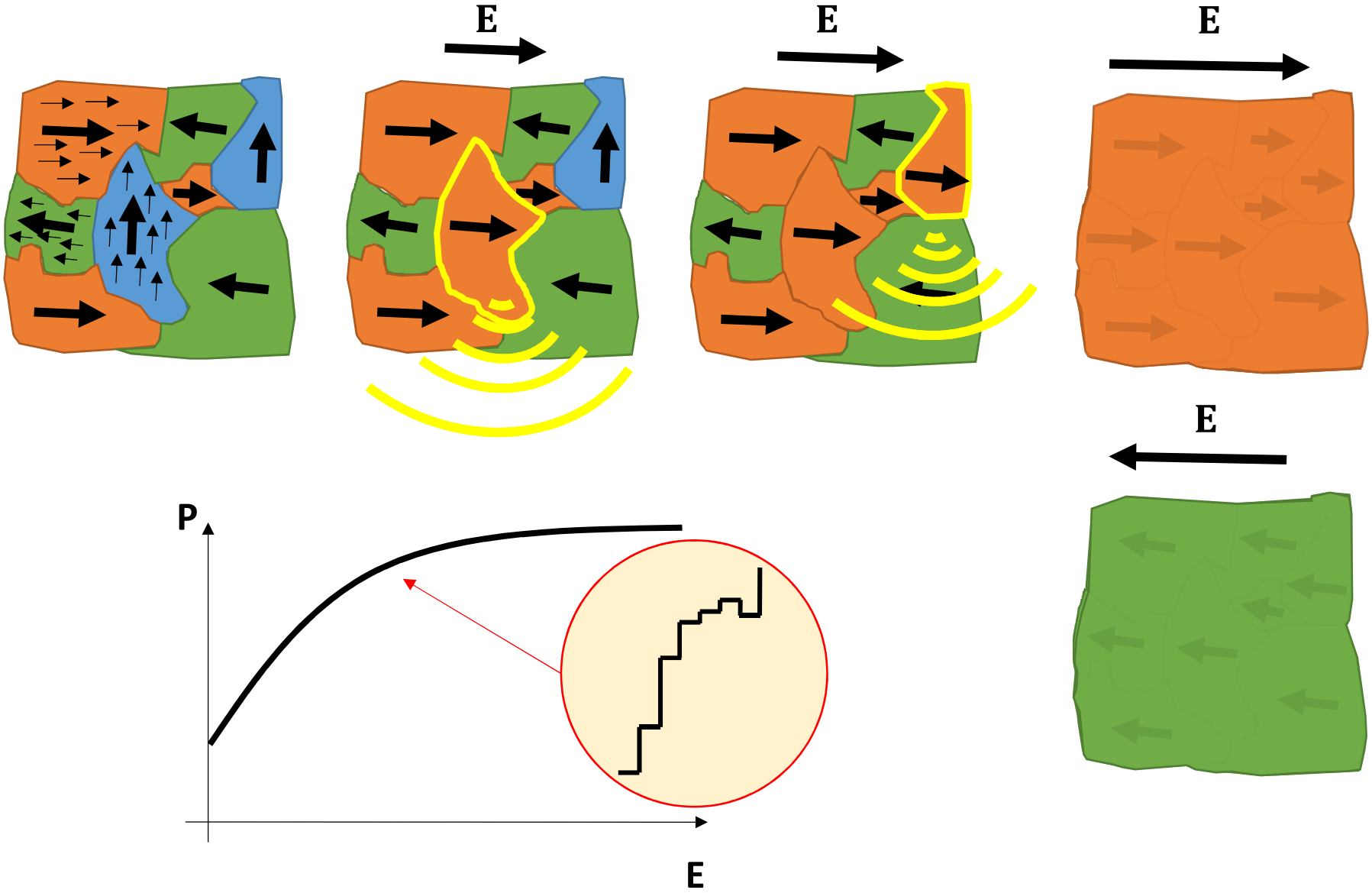


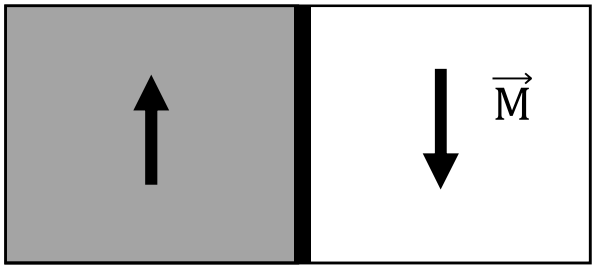
Ferroics and Barkhausen

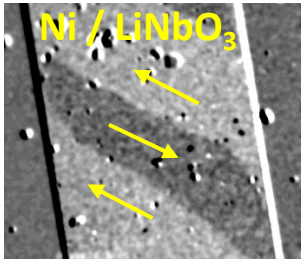


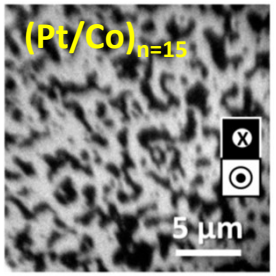
Domain wall (DW) by optics

DW

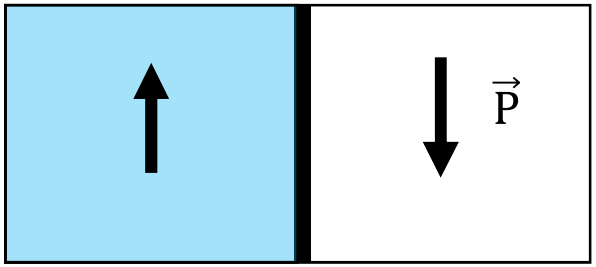
Ferromagnetic

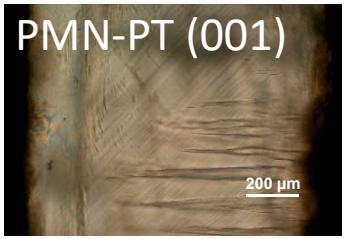




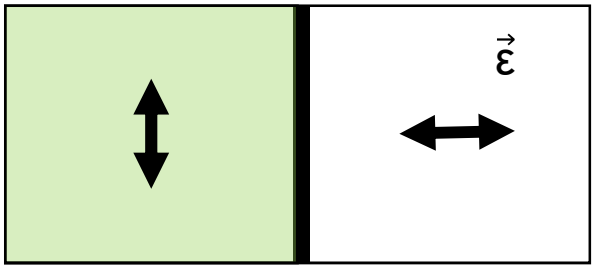


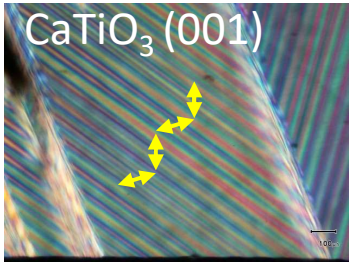
Ferroelectric

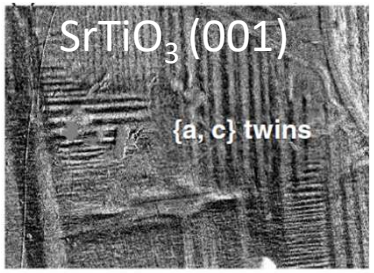




Ferroelastic







...

Summary and conclusions

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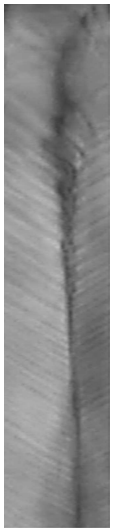
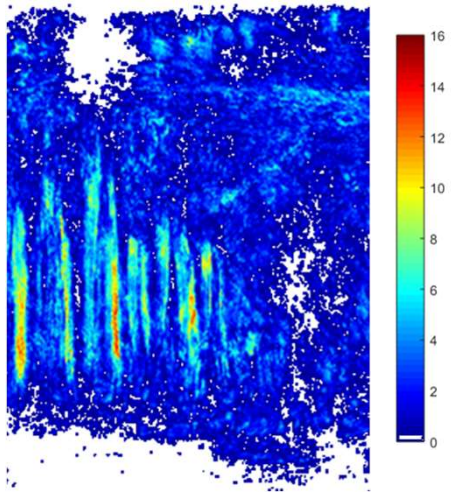
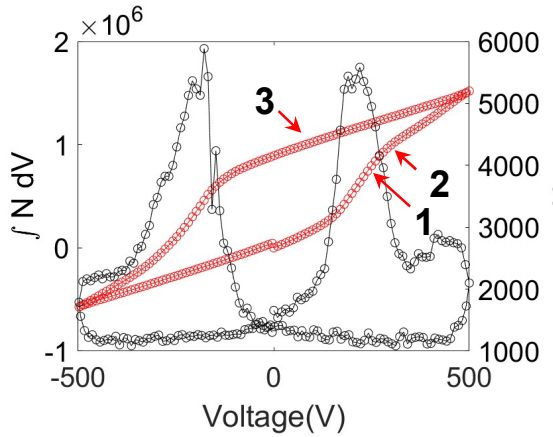
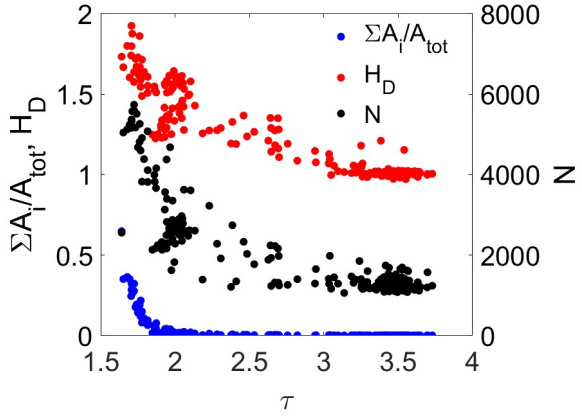


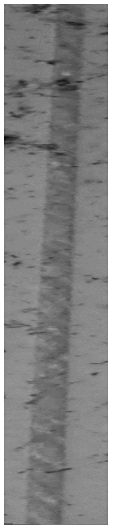
PMN-PT

Avalanches accumulation map

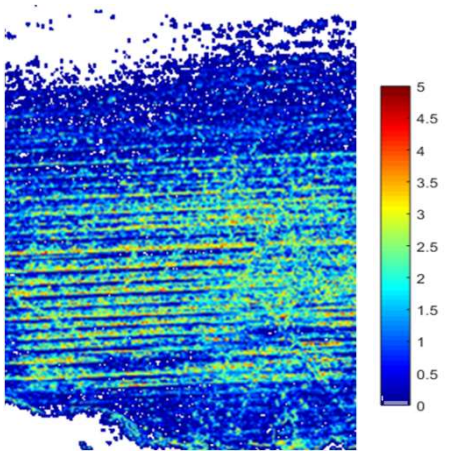


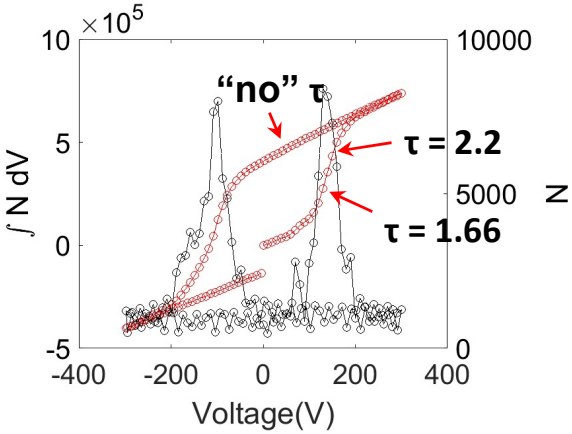


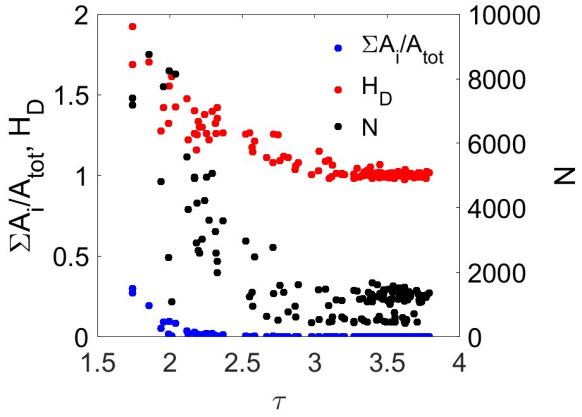




$BaTiO_3$







Anticorrelation between τ and H_D

τ = 1.66 (unrelaxed mean-field) at the Coercive field,

τ = 2.2 (integrated mean-field) before and after Ec.

Avalanches during FE switching, $BaTiO_3$

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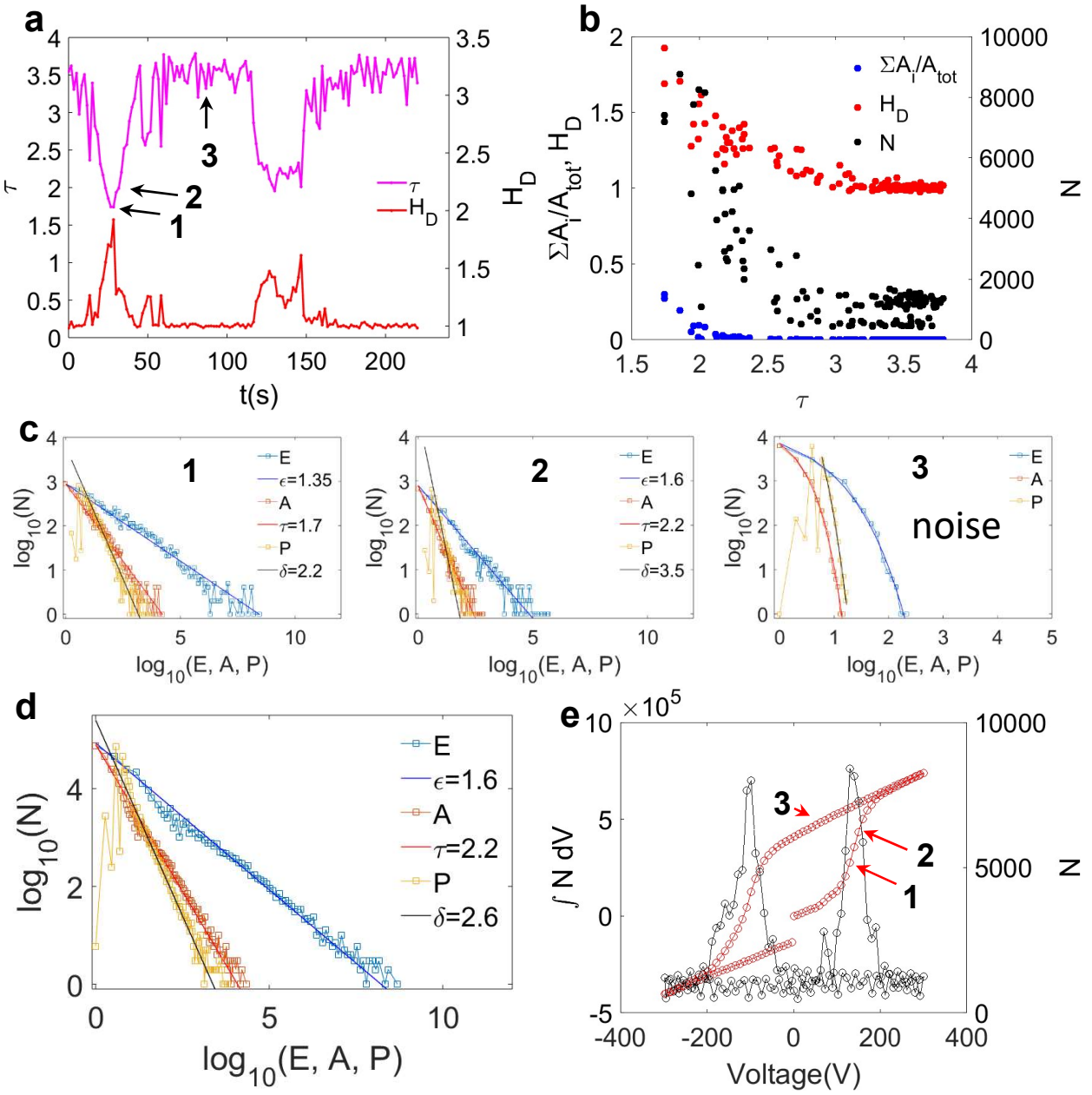

Hysteresis from avalanches

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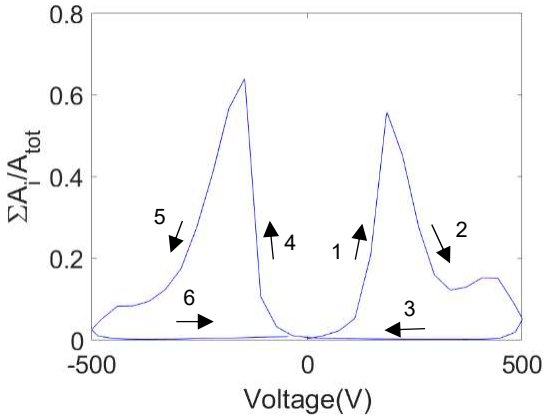
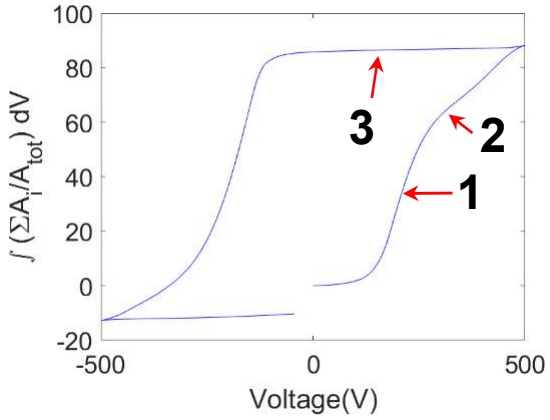
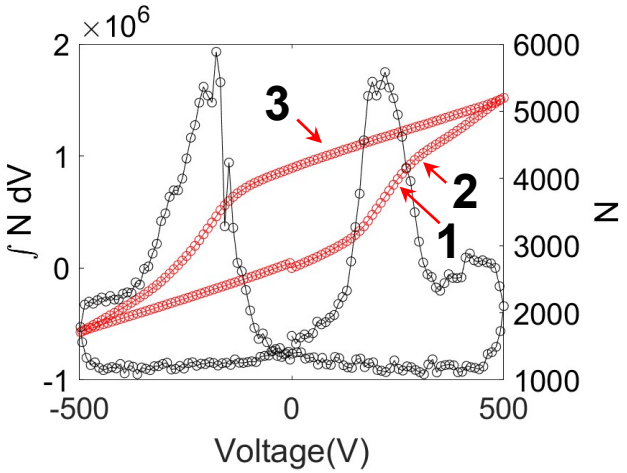
Area occupied by avalanches $\Sigma(A_i)/A_{tot}$

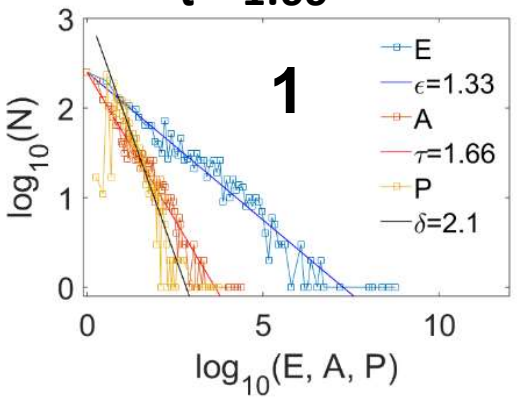




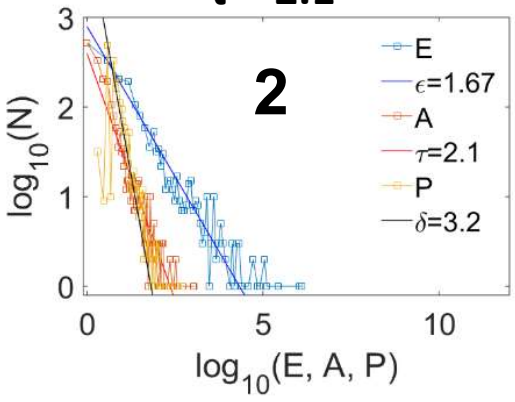
Activity N



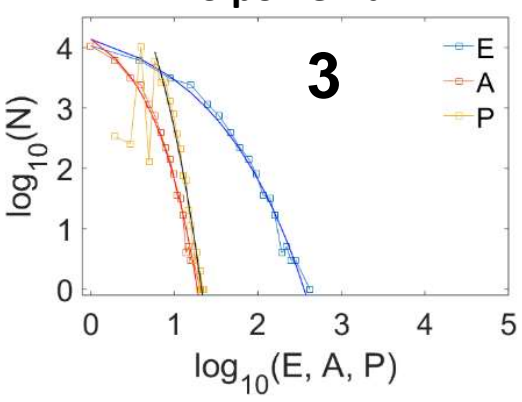
τ = 1.66



τ = 2.1



No power law



τ = 1.66 (Mean-field) at the coercive field

Same dynamics

APL Materials ARTICLE scitation.org/journal/apm

Avalanches from charged domain wall motion in $BaTiO_3$ during ferroelectric switching

Cite as: APL Mater. 8, 011105 (2020); doi: 10.1063/1.5128892
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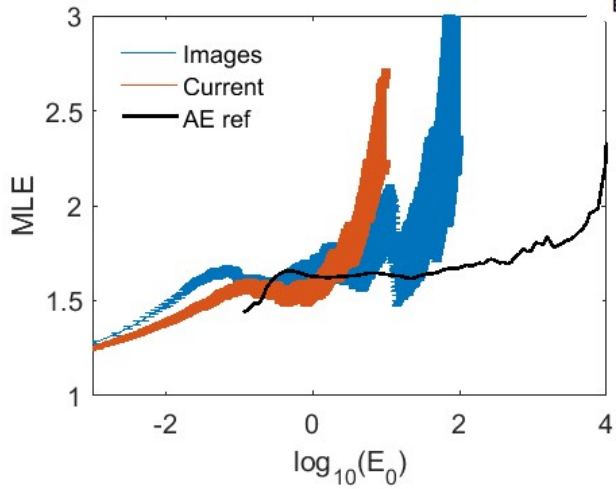
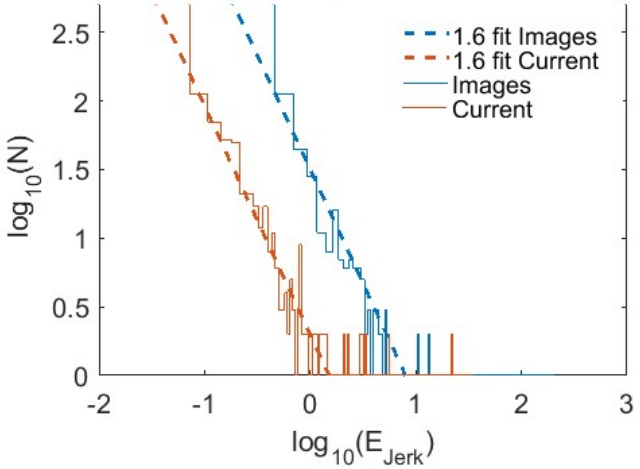
Blai Casals,[1] Guillaume F. Nataf,[2] David Pesquera,[1] and Ekhard K. H. Salje[1]





$$PDF(E) \sim E^{-\varepsilon}$$

"Listen"

Acoustic Emission

Strain changes

ε=1.6

"Touch"

Displacement current

Polarization changes

ε=1.6

"Watch"

Imaging patern changes

Polarization and Strain

ε=1.6

Ferroelectrics, beyond memories

FeRAM

Neuromorphic computing

Ferroelectric materials for neuromorphic computing EP

Cite as: APL Mater. 7, 091109 (2019); doi: 10.1063/1.5108562
Submitted: 30 April 2019 • Accepted: 5 September 2019 •
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S. Oh, H. Hwang, and I. K. Yoo[a)]

nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-020-16548-3 OPEN

Check for updates

Control of criticality and computation in spiking neuromorphic networks with plasticity

Benjamin Cramer[1✉], David Stöckel[1], Markus Kreft[1], Michael Wibral[2], Johannes Schemmel[1], Karlheinz Meier[1] & Viola Priesemann[3,4,5✉]

Relation between criticality, task-performance

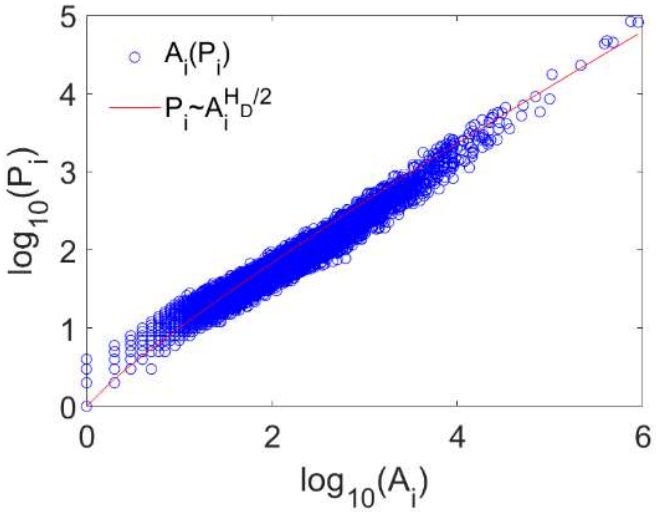
Pixel by Pixel analysis

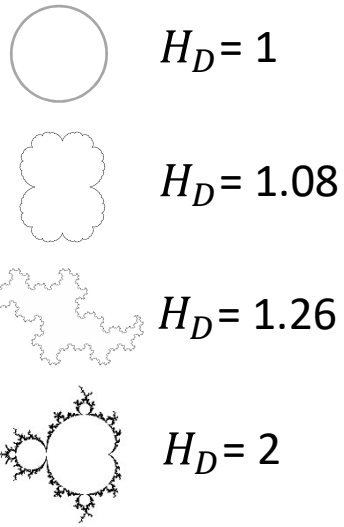
UNIVERSITY OF CAMBRIDGE

$$J_{ij} = \left(\frac{dB_{ij}}{dt}\right)^2$$ J_{ij} > threshold

$$P \sim A^{H_D/2}$$

Fractal dimension (Hausdorff dimension) $\boldsymbol{P \sim A^{H_D/2}}$





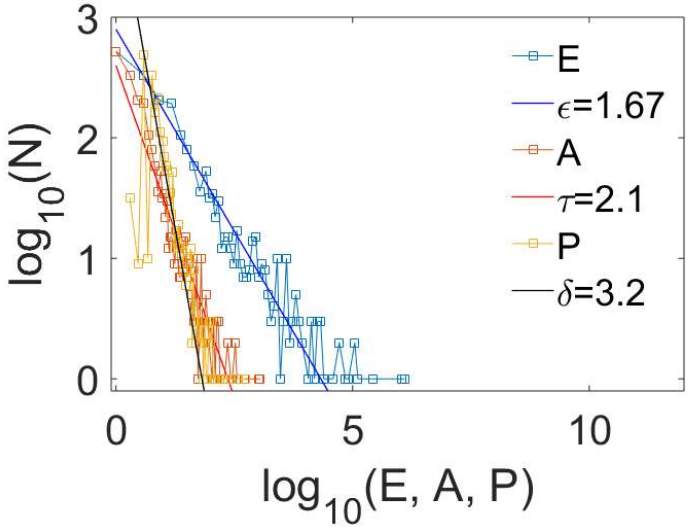
H_D= 1

H_D= 1.08

H_D= 1.26

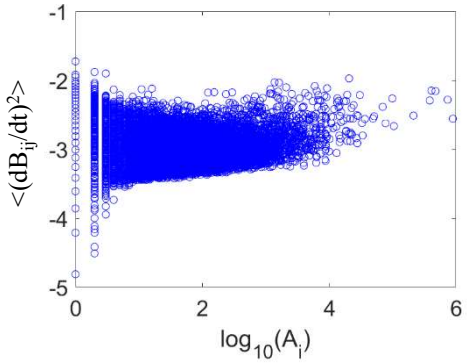
H_D= 2

Distribution of Areas A, Perimeters P and Energies E.



$\mathrm{PDF(A)} \sim \mathrm{A}^{-\tau}$

The spread of J_{ij} at each A_i is similar



Avalanche statistics, power law distribution

Snow avalanches

Earthquakes

Landslides

Neuronal activity

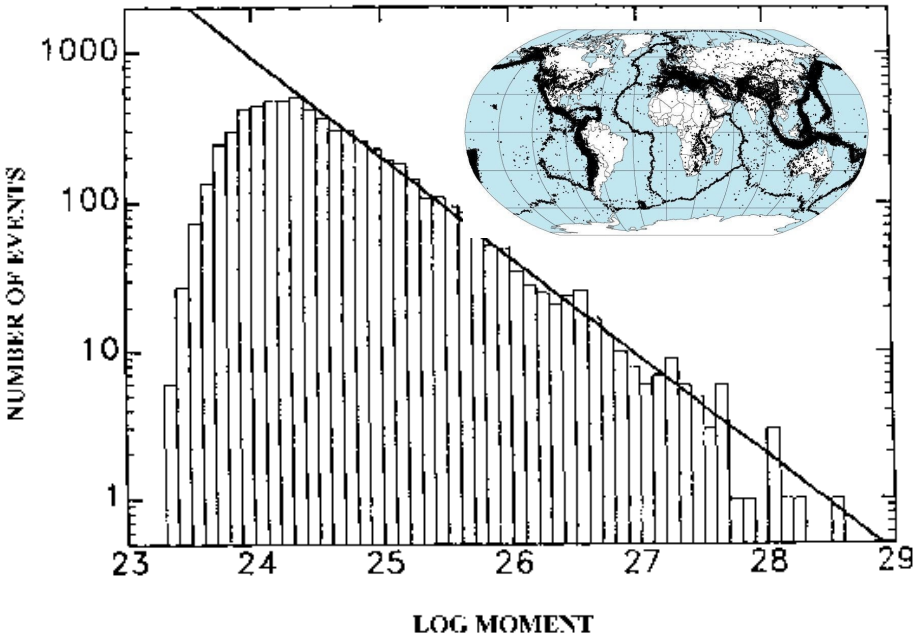


FIGURE 6