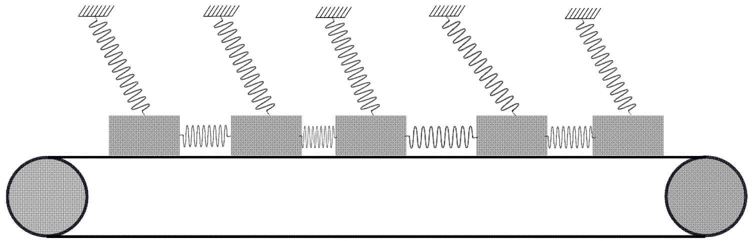
Avalanche-size statistics in a Burridge-Knopoff type spring-block model

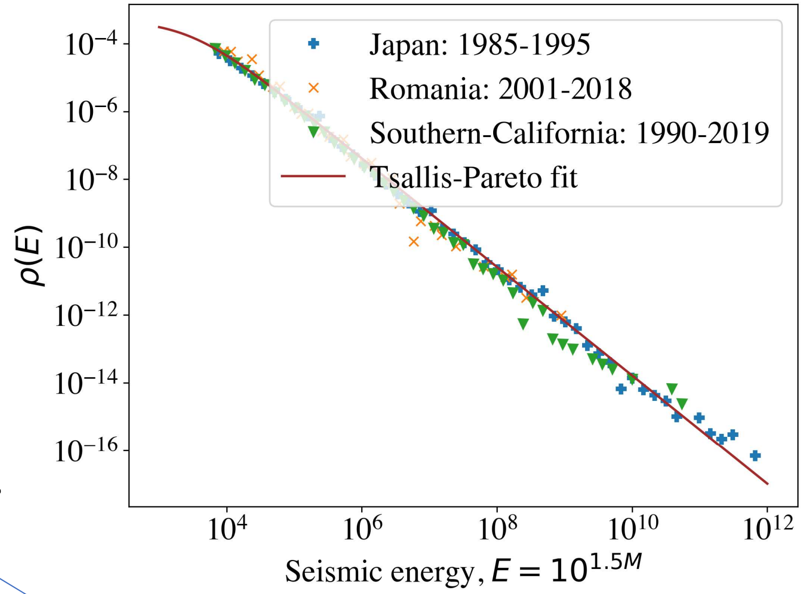
Attila Gergely, András Kuki, Sándor Lipcsei and Néda Zoltán

Department of Physics, Babeş-Bolyai University, M. Kogălniceanu
nr. 1, 400084, Cluj-Napoca, Romania

Aims:

- Development of a treadmill-based setup for the experimental study of the Burridge-Knopoff type spring-block system
- Studying the system to find interesting dynamics (avalanches,…)
- Reproduction of the observed behaviors using a simple computational model

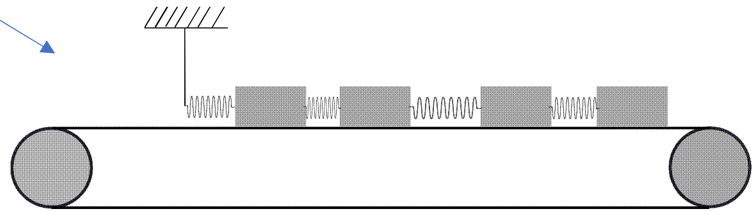




Similar study:

- B. Sándor, F. Járai-Szabó, T. Tél and Z. Néda, Phys. Rev. E 87, 042920 (2013).

Interesting chaotic, periodic and quasiperiodic behaviors

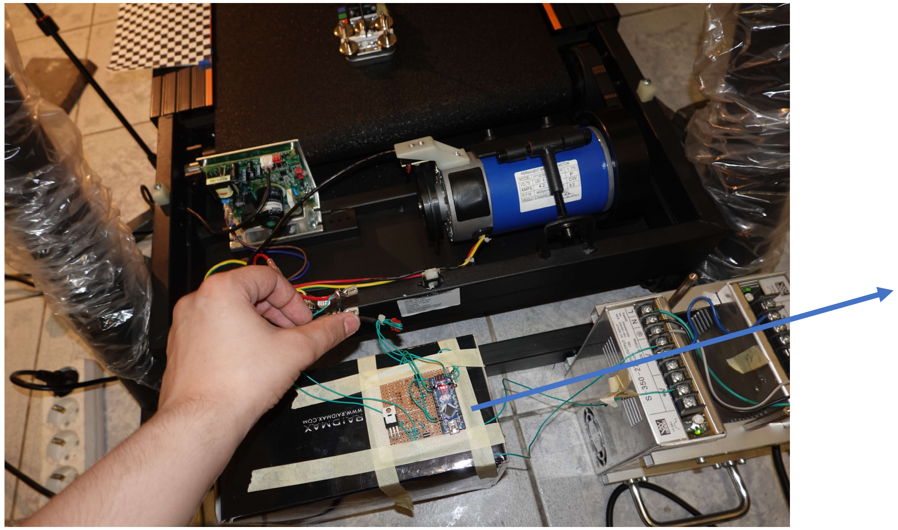


- Varamashvili, Nodar, et al. "Mass-movement and seismic processes study using Burridge-Knopoff laboratory and mathematical models." *JOURNAL OF THE GEORGIAN GEOPHYSICAL SOCIETY* 18.18 (2015).

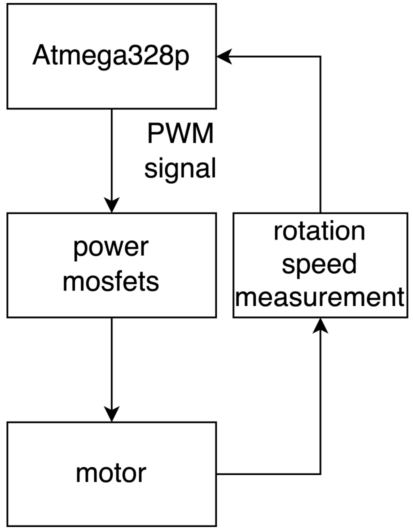
Experimental setup

Treadmill with blocks and springs

Treadmill speed controller



controller block diagram



snapshot of the dynamics

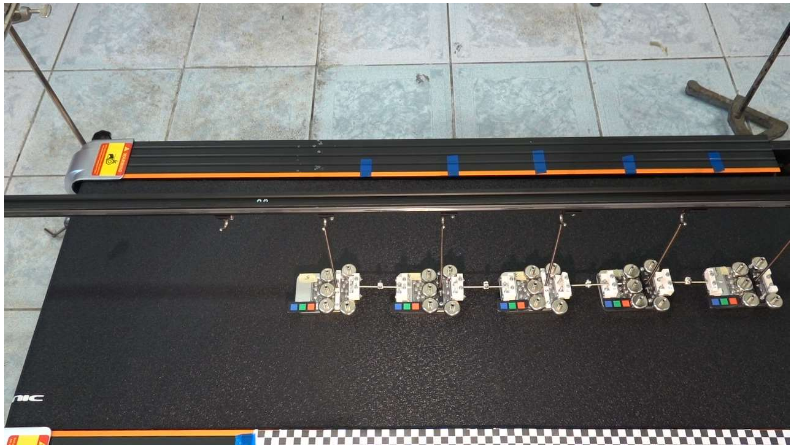
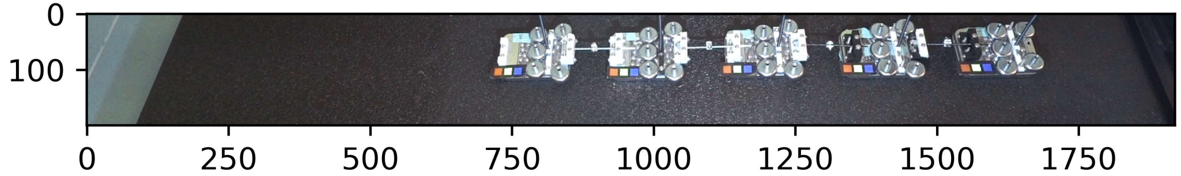
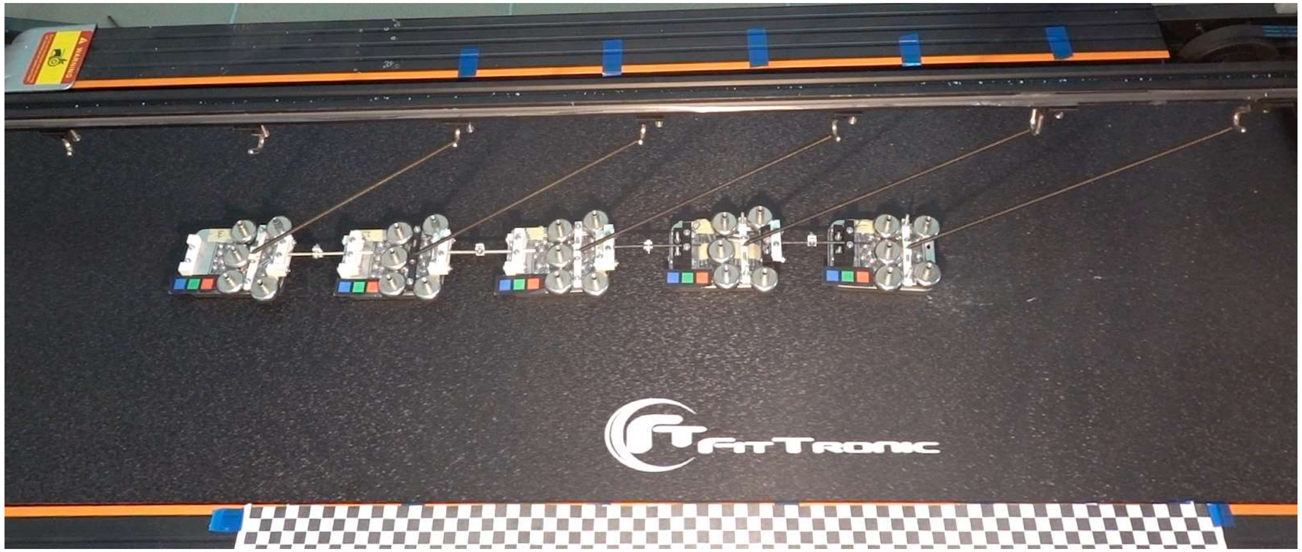


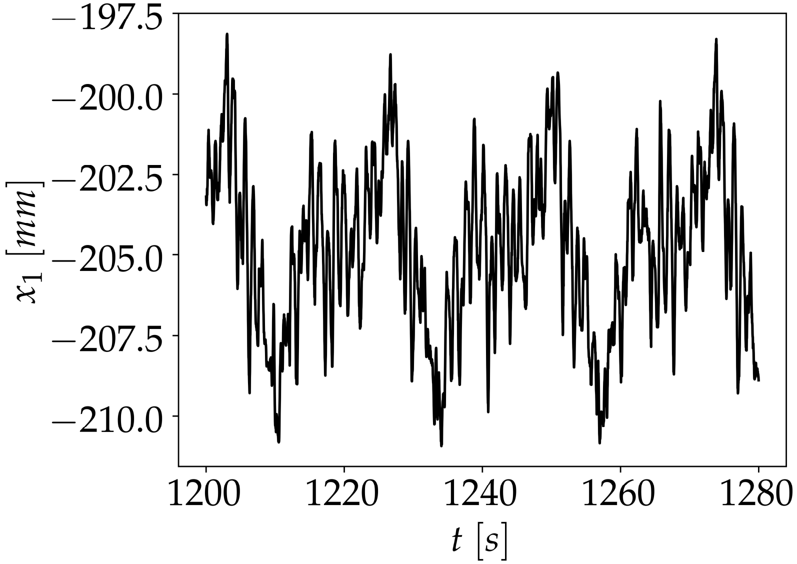
image processing algorithm to locate the blocks



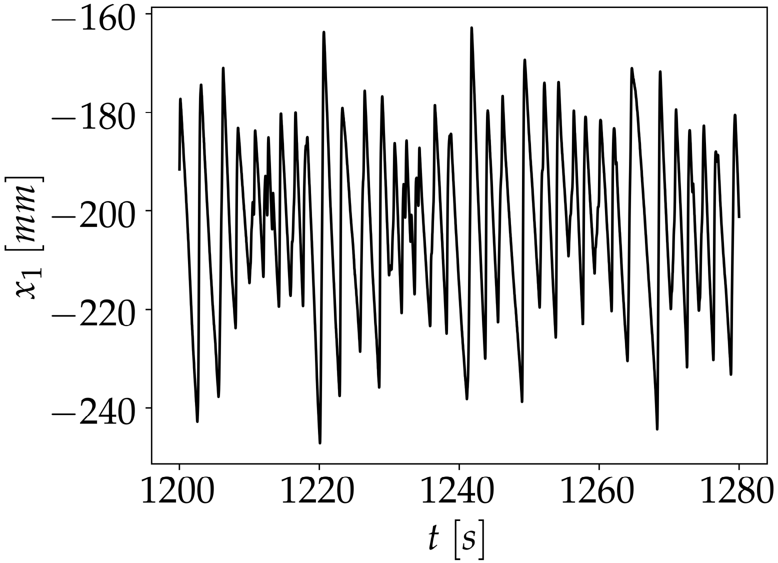
quasi-periodic behavior -> v=0.4km/h



The position of the first block



Stick-Slip behavior (avalanches) -> v=0.1km/h

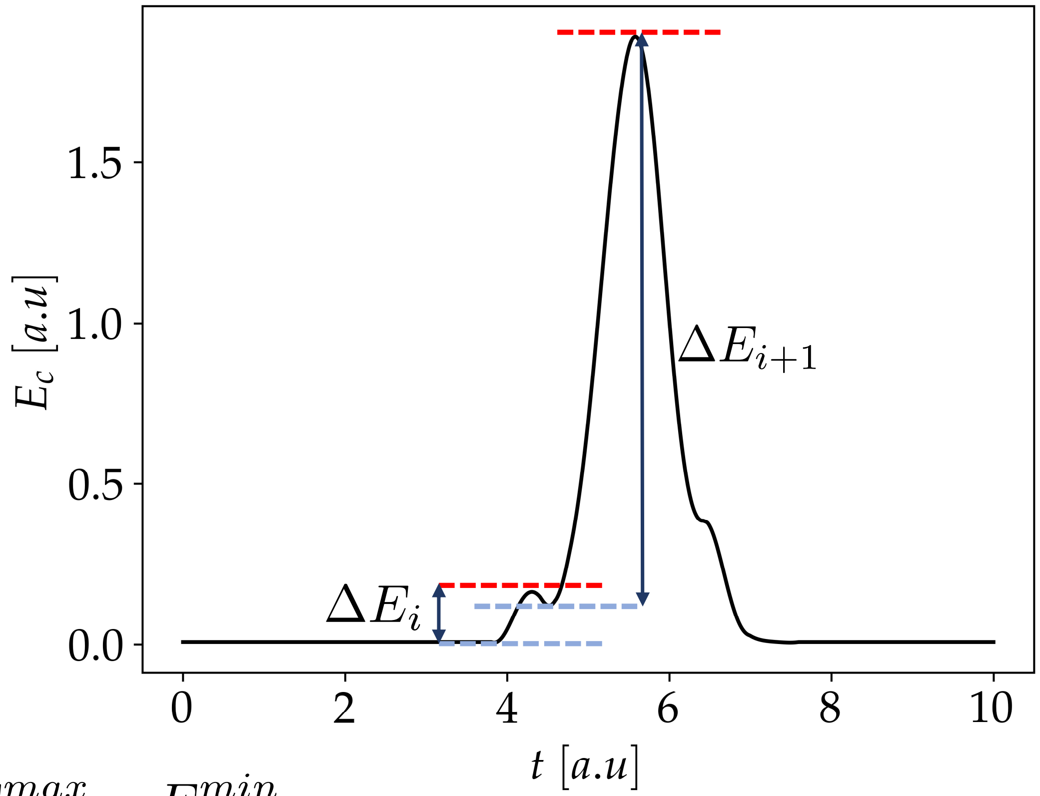


Avalanche definition

- Consecutive time steps where the kinetic energy of the blocks increases
- The energy of the avalanche is the difference between the initial and final kinetic energies of the blocks

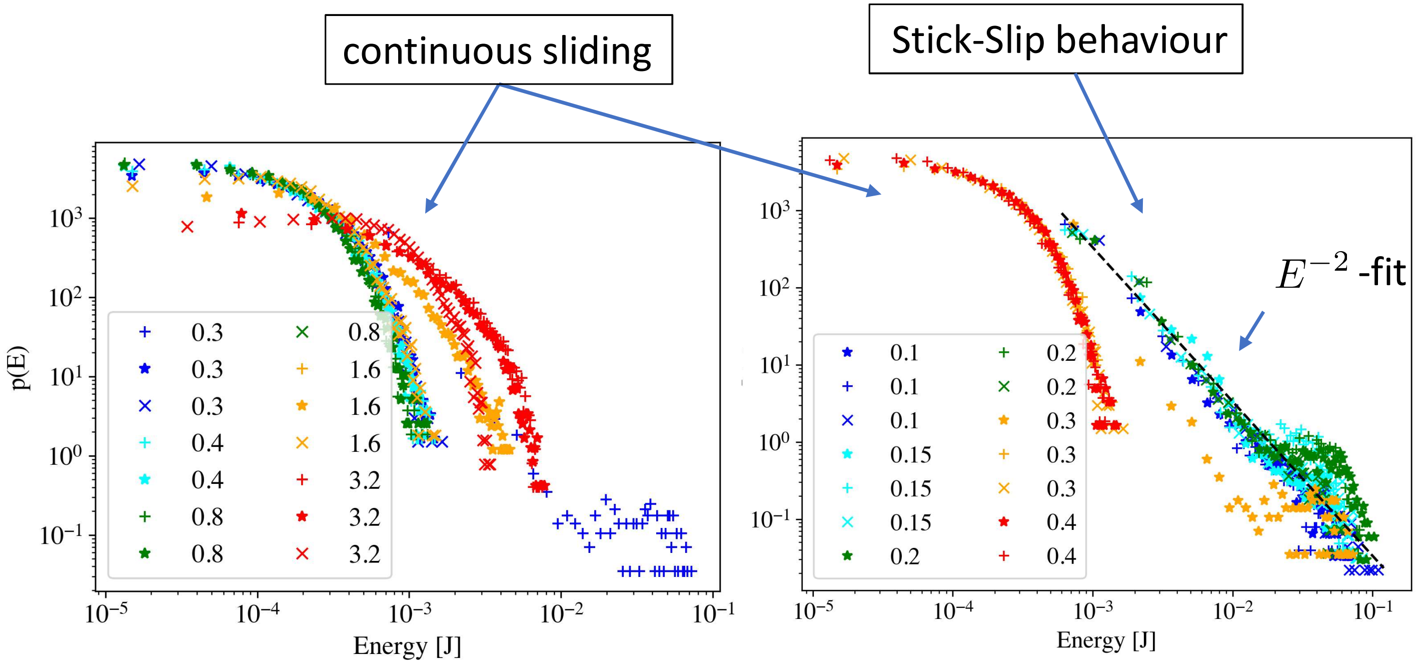
Implementation:

- Identification of the i-th local minimum in kinetic energy → E_i^{min}
- Identification of the first local maximum after the i-th minimum in the kinetic energy → E_i^{max}
- The energy assigned to the avalanche → $\Delta E_i = E_i^{max} - E_i^{min}$

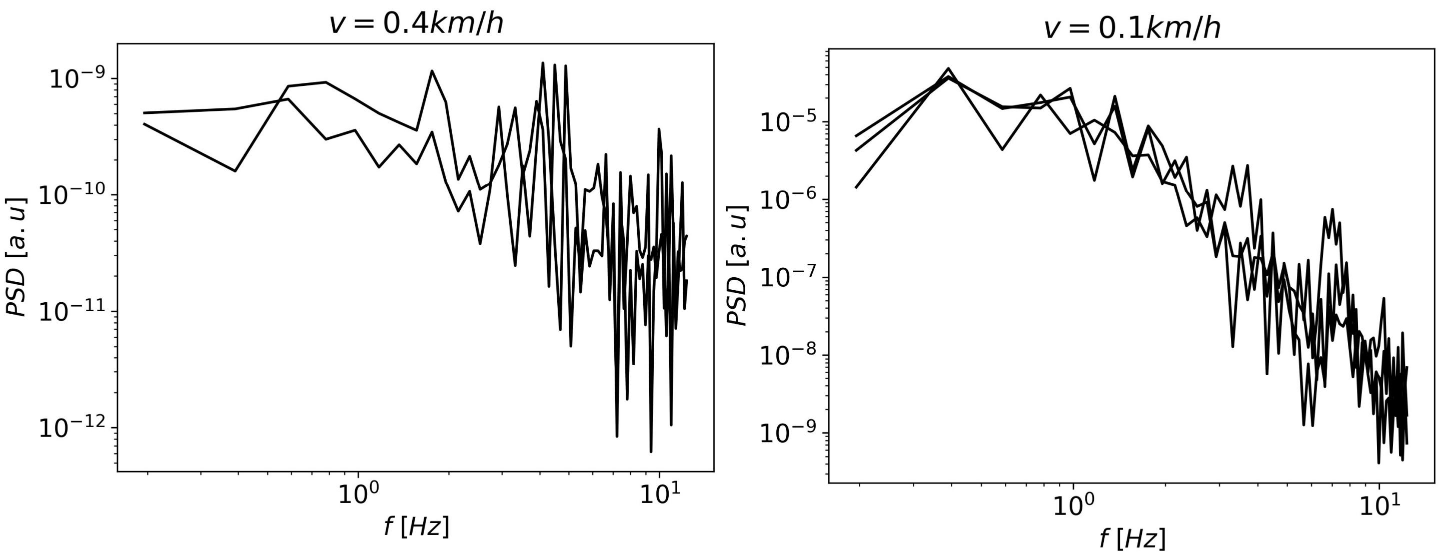


Experimental results

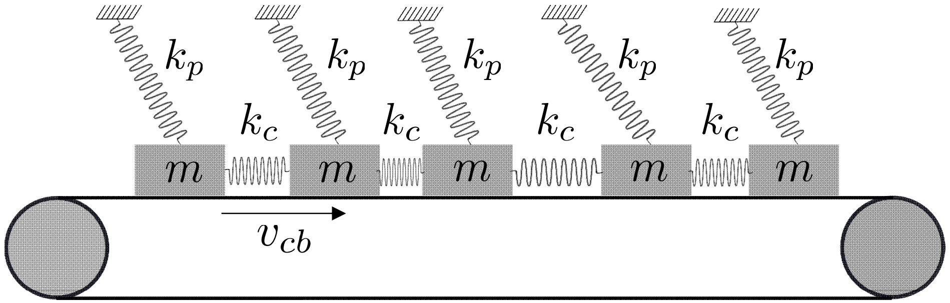
Avalanche statistics



Power spectral density from $E_c(t)$



dynamical model



The spring force acting on the blocks

$$f_s = \begin{cases} i = 0 \ : \ -k_p \cdot x_i + k_c \cdot (x_{i+1} - x_i) \\ i = \overline{1, N-1} \ : \ -k_p \cdot x_i + k_c \cdot (x_{i+1} - 2 \cdot x_i + x_{i-1}) \\ i = N \ : \ -k_p \cdot x_i + k_c \cdot (x_{i-1} - x_i) \end{cases}$$

$k_p \longrightarrow$ pooling spring spring constant

$k_c \longrightarrow$ coupling spring spring constant

$m \longrightarrow$ mass of bodies

$F_c \longrightarrow$ static friction force

$F_c \cdot \alpha \longrightarrow$ dynamic friction force

$v_{rstick} \longrightarrow$ maximum relative speed for sticking

Total resultant force acting on the blocks

$$f_i = \begin{cases} |f_s| > F_c \ : \ f_s - sign(\dot{x}_i - v_{cb}) \cdot F_c \cdot \alpha \\ \dot{x}_i - v_{cb} = 0 \ and \ |f_s| <= F_c : \ 0 \\ |\dot{x}_i - v_{cb}| > v_{rstick} \ and \ |f_s| <= F_c : \ f_s - sign(\dot{x}_i - v_{cb}) \cdot F_c \cdot \alpha \\ |\dot{x}_i - v_{cb}| < v_{rstick} \ and \ |f_s| <= F_c \ and \ sign(\dot{x}_i - v_{cb}) = sign(f_s) : \\ \ f_s - sign(\dot{x}_i - v_{cb}) \cdot F_c \cdot \alpha \\ |\dot{x}_i - v_{cb}| < v_{rstick} \ and \ |f_s| <= F_c \ and \ sign(\dot{x}_i - v_{cb}) \neq sign(f_s) : \\ \ 0 \ (\text{and } \dot{x}_i \to v_{cb}) \end{cases}$$

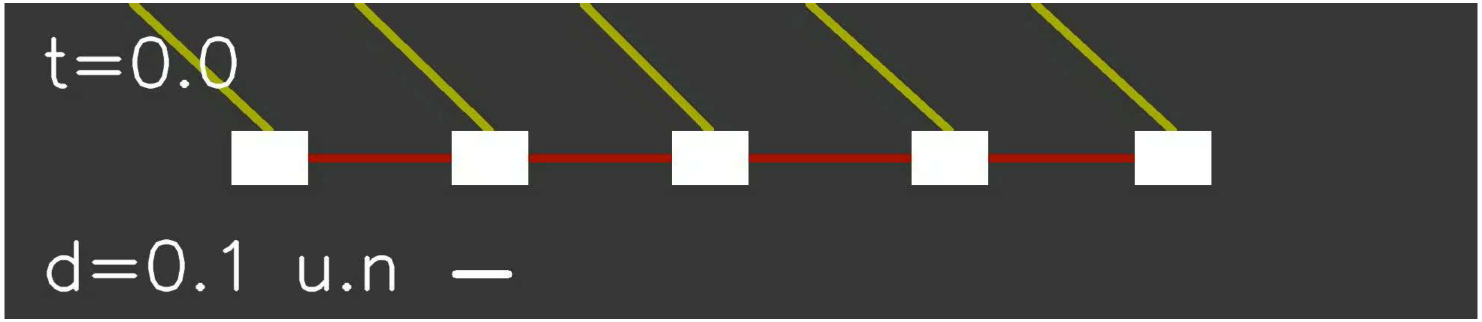
$$\ddot{x}_i \cdot m = f_i$$

Numerical implementation

- The simulations were performed in C++.
- The equations of motion were integrated using the 4 order Runge Kutta method
- The experimentally obtained Stick-Slip and periodic behaviors were successfully reproduced

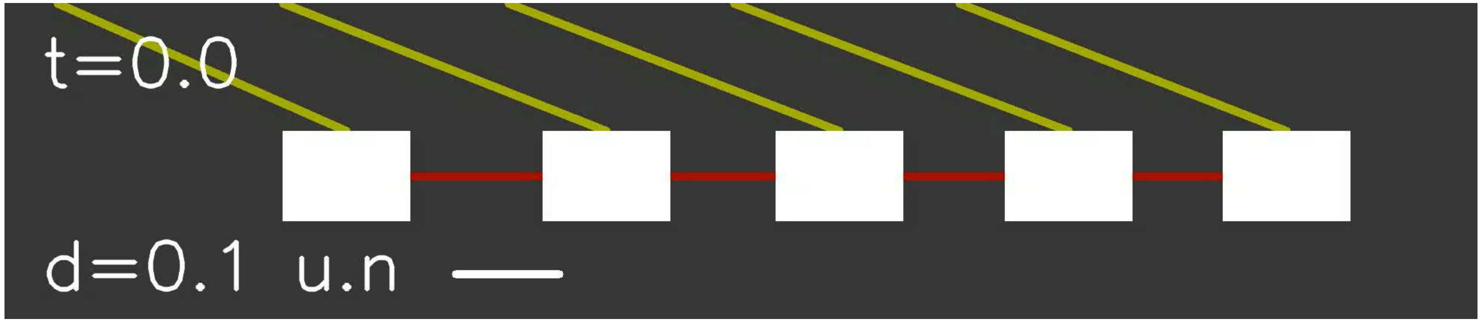
Stick-Slip behavior

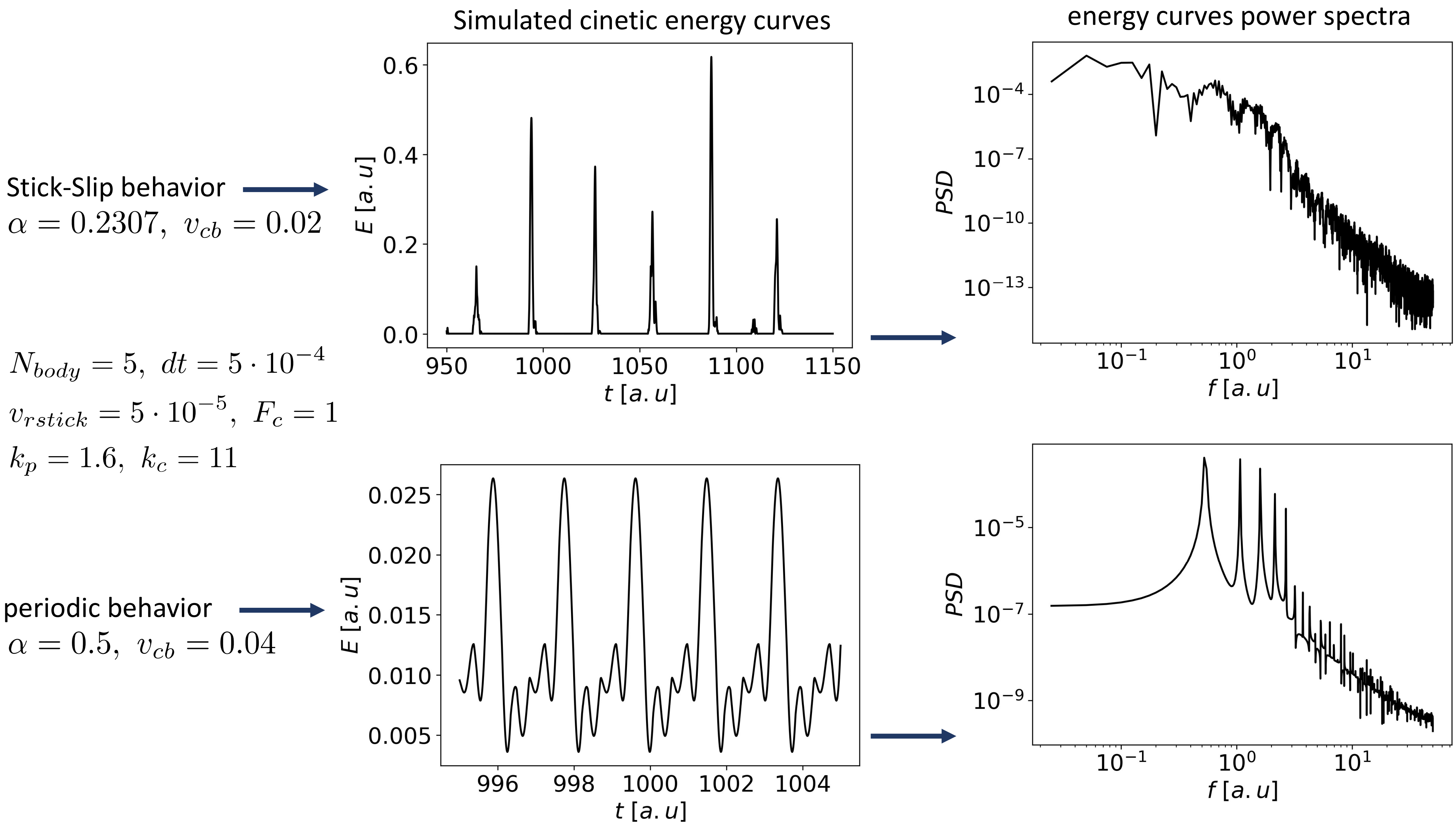
$\alpha = 0.4, \; v_{cb} = 0.02$



periodic behavior

$\alpha = 0.5, \; v_{cb} = 0.05$




Simulated cinetic energy curves
energy curves power spectra
Stick-Slip behavior
$\alpha = 0.2307,\ v_{cb} = 0.02$
$N_{body} = 5,\ dt = 5 \cdot 10^{-4}$
$v_{rstick} = 5 \cdot 10^{-5},\ F_c = 1$
$k_p = 1.6,\ k_c = 11$
periodic behavior
$\alpha = 0.5,\ v_{cb} = 0.04$
$E\ [a.u]$
$t\ [a.u]$
PSD
$f\ [a.u]$
0.0
0.2
0.4
0.6
950
1000
1050
1100
1150
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-1}
10^{0}
10^{1}
0.005
0.010
0.015
0.020
0.025
996
998
1000
1002
1004
10^{-5}
10^{-7}
10^{-9}

spectral entropy

Spectral entropy (SE) is the Shannon entropy of the normalized power spectral density (PSD) of the time series :

$$P(f) = \frac{PSD(f)}{\sum_{f=0}^{f_s/2} PSD(f)}$$

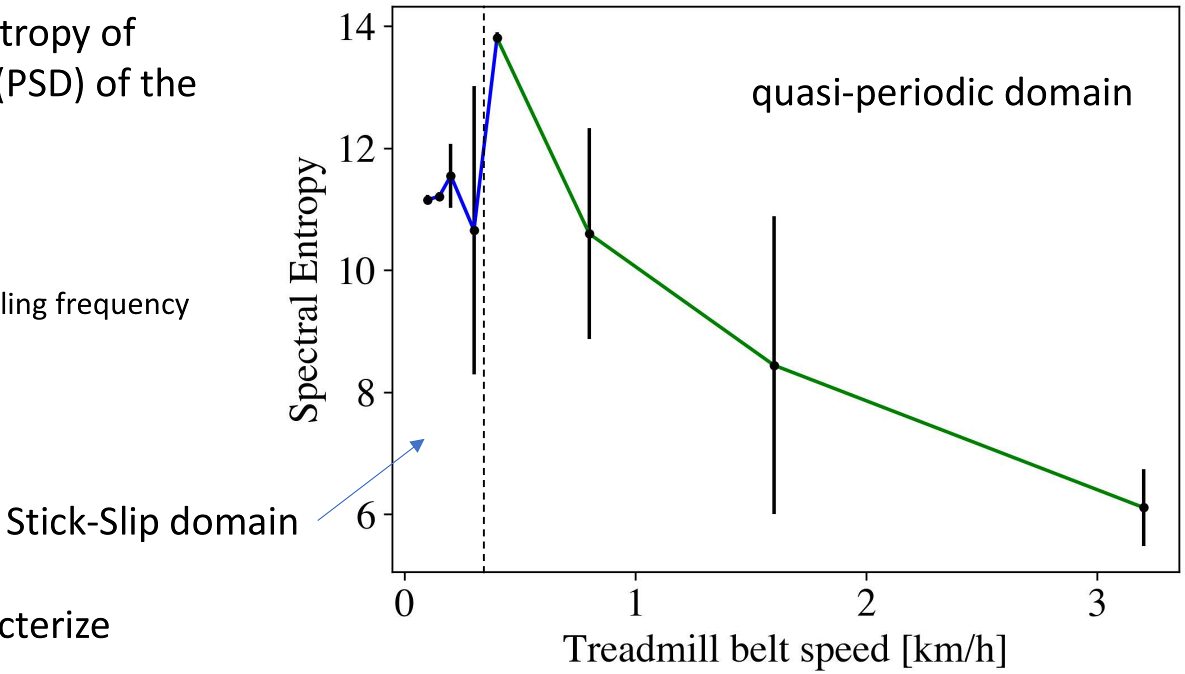
f_s → sampling frequency

$$SE(f_s) = -\sum_{f=0}^{f_s/2} P(f) \cdot log_2[P(f)]$$

The spectral entropy was used to characterize the periodicity of the time series

Low SE -> periodic behaviour

spectral entropy calculated for experiments



Powell G.E., Percival I.C. // J. Phys. A: Math. Gen. 1979. V. 12. № 11. P. 2053–2071.http://dx.doi.org/10.1088/0305-4470/12/11/017

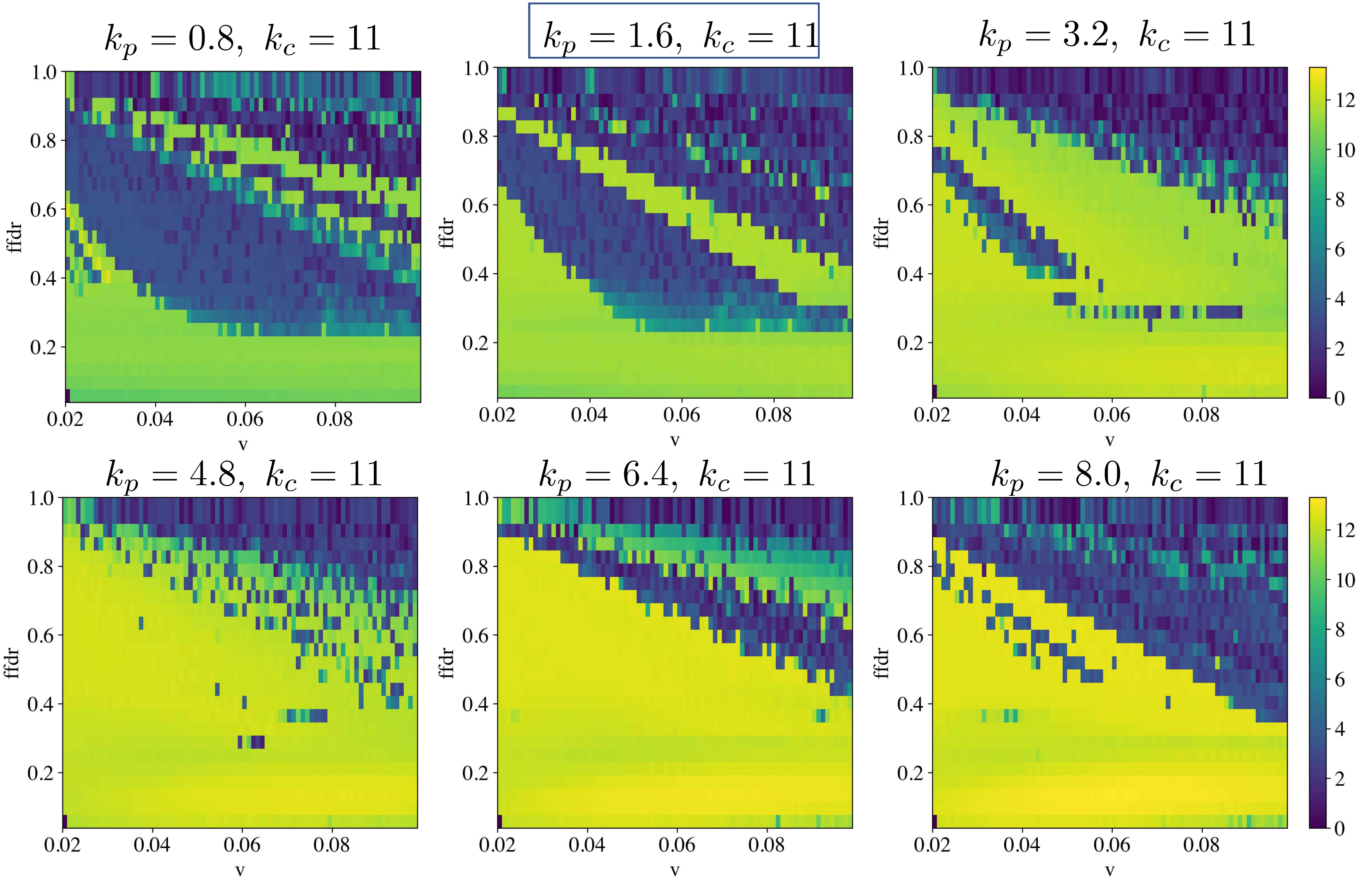
Spectral Entropy

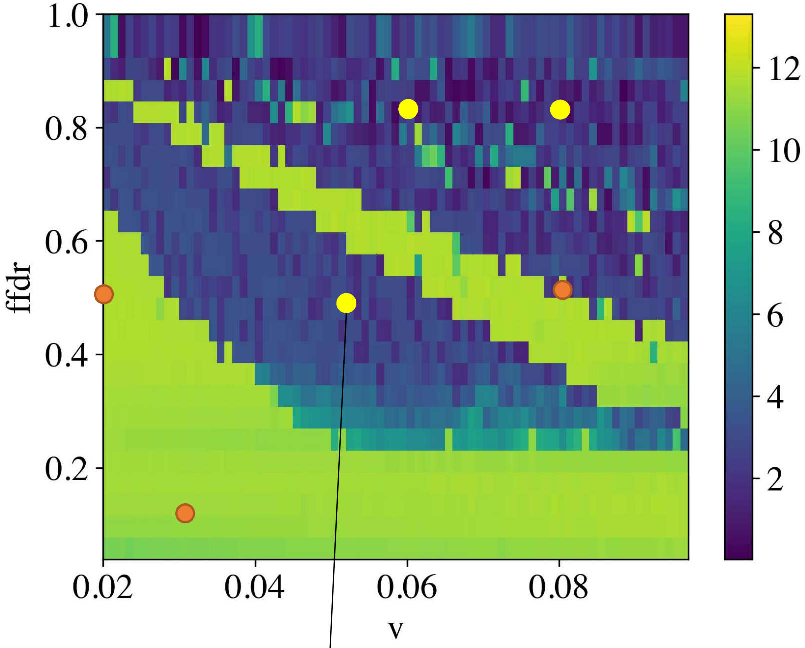
$N_{body} = 5,\ dt = 5 \cdot 10^{-4}$

$v_{rstick} = 5 \cdot 10^{-5},\ F_c = 1$

Periodic behavior ->
low spectral entropy

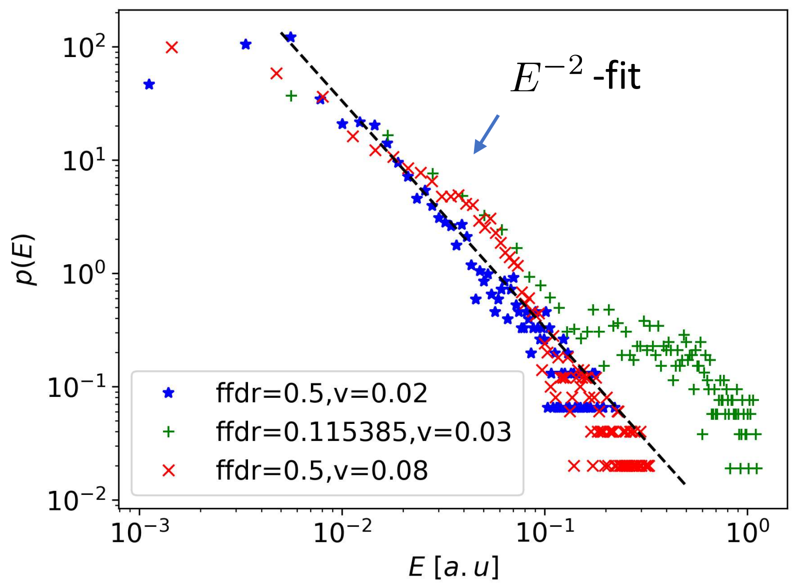
Stick-Slip behavour ->
high spectral entropy

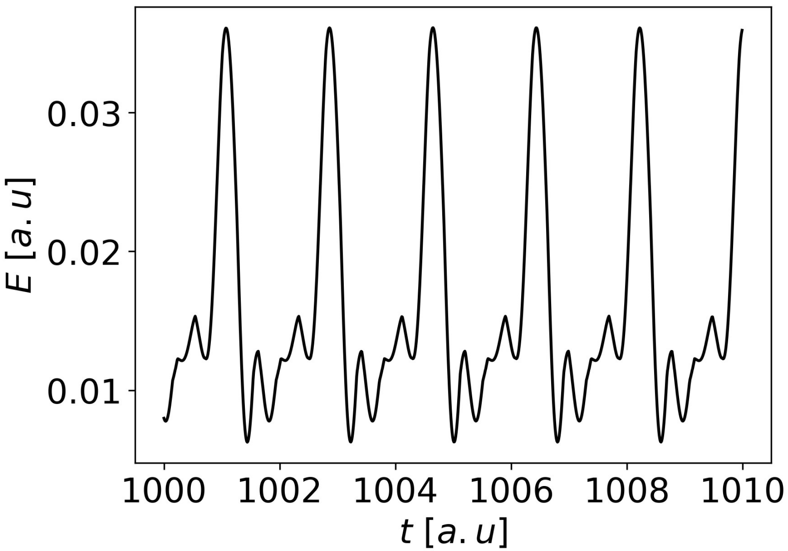




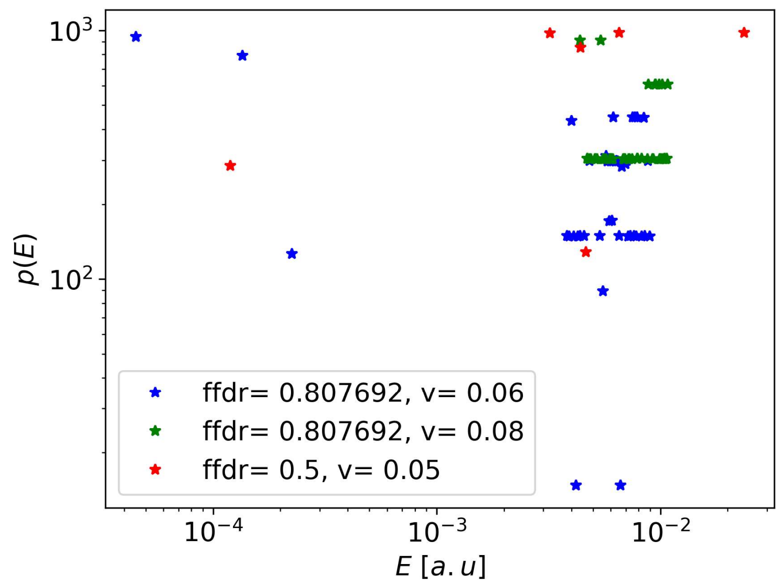
$$N_{body} = 5,\ dt = 5 \cdot 10^{-4}$$
$$v_{rstick} = 5 \cdot 10^{-5},\ F_c = 1$$

Similar behavior in the Stic-Slip domain





Behavior different from experiments in the periodic domain

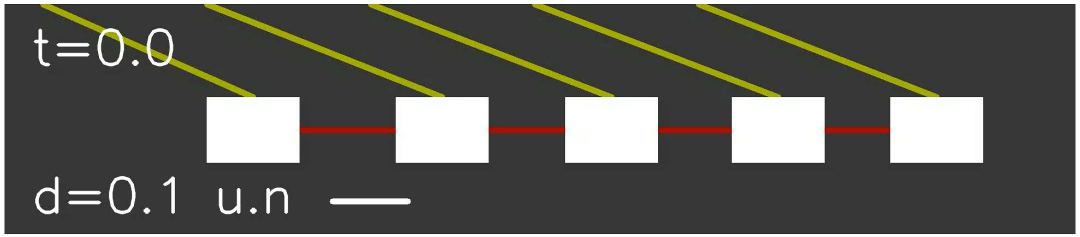


Conclusions

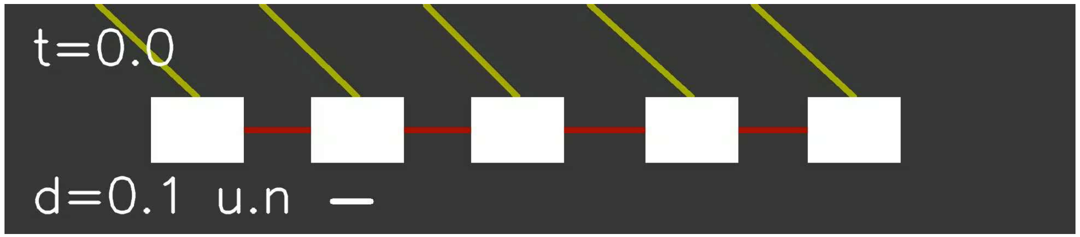
- simple experimental realization of the Burridge-Knopoff type spring-block model by using a spring-block chain on a treadmill
- The experiments show qualitatively different behavior in different speed regimes
- Power-law type scaling for the avalanche size distribution as a function of avalanche energy
- The simple computer model reproduces the main features of the observed dynamics
- Similar avalanche size scaling to the one observed for earthquakes

Numerical implementation

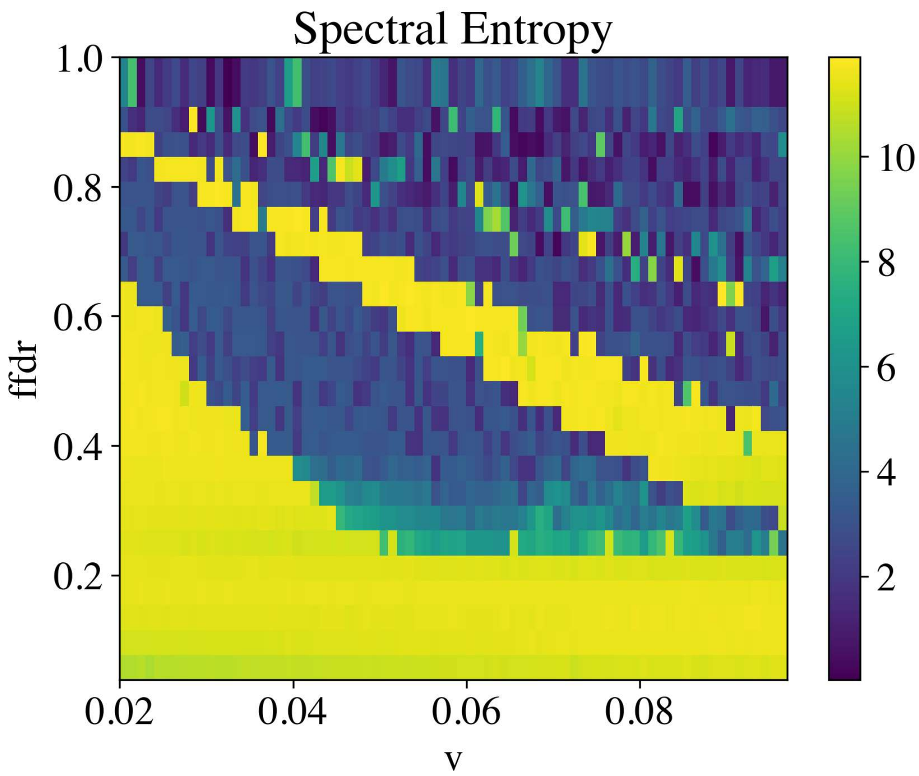
periodic behavior



Stick-Slip behavior



Numerical results



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