FAILURE AVALANCHES OF THE FIBER BUNDLE MODEL ON COMPLEX NETWORKS

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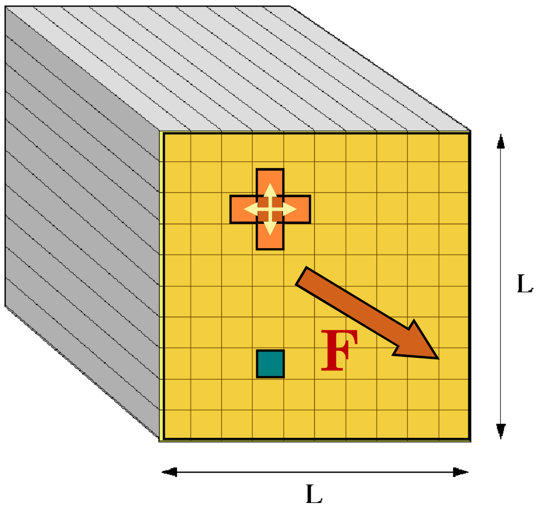
Fiber bundle model of fracture phenomena

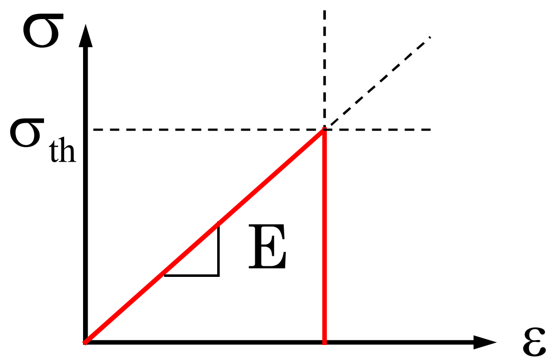
A simple model of the fracture of heterogeneous materials



- Parallel fibers on a lattice
- Load parallel to fibers
- Perfectly Brittle response

  Two parameters: E, σ_{th}

  Distribution of failure thresholds $p(\sigma_{th})$



- Load redistribution

  Two limiting cases: ELS ⟺ LLS

S. Pradhan, A. Hansen, B. K. Chakrabarti, Rev. Mod. Phys. **82**, 499 (2010).

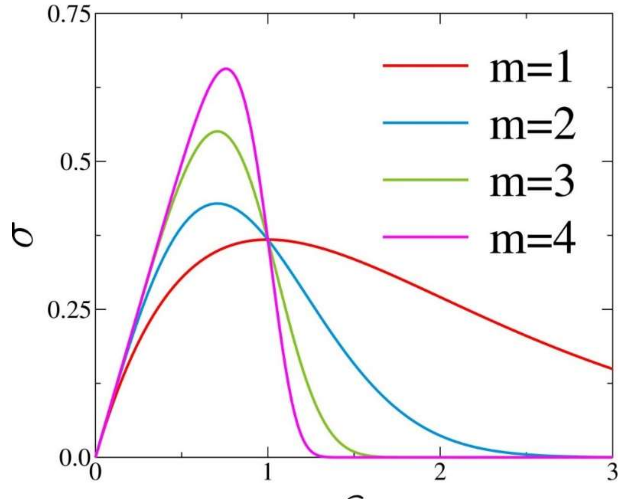
THE MEAN FIELD UNIVERSALITY CLASS OF FBMS

Equal load sharing (ELS)

- The excess load is shared equally ➡ No stress fluctuations
- Topology of fibers' lattice is irrelevant
- Mean field limit of FBMs

Macroscopic response

$$\sigma(\varepsilon) = E\varepsilon[1 - P(E\varepsilon)]$$



Weibull distribution: $P(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^m}$

Lower disorder implies a higher brittleness

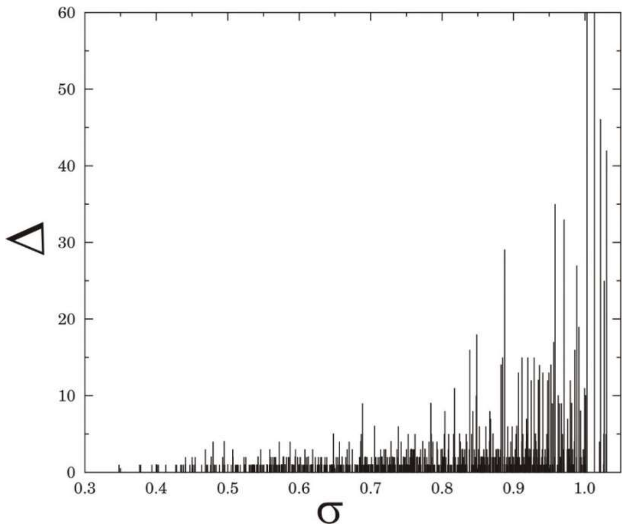
The mean field universality class of FBMs

Microscopic failure mechanism under stress controlled loading

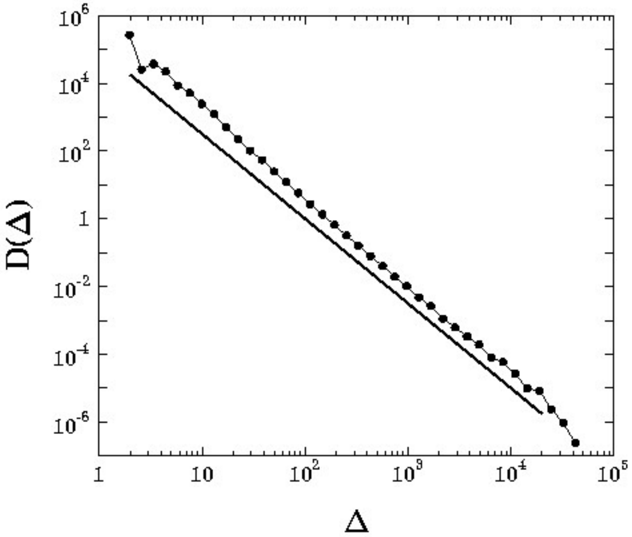
- Failure of a single element
- Local failure is followed by load redistribution
- Secondary failure events are induced

} Failure cascade/avalanche

Accelerating dynamics



Universal avalanche size distribution

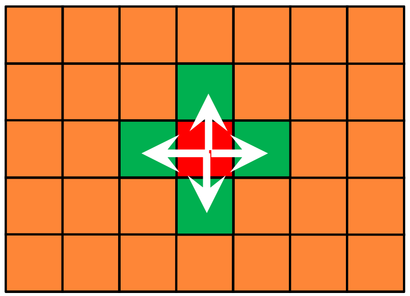


$$p(\Delta) \propto \Delta^{-\tau}$$

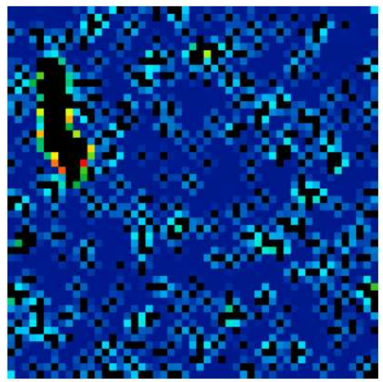
$$\tau = \frac{5}{2}$$

The LLS universality class of FBMs

Nearest neighbor interaction

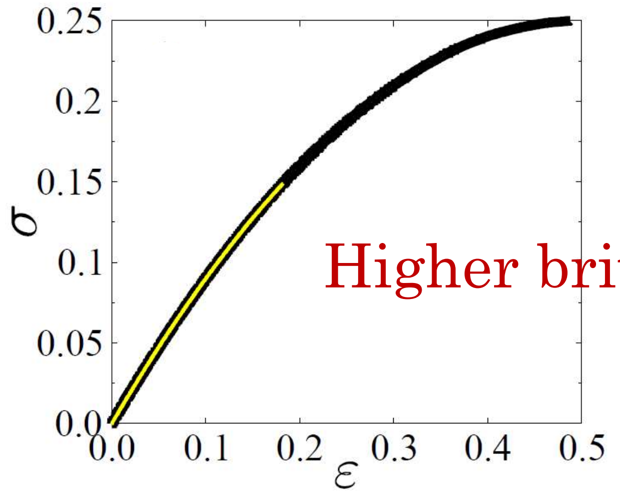


Complex spatial structure



- Small size clusters
- Highly stressed perimeter
- Inhomogeneous stress field

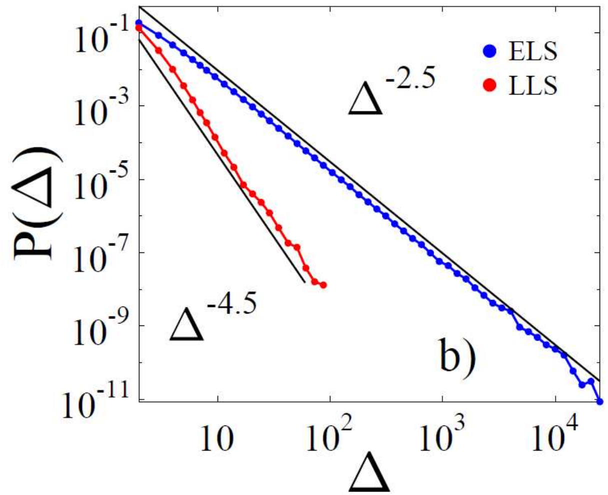
Macroscopic response



Higher brittleness

Non-universal avalanche size distribution

$$p(\Delta) \propto \Delta^{-\tau}$$



FAILURE PHENOMENA IN COMPLEX SYSTEMS

Governing failure mechanism in FBM is cascading failure driven by load redistribution

Complex systems: A large number of interacting elements

Fracture of heterogeneous materials

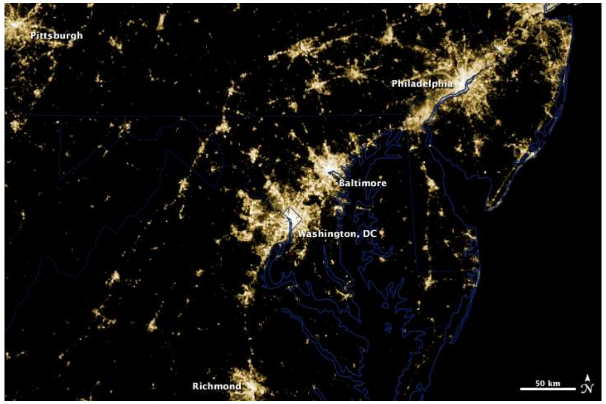
Power outages

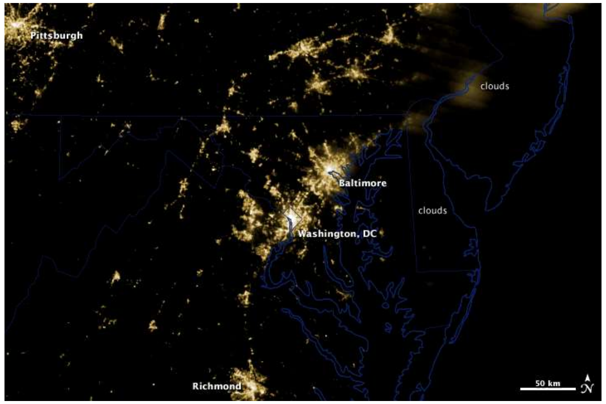
Breakdown of urban traffic

Widespread applications of the fiber bundle model

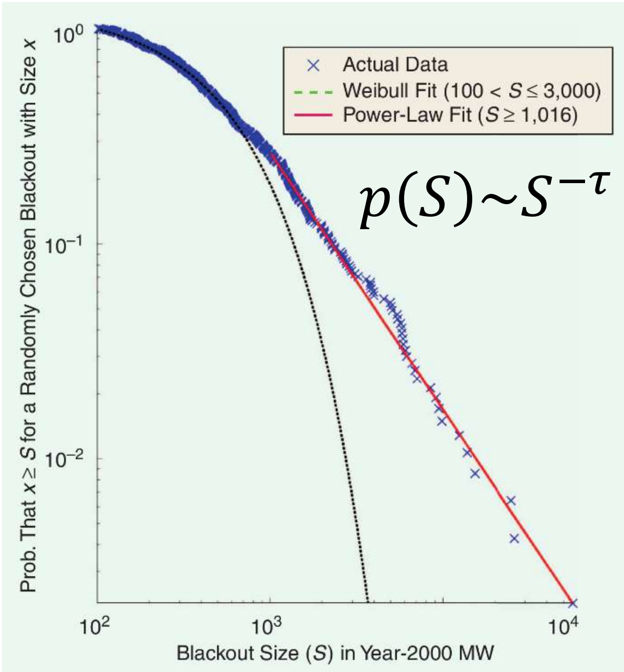
CASCADING FAILURE IN HIGH VOLTAGE POWER GRIDS

Power outages
in Washington DC area





Size distribution of electric blackout events during North American blackouts from 1984 to 1998



I. Dobson et al., Chaos **17**, 026103 (2007).

Importance of network topology

N. Friedman et al., *Phys. Rev. Lett.* **108**, 208102 (2012).

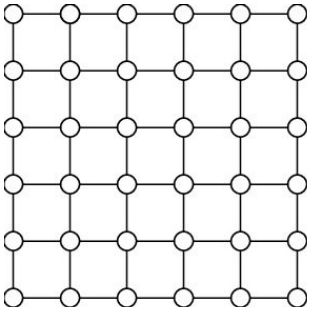
OBJECTIVES

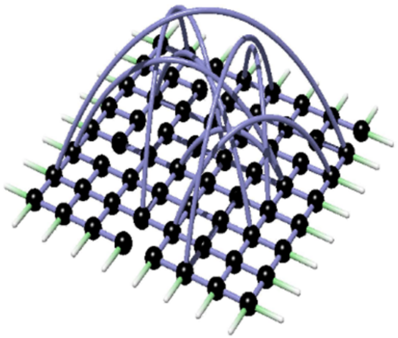
Cascading failure mechanism of the fiber bundle model on complex networks

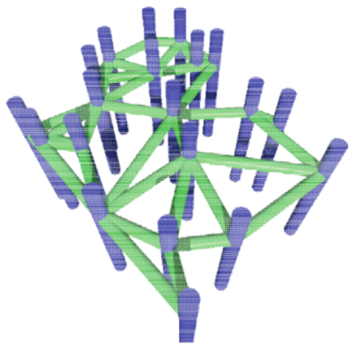
Effect of network topology of load transmitting connections and strength disorder on

- the fracture strength of bundles on the macroscale
- the statistics of avalanches on the microscale
- temporal evolution of failure cascades

Complex network of load transmitting connections





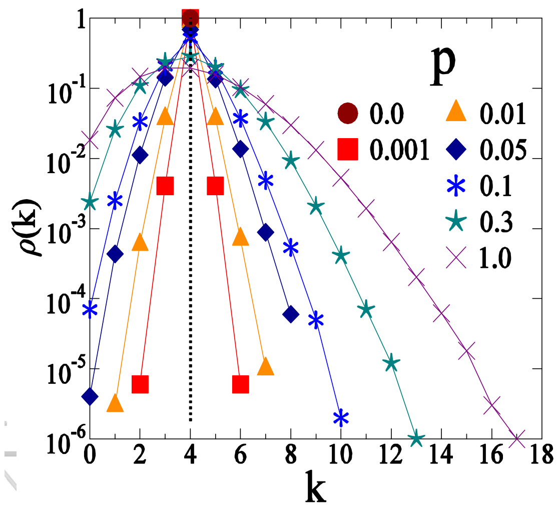


Watts-Strogatz rewiring technique to randomize connections with probablity p.

Regular Square lattice ➡ Randon network

Fibers are oriented perpendicular to the plane of orignal lattice

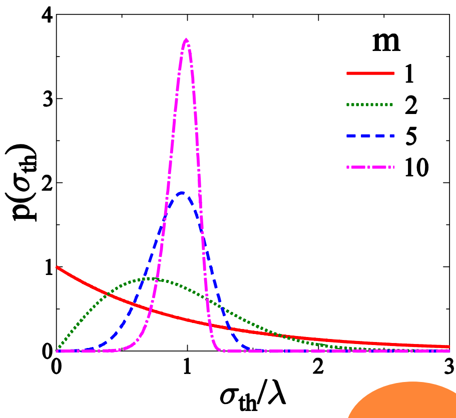
Degree distribution of WS networks



Structural randomness &
Strength disorder

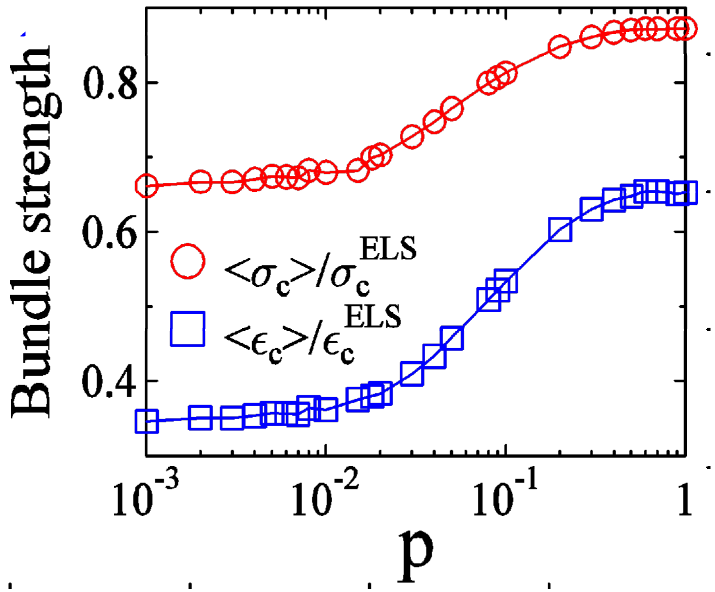
Weibull distribution: $\sigma_{th} = m \frac{\sigma_{th}^{m-1}}{\lambda^m} e^{-(\frac{\sigma_{th}}{\lambda})^m}$

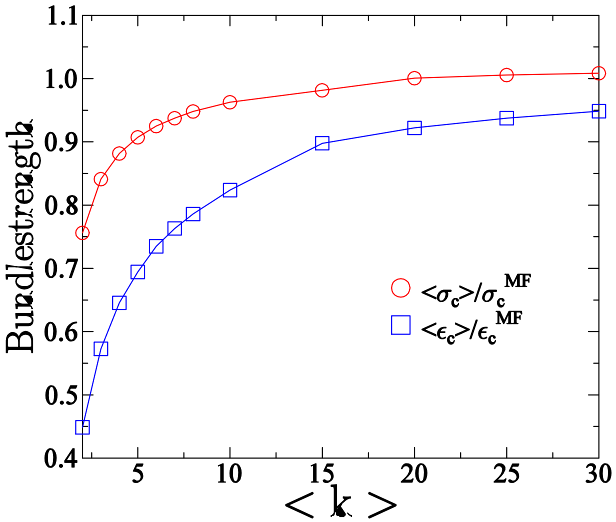
$1 \le m \le 22$



Localized load sharing on complex networks

Macroscopic strength

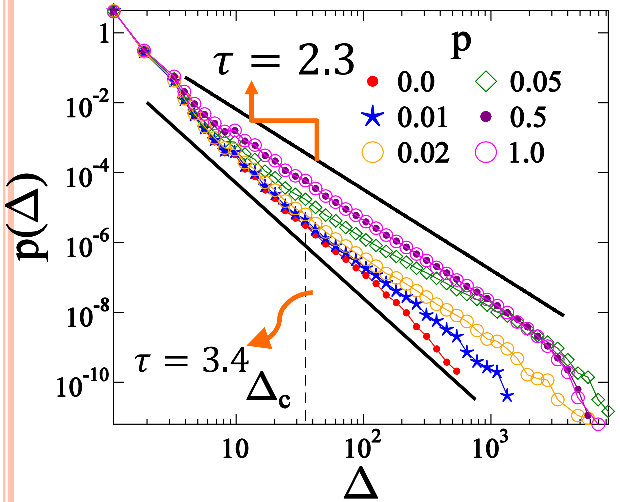


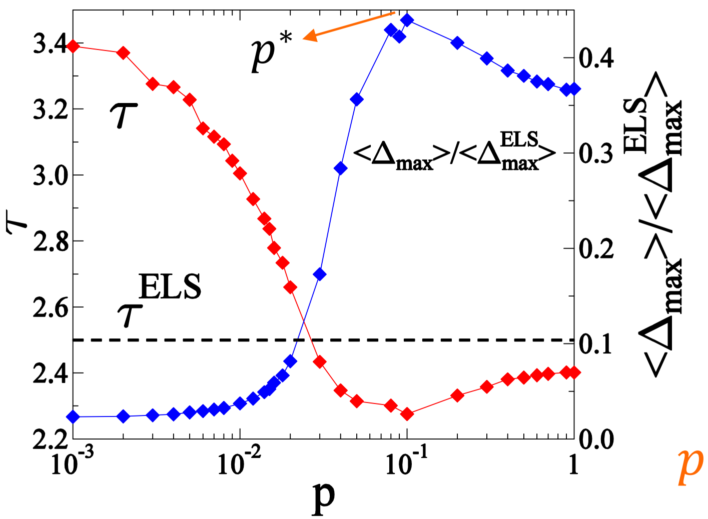


- Monotonically increasing critical load and strain with increasing randomness
- The transition sets on at a threshold rewiring probability p_l
- Convergence towards the mean field (ELS) limit

Localized to mean field transition ($LLS \rightarrow ELS$)

Cascading failure on complex networks





Power law avalanche size distribution

$$p(\Delta) \propto \Delta^{-\tau}$$

For $p \leq p_l$:

- Small avalanches
- Constant $< \Delta_{max} >$
- High exponent τ

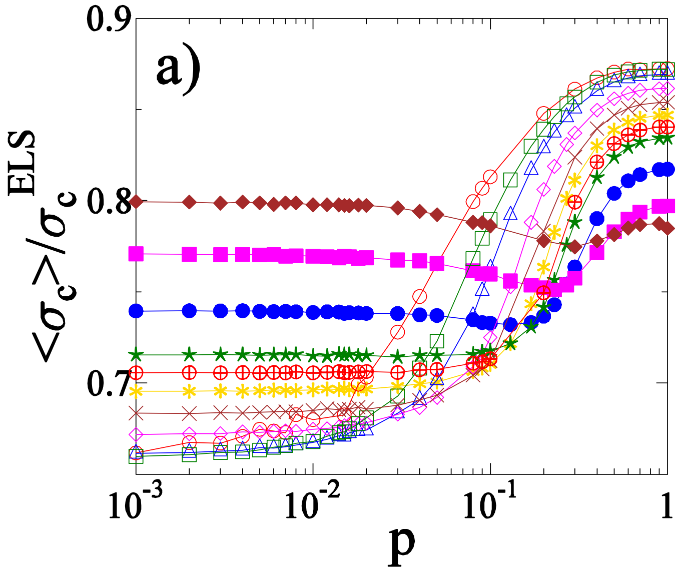
For $p > p_l$:

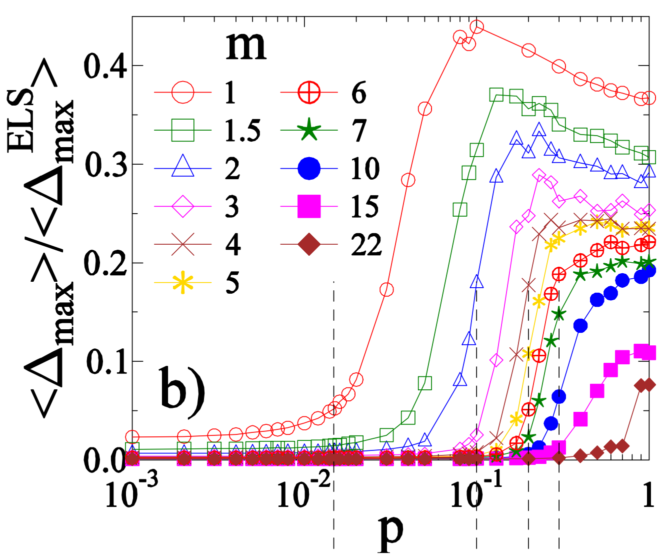
- large avalanches
- Increasing $< \Delta_{max} >$
- Crossover to a second power law regime

p^*Network topology of highest robustness of failure cascade

STRONG EFFECT OF THE STRENGTH DISORDER OF FIBERS

By decreasing strength disorder:





- ➢ Transition regime shifts and shrinks.
- ➢ Less improvement in load bearing capacity.
- ➢ No improvement in avalanche tolerance

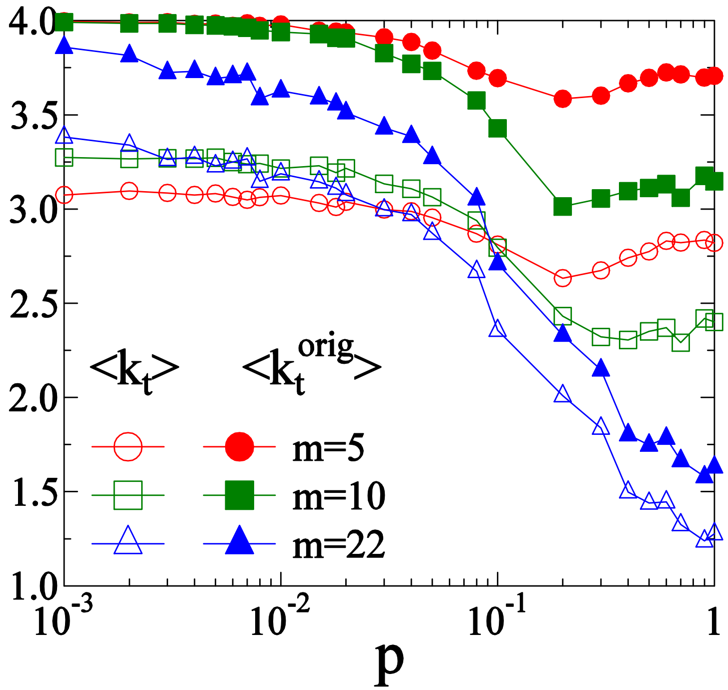
Transition is restricted to a certain range of strength disorder

FAILURE DRIVEN BY LOW DEGREE FIBERS

- Long range connections
- low load localization
- High avalanche tolerance

Increasing p:

- Increasing low degree nodes
- High load localization
- Low avalanche tolerance

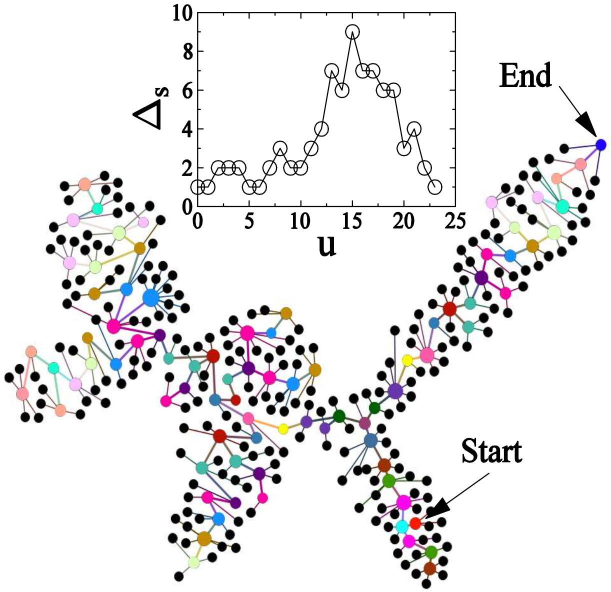


$$m > \frac{\ln(N \ln N)}{\ln 2} \;\Rightarrow\; m > 20.9$$

When strength disorder is low, at high structural randomness the nodes with lowest degree trigger the catastrophic failure

Avalanche size and duration

Evolution of an avalanche



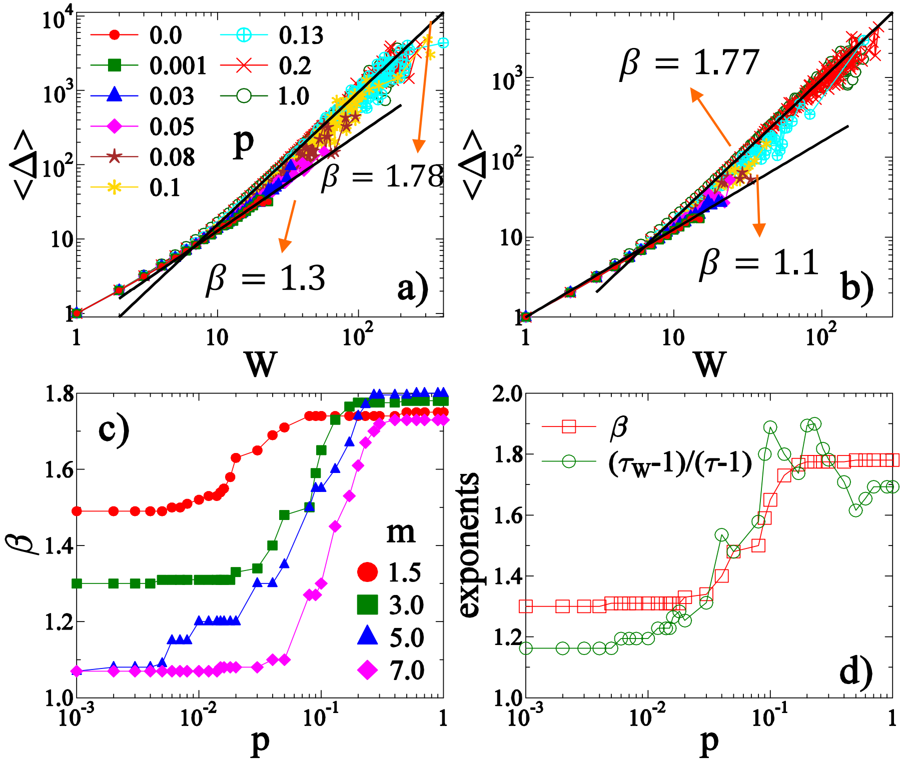
Avalanche size: Δ

Avalanche duration: W

$\Delta = \sum_{u=1}^{W} \Delta_s(u)$ $\quad W \leq \Delta$

Power law duration distribution $p(W) \propto W^{-\tau_W}$

Avalanches spread as a sequence of sub-avalanches of size $\Delta_s(u)$.



Longer avalanches have large size

$\langle \Delta \rangle \sim W^{\beta}$

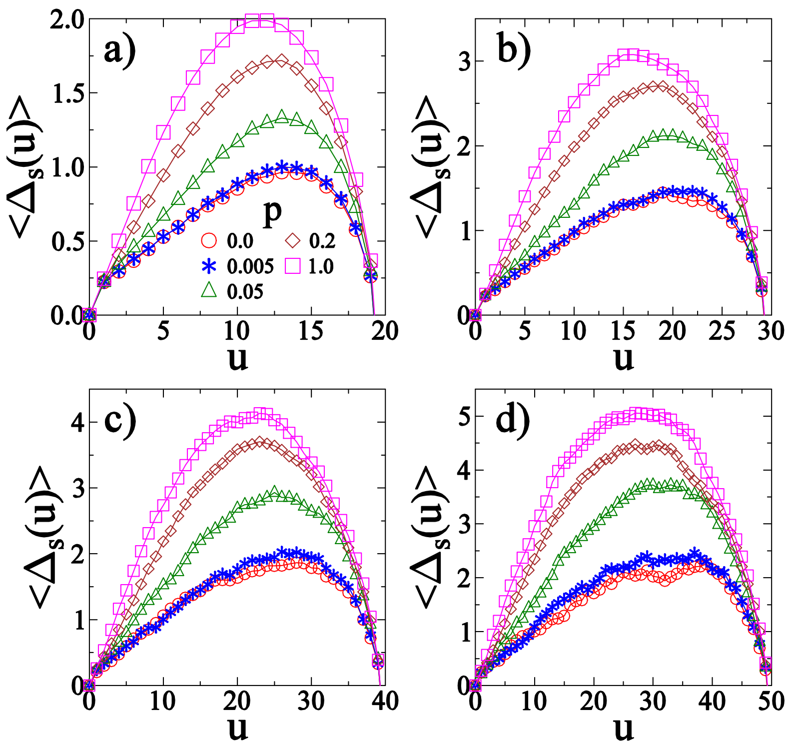
β depends on p and m

Scaling relation

$$\beta = \frac{\tau_W - 1}{\tau - 1}$$

TEMPORAL PROFILE OF AVALANCHE SPREADING

The average profile $<\Delta_s(u)>$ has a well defined parabolic form



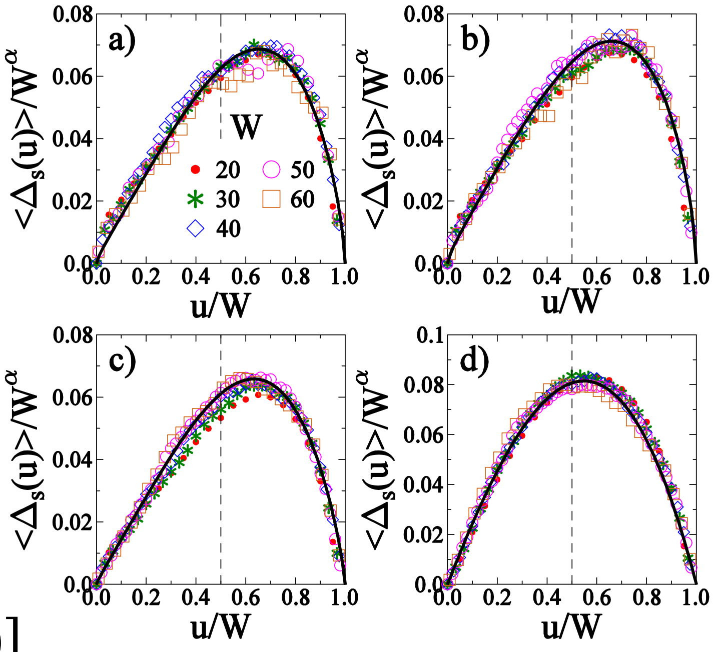
- At a fixed duration, $\Delta_s(u)$ increases with increasing p
- Avalanches start slowly, gradually accelerate and stop suddenly.
- For low p, profiles have strong right handed asymmetry for all durations
- Degree of asymmetry decreases by increasing p but even at $p = 1$ some asymmetry still remains

Temporal profile of avalanche spreading

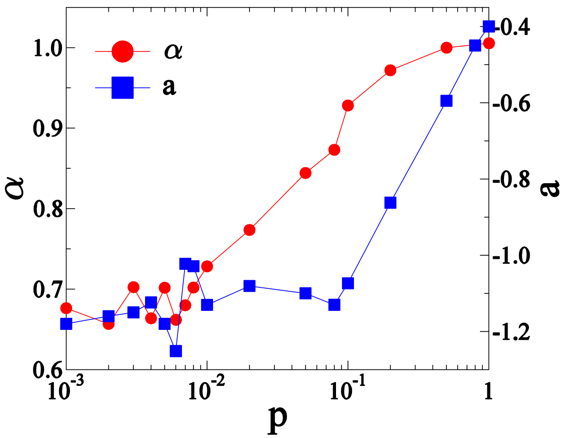
Scaling behaviour of avalanche profiles

Profiles for a fixed p with different W collapsed on each other by rescaling with an apropriate power of W

$$< \Delta_s(u, W) >= W^{\alpha} f(u/W)$$



The fitting function has the form:



$$f(x) \approx [x(1-x)]^{\alpha}[1-a(x-1/2)]$$

The qualitative behaviour of α is similar to the behaviour of exponents β, τ and τ_w confirming the localized to mean field transition

SUMMARY

- Structural randomness results in localized to mean field transition
- Transition is limited to a range of strength disorder
- A special network topology with the highest robustness
- Parabolic avalanche profiles with right handed asymmetry
- The degree of asymmetry is determined by the network topology

PUBLICATIONS

- A. Batool, G. Pal, Z. Danku, and F. Kun, *Transition from localized to mean field behaviour of cascading failures in the fiber bundle model on complex networks*, Chaos Solitons & Fractals **159**, 112190 (2022).

- A. Batool, G. Pal, Z. Danku, and F. Kun. *Temporal evolution of failure avalanches of the fiber bundle model on complex networks***.** Chaos: An Interdisciplinary Journal of Nonlinear Science **32**, 063121 (2022).

THANK YOU FOR YOUR ATTENTION